The Assistant Engineer

BY

Prof. JEAN P. GENTHON
Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK I

THE AXEMAN

WRITTEN FOR

THE CHIEF
Journal of the Civil Service

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THE ASSISTANT ENGINEER.

Book I.

PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with that most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3°. The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examinatin in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON,
New York.

July 1, 1907.

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1°. Aptitude for mathematics.

2°. Habit of observation.

3°. Good memory

4°. System.

5°. Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials, or new appliances and processes are used for old or new purposes. Notice the several kinds of labor and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results: principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figures and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations; in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public

works have a Chief Engineer who prepares the work and directs its construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an Assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

Chief Engineer.	Division Engineers.	Assistant Engineers.	Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist.
			Inspectors.	
			Transitmen.	Chainmen. Axemen.
			Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.	Memphis, Tenn.	St. Louis, Mo.
Boston, Mass.	Milwaukee, Wis.	St. Paul, Minn.
Buffalo, N. Y.	Mobile, Ala.	San Francisco, Cal.
Charleston, S. C.	Nashville, Tenn.	Savannah, Ga.
Chattanooga, Tenn.	New London, Conn.	Seattle, Wash.
Chicago, Ill.	New Orleans, La.	Sioux City, Iowa.
Cincinnati, Ohio.	Newport, R. I.	Tampa, Fla.
Cleveland, Ohio.	New York, N. Y.	Vicksburg, Miss.
Detroit, Mich.	Norfolk, Va. ,	Washington, D. C.
Galveston, Tex.	Philadelphia, Pa.	Wheeling, W. Va.
Grand Rapids. Mich.	Pittsburg, Pa.	Wilmington, Del.
Jacksonville, Fla.	Portland, Me.	Wilmington, N. C.
Little Rock, Ark.	Portland. Oreg. ..	Yellowstone Park, Wyo
Louisville, Ky.	Rock Island, Ill.	

where examinations may be taken except that of Assistant Engineer which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken in

Albany,	Ithaca,	Ogdensburg,
Amsterdam.	Jamestown,	Olean,
Auburn,	Kingston,	Plattsburg,
Binghamton,	Lockport,	Poughkeepsie,
Buffalo,	Malone,	Rochester,
Elmira,	Newburg.	Syracuse,
Hornellsville,	New York,	Utica,
		Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewers, or construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman; topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service appear regularly in "THE CHIEF."

BOOK I

THE AXEMAN

AXEMAN.—An axeman is one who carries and uses the axe.

WHO HIS SUPERIORS ARE.—On reconnaissance he is under the orders of the Assistant Engineer.

On survey work he is under the orders of the Head Chainman; sometimes under those of the Transitman.

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service—Panama Canal.

Title: Helper.
Age limits: 18 to 40 years.
Salary: $50 per month—Traveling expenses

Written Examination.

	Subjects.	Relative weights.
1.	Spelling (20 words of average difficulty in common use)	20
2.	Arithmetic (simple tests in addition, subtraction, multiplication, and division of whole numbers, and in common and decimal fractions, and United States money)	20
3.	Letter-writing (a letter of not less than 125 words on some subject of general interest. Competitors will be permitted to select one of two subjects given)	20
4.	Penmanship (the handwriting of the competitor in the subject of copying from plain copy will be considered with special reference to the elements of legibility, rapidity, neatness, general appearance, etc.)	15
5.	Copying from plain copy (a simple test in copying accurately a few printed lines in the competitor's handwriting)	15
6.	Training and experience	10
	Total	100

Applicants must not have less than one year's practical experience in similar work, provided that two years' study in a school of civil engineering will be accepted as equivalent to this experience.

Persons appointed to the position of helper will be eligible for promotion to the grade of chainman without examination.

One day will be required for this examination.

New York State and County Civil Service.

Silent.

New York Municipal Civil Service.

Title: Axeman.
Age limit: Not less than 18 years of age.
Grade: Group 3—First grade.
Salary: Annual compensation of $900 or less.

Written Examination: Schedule D—Part I.

Subjects.	Relative weights.
1. Handwriting (as shown in examination papers)	1
2. Arithmetic, viz.: Addition, subtraction, multiplication and division	1
3. Questions relating to the technical knowledge required for the position of axeman	6
4. Experience tending to qualify him for that position	2
Total of weights	10

QUESTIONS GIVEN AT CIVIL SERVICE EXAMINATIONS.

Arithmetic.

1. Write in figures the whole number, one million seventy thousand and five hundred and seven, followed by the decimal number fourteen thousandths.
2. Write in words the following number, 300007.7009.
3. Write the following number (not numbers) decimally, ten thousand and seven and forty-three ten thousandths.
4. Write decimally the following: One million, twenty-three thousand and eighty and thirty thousandths.
5. Add 7305098, 6121774, 17853, 970007, 106, 9651.
6. Add 219 ft. 7 in., 847 ft. 9 in., 796 ft. 3 in., 654 ft. 11 in., [illegible] ft. 5 in
7. Add the following fractions: 3-5, 5-8, 11-16, 7-15, 3-10.
8. Add the following fractions 2-3, 5-8, 4-9. 11-16, 17-18.
9. Multiply 690.875 by 78.096.
10. Multiply 3 and 3-8 by 2 and 5-7.
11. Find the amount of $\frac{194.5}{167.3} \times \frac{205.9}{543.7}$

12. Divide 322,622,362 by 78.746.

13. Divide 37 and 1-7 by 6 and 4-5.

14. Divide 684.007 by 79.6.

15 A church is 85.64 ft. long, 51.28 ft. wide and 31.5 ft. high to the eaves. Find the cost of painting the outside walls at 34 cents a square yard.

16. What number added to 7-9 of 18 and 3-4 will equal 9-10 of 41 and 2-3.

17. If 3¾ is 2½ times a certain number, what is 3⅞ times the same number?

18. If a laborer can reap a field of grain in 4 and 4-5 days, how long would it take 4 laborers to reap a field 6 and 1-4 times as large?

19. If a man can walk 200 miles in 9.375 days, how far, at the same rate, can he walk in 15.625 days?

20. A field is 56 rods wide, and contains 25 acres 88 square rods. Find the cost of fencing it at 66 and 2-3 cents a yard.

21. If 9 and 3-8 yards of cloth cost $26 and 1-4, how many yards could be bought for $24 and 4-5?

22. Write in words 1006.306.

23. Write in figures seventy and three hundredths.

24. What is the difference in area between 4 square feet and four feet square?

25 How many tenths of a foot are there in 30 inches?

26. Add 49 feet and 8 inches; 6 feet and 9 inches; 13 feet and 11 inches; and 7 feet and 8 inches.

27. Add 4.05 feet; 19.982 feet; 17.33 feet and 2.638 feet.

28. Subtract by fractions and by decimals 3-8 from 12-16.

29. Divide 2 and 1-2 by 4-5; multiply by 3-8; add 3-4 and subtract 1-2. What is the result?

30. Four men working 8 hours each can shovel 128 cubic yards of earth; how many cubic yards of earth can 8 men shovel if they work 12 hours each at the same rate.

Technical.

1. What are the duties of an axeman as you understand them?

2. How do you fix points in soft, marshy ground?

3. (a) What do the letters T. P. mean when used in connection with a set of levels? (b) What do the same letters mean when used in an extensive survey of a portion of the city.

4. May a stake drive crooked from improper driving? If so state in what way?

5. What other causes may make a stake drive badly?

6. (a) How far apart are center stakes ordinarily placed on the line of a street to be graded? (b) How far apart are stations made on sewer work?

7. (a) In what way can an axeman assist a chainman in windy weather? (b) How can he assist the transitman in such weather?

8. Show and describe how you would cut a bench mark on the root of a tree.

9. What does an axeman have to do besides carry and drive stakes?

10. Describe the shaping and driving of a stake so as to secure the best results as to solidity and accuracy of position.

11. (a) Describe fully the work of setting a center stake on a transit line. (b) How are such stakes marked?

12. In what other ways besides the use of stakes are stations marked on a line?

13. Describe the method you would pursue to cut straight on a line through brush.

14. Describe the operation of setting reference stakes or otherwise referencing a point which may be lost unless so marked.

15. In doing street grading, where slopes have to be provided for, state the position of all stakes that would be set at a cross-section in embankment, and also what marks would be placed on them.

16. How should stakes be set for a job of sewer work?

17. What conditions control the sizes and lengths of stakes?

18. If called upon to hold the rod, state all you would do in case where the rod had to be extended.

19. If called upon to hold the front end of a chain, state all the things you must attend to, in order to get correct measurements.

20 State the operation of setting up a transit instrument in the field not using the box.

21. What other duties ordinarily fall upon an axeman besides those mentioned in previous questions?

22. (a) How are stations marked on rocks? (b) How on a stone back pavement? (c) How on an asphalt pavement?

23 (a) What marks are placed on center line stakes for grading a street? (b) Are stakes ever placed between regular stations, and if so how are they marked?

24. If for any reason you were told to set an offset stake at the end of a line, how would you do it?

25. If called upon to set up a transit instrument over a point, state the important things you should pay attention to.

26. In carrying an instrument on the tripod where there are brushes and fences, what is the safest way?

27. In what ways can an axeman assist a transitman while he is setting up a transit.

28. In the transit work (not the level work) of running the center line of a street in the city, how far apart are the stakes usually placed and what marks are placed upon them?

29. What additional marks are placed upon them to indicate what grading is to be done?

30. When heavy filling or cutting are to be done, what other stakes are set at each station besides the center stakes, and how are they marked?

31. In doing leveling where points have to be set on between stations, what are such points called?

32. (a) If called upon to set up a transit instrument, what two points are to be particularly attended to? (b) Would you screw the leveling screw very tight or how?

33. What is the difference between a rod used for a transit survey and one used for a level survey?

34. In cutting through brush, what is the best way to keep on line?

35. Assume that the engineer sent you to the office to bring everything necessary for a transit survey, what would you bring?

36. How is a stake referenced when it is likely to be destroyed?

37. How many stakes would be required to give stations in a transit line half a mile long, allowing 7 for plus stations including the last stake?

38. Where a railroad cut is to be made, where are the stakes set?

39. Write in plain letters (about ½ inch high) the abbreviations for the following as written on stakes: (a) Forty-one plus fifty, center-line; (b) Slope stake, 15 numerator, 20 denominator; (c) Bench mark, plus seventy-one, point, two, six, three.

40. (a) How is a transit point accurately marked on a stake; (b) How when it falls on a rock ledge?

The following questions were given at an examination for rodman and axeman in Buffalo, Feb. 26, 1898.

1. (a) Make a sketch and give description of a level rod, also describe a vernier and its use. (b) Describe the manner of using a level rod and its relation to a level.

2. (a) What is a "bench-mark?"
   (b) What is meant by a "turning-point?"
   (c) What is meant by a "back-sight?"
   (d) What is the meaning of "datum" or "datum-plane?"

3. (a) In measuring any distance over rough or uneven ground, describe how the measurement should be made to secure accuracy. (b) If a tape line is divided into feet and tenths of feet, how many tenths will there be in 157¾ feet?

4. A man had climbed a mountain in 10½ hours, traveling at the rate of 2¾ miles an hour and at the top is 6098 feet higher than when he started. What is the average grade he had ascended?

5. (a) A farmer has an orchard which measures 3 chains, 25 links on each side; how many trees does it contain, if the trees are placed one rod apart and one rod from the fences? (b) How many acres in the orchard?

6. How many cubic feet of space in a cellar, the outside dimensions of which are 39 feet in length, 23 feet in width and 7 feet in depth, the walls being 15 inches thick, and the floor 6 inches thick?

7. Describe in detail the method of cross-sectioning and setting slope-stakes. Write the notes of some imaginary cross-sections on rough ground.

8. (a) In what general direction, north, south, east or west does the City Hall stand from the new Post Office building? (b) What is the approximate distance in feet from the entrance of the Elliott Square Building to the Genesee Hotel? (c) How many feet in height would you estimate the Elliott Square Building to be?

Experience.

1. What is your age?

2. (a) State what your education has been, giving dates and places. (b) State particularly whether you have pursued any engineering studies, and if so state the length and character of the course.

3. State fully the practical experience you have had with engineers, giving length and character of service.

4. Give the names and addresses of two or more persons to whom application may be made if necessary, for verification of your statements.

SCIENTIFIC REQUIREMENTS OR WHAT HE SHOULD KNOW.

The scientific requirements of an axeman are good handwriting, correct spelling, sure figuring.

Good Handwriting.—A good, plain, broad business handwriting, without unnecessary flourishing; the height of the letters uniform and the capitals in proportion with the small letters. The writing tolerably parallel with the top and bottom edges of the book or sheet, and the general slant uniform throughout are the principal conditions of a good handwriting. Attention should be paid to the plain writing of figures and numbers which have sometimes to be read or copied by others from a distance.

Correct Spelling.—The axeman should be not only a fair but a very good speller and able to catch and correct a mistake in letters, reports, etc., which he may have to compose or check. Under this head it may be stated that he should be able to write a letter or a little composition on a given subject with facility and accuracy of expression. We here advise the student engineer to avoid the use of foreign terms when equally good equivalents can be found in the domestic language and dictionary; however, he is supposed to be or become familiar with these terms as he may find them in scientific books, treatises, etc.

Sure Figuring.—Although not expected to be a lightning calculator he is to be rapid, but above all, sure in his figuring. The four fundamental operations, addition, subtraction, multiplication and division should be very familiar to him, not only when whole numbers are involved, but also when decimals or fractions are concerned. Addition of many numbers is often a stumbling block and should be practiced by the beginner because it is of frequent occurrence.

He shall also try to become proficient in factoring and in reducing fractions or fractionary expressions to lower terms, this will always simplify his work. A certain knowledge of mensuration is desirable, and his examination papers may contain questions involving areas of simple figures or volumes of elementary solids.

ARITHMETIC.

Arithmetic is the Science of Numbers.

A **Magnitude** or **Quantity** is anything that can be supposed to be greater or smaller than it is.

Mathematics treat of measurable magnitudes and quantities only.

A **Magnitude** is a whole the parts of which are not separated, as time, weight, length, etc.

A **Quantity** is formed of separate parts, as a gathering of persons, an aggregate of objects, etc.

A **Number** is the result of the comparison (also called measurement) of a magnitude or quantity with another magnitude or quantity of the same kind supposed to be known.

A **Unit** is a known magnitude or quantity with which we measure unknown magnitudes or quantities of the same kind.

Standard.—When a unit is established by law or usage it is called a Standard.

Arbitrary Unit.—In the measurement of magnitudes, the unit may be assumed at will or convenience, greater in one case, smaller in another; it is called arbitrary.

In the measurement of magnitudes, the aim should be to employ a unit that shall be neither too large nor too small, in order that the resulting numbers be neither too small nor too large, so that we may have a better conception of those numbers.

Natural Unit.—In the measurement of quantities, the unit is necessarily one of the separate parts the aggregate of which compose the quantity under consideration; such a unit is called natural.

A Concrete Number is one the nature of the unit of which is known.

Denominate Number.—A concrete number the standard of which is fixed by law or established by long usage.

An Abstract Number is one of which the nature of the unit is unknown.

Generalization of the Term Quantity.—This prefixed and for the sake of brevity, both magnitudes and quantities are generally included under the single term of quantities.

Further Extension of the Term Quantity.—The expression quantity is often applied not only to the things themselves, but to the Numbers which are their measurements.

When measuring a quantity there may be **three** cases:

1°. **Whole Number.**—The quantity is equal to or is greater than the unit and contains it an exact number of times. The resulting number is a whole number.

2°. **Fraction.**—The quantity is smaller than the unit and contains only a portion of that unit. The resulting number is a Fraction.

3°. **Mixed Number.**—The quantity is greater than the unit; it contains it a certain number of times and there remains a portion smaller than the unit. The resulting number is a Mixed Number

The **Hyphen.**—When writing numbers in words, place a hyphen between the parts of all numbers from twenty-one to ninety-nine, both inclusive, when composed of more than one word.

Cue Numbers.—Numbers are often written in words in legal documents. They are generally followed by the same number written in Arabic notation and placed between parenthesis as: Three hundred sixty-five and three tenths (365 3) feet. These Arabic numbers may be called Cue Numbers and are so used for purposes of quick reading and checking.

How to Read Numbers.—The right way to read 101;274, etc., is one hundred one, two hundred seventy-four, etc.

The **Comma.**—The comma is placed between the classes of units of a number. It is often omitted however.

Classes of Units.—The classes are: Simple units, thousands, millions, etc.; also thousandths, millionths, etc.

Orders of Units.—Each class contains three (3) orders: Units, tens and hundreds running from right to left.

The **Decimal Point.**—A period, called decimal point, is placed in a mixed number between the integral part and the decimal portion which follows. It should never be omitted.

Roman Numbers.—I stands for 1, V for 5, X for 10, L for 50, C for 100, D for 500 and M for 1,000.

Abbreviations.—A smaller unit, written to the left of a greater one, is subtracted from the latter, as: IV = 4 (IV is marked IIII on clock and watch dials); IX = 9; XC = 90; CD = 400, etc. Sometimes a Roman number is surmounted by a dash or vinculum; it then expresses thousands as $\overline{IX}$ = 9,000.

Addition.—Operation which consists in taking in any order all the units and portions of units of several numbers and forming with them a single number called their Sum or Total.

Addition of Long Columns of Numbers.—When long columns of numbers are to be added, the student should endeavor to add more than one figure at a time. He may pick those figures which aggregate 10, 15, 20, etc., and add the intermediate figures when convenient.

Addition of Denominate Numbers.—Begin by adding the smallest subsidiary units on the right and divest their sum of all the next higher units it contains writing down only the remainder and carrying the higher units to those on the left in the numbers to be added. Continue until the primary or principal units have been added. In the addition of a column, when the sum becomes greater than the number required to make one unit of the next higher grade, divest that sum at once of one such unit by marking a dot on the paper or pad and continue the addition with the remainder; at the end of the column after writing the remainder down, count the number of dots so marked and add their number to the next higher column on the left.

Sign of Addition.—The sign of addition is the horizontal-vertical or Roman cross + placed between all the numbers to be added; it is read Plus.

To Prove an Addition.—The shortest way to prove an addition is to do it over again from bottom to top.

Sign of Equality.—The sign of equality is two short equal horizontal parallels = ; it is read Equal.

Equality.—An equality is the indication that a quantity, written on the left of the sign = has the same value as another quantity written on the right of that sign.

Identity.—An identity is an equality between known quantities.

Equation.—An equation is an equality between quantities some of which are unknown.

Solving an Equation.—To solve an equation is to find the value of the unknown quantities which it contains.

A simple example will illustrate this: Take for instance the sum of the numbers 79, 84, 39 and represent it by S; the relation between these numbers is $79 + 84 + 39 = S$; this is an equation; but perform the addition and find the sum 202 and the above relation becomes $79 + 84 + 39 = 202$ which is an identity. Thus we may say that to solve an equation is to find the numerical values which, being substituted for the unknown quantities will transform the equation into an identity.

Formula.—A formula is an equality indicating the series of operations to be performed on certain quantities in order to obtain a certain result.

Subtraction.—An operation which consists in taking from a number called **minuend** (m) all the units and parts of units contained in another number called **subtrahend** (s). The result is called the **difference** (d) of the two numbers or the **remainder** of their subtraction.

Sign of Subtraction.—The sign of subtraction is a horizontal dash — placed between the minuend written first and the subtrahend. Thus: $84 - 38 = d$; $84 - 38 = 46$. Generally $m - s = d$.

To Prove a Subtraction.—Add from bottom to top the difference and the subtrahend; the sum must equal the minuend.

Multiplication.—An operation which consists in repeating a number called **multiplicand** (M) as many times as there are units in another number called **multiplier** (m); the result is called the **product** (p) of the numbers, and the numbers themselves are called **factors of** the product. This definition may be extended to the case where the factors are not whole numbers.

Sign of Mutiplication.—The sign of multiplication is the oblique or St. Andrew's cross ×, called multiplied by, and placed between the factors written one after the other.

Thus: $35 \times 7 = p$; $35 \times 7 = 245$. Generally $M \times m = p$.

Multiple of a Number.—Common Multiple. The product of a whole number by a whole number is a multiple of either.

In $7 \times 3 = 21$; 21 is a multiple of 7 and also a multiple of 3; 21 is therefore a common multiple of 7 and 3.

To prove a Multiplication.—Multiplication may be proved by a second multiplication in which the factors are inverted.

This is the surest but the longest method.

A shorter method is given below.

Find the residue of a number by 9.—Let 3098657 be a given number. The residue of that number by 9 is the remainder of its division by 9 (see Division). But this remainder may be easily obtained as follows without performing the division. Add all the digits of the number, and when in the course of that process a partial sum greater than 9 is obtained, add its figures together before continuing, and so continue until the last digit has been used, and the final result is a single figure. Skip all 9's on the way. Thus:

3 + 8 = 11 (1 + 1 = 2) ; 2 + 6 = 8 ; 8 + 5 = 13 (1 + 3 = 4) ; 4 + 7 = 11 (1 + 1 = 2)
This final result 2 is the Residue by 9 of 3 098 657, and the operation which leads to it is made mentally.

Another Proof of the Multiplication.—Find the residue of the multiplicand and multiplier. Multiply them and find the residue of their product: this is equal to the residue of the product of the multiplication.

64327	4		Residue of the multiplicand.
781	7		Residue of the multiplier.
64327	28	1	Residue of the product of the residues.
514616			
450289			
50239387		1	Residue of the product of the multiplication.

Proof not Absolute.—Practically a proof is not absolute because an error may be committed in its use, and also it may not work well in all cases.

Simplification when many Numbers have to be multiplied by the same Multiplier.—When we have to multiply many numbers by the same multiplier we may simplify the operation.

Example: 8 tanks have the following capacities: 6323, 8476, 2687, 9734, 7718, 8792, 3095 and 4893 cu. ft.; what is their capacity in gallons?

If only the total capacity of the 8 tanks were required, the quickest way would be to find the sum of the given capacities in cu. ft. and multiply it by 7.48052 (number of gals. per cu. ft.), thus reducing the problem to one addition and one multiplication; but if the capacity in gals. of each tank were also required, it would be advantageous to proceed as follows:

Form a table of the first nine (9) multiples of 748052, adding it to itself then to the last multiple obtained in order to get the next one. Now, to find the capacity in gals., say of the 4th tank, write the capacity in cu. ft. and a line under it. Pick from the table

1— 748052	
2—1496104	9734 cu. ft.
3—2244156	
4—2992208	2992208
5—3740260	2244156
6—4488312	5236364
7—5236364	6732468
8—5984416	
9—6732468	72815.38168 gals.
Proof. 10—7480520	

of multiples as formed the products by 4, 3, 7 and 9; place them where they rightly belong in the multiplication; add for final result and place the decimal point where it should be.

Power of a Number.—When the factors of a product are equal, the product is called a power of the factor.

Square of a Number.—A power is a square when it is the product of two (2) equal factors as $7 \times 7 = 49$ in which 49 is the square of 7. The term square is derived from the fact that the area of a square is obtained by multiplying the length of its side by itself, or taking it twice as a factor.

Cube of a Number.—A power is a cube when it is the product of three (3) equal factors as $5 \times 5 \times 5 = 125$, in which 125 is the cube of 5.

The term cube is derived from the fact that the volume of a cube is obtained by multiplying the length of its side by itself and again by itself, or by taking it three times as a factor.

A product, for instance, of 4, 9, etc., equal factors would be called the 4th or the 9th, etc., power of that number.

Exponent.—An abbreviation is used. Instead of writing all the equal factors of a power, it is agreed to write only one of them and to place on its right and a little above a small figure indicating how many times it is to be taken as a factor. This small figure is called an exponent.

Thus: $7 \times 7 \times 7 \times 7 \times 7 = 7^5$.

Product of Powers of a Number.—*Add the exponents.*

Example. $4^3 \times 4^2 = 4^{3+2} = 4^5$

Division.—An operation by means of which we find one of two factors of a product when that product and the other factor are given. The given product is called **Dividend** (D) of the division;

the known factor is called the **Divisor** (d), and the unknown factor which is sought is called **Quotient** (q). We know that a quotient is seldom exact and that there is generally a **Remainder** (r) or **Residue.**

Sign of Division.—The sign of division is a small dash with a point above and one below ÷ it is read divided by, is placed after the dividend and is followed by the divisor. For instance, to indicate the division of 72 by 8 which we know gives the quotient 9, we write $72 \div 8 = 9$; generally $D \div d = q$.

Other Sign of Division.—In the study of fractions it is shown that a fraction expresses the quotient of its numerator by its denominator, so that the preceeding identity may be written $\frac{72}{8} = 9$ or more generally $\frac{D}{d} = q$, and another sign of division is a horizontal line separating the dividend written above it from the divisor written below it.

Proof of the Division.—We prove a division by multiplying the divisor by the quotient and adding the remainder, if there is any; the result thus obtained must equal the dividend. When there is a remainder, the formula of division is $D = dq + r$.

Another proof much shorter consists in finding the residue by 9 of the remainder and subtracting it from any figure or sum of figures in the dividend, then finding the residue of whatever is left of the dividend. This must equal the product of the residues of the divisor and quotient.

```
Residue of the    Residue of the
Divisor           Dividend after
= 3               subtracting residue
                  of Remainder        2 = Residue of the Quotient.
                  = 6
   75 | 3 140 978 | 41879
      | 3 00
          140          2 x 3 = 6 = Residue of the product of the residues,
           75                      of d and q.
           659
           600
            597
            525
             728
             675
              53   8 = Residue of the Remainder
```

Abbreviation.—Many calculators do not write the partial products 300, 75, 600, 525 and 675 (in the above example), but subtract them mentally from the partial dividends 314, 140, 659, 597 and 728, saying for instance at the beginning of the operation: 4 times 5, 20; from 24 leaves 4 (which write) and carry 2; 4 times 7, 28 and 2, 30; from

```
    3             6          2
   75 | 3 140 978 | 41879
          140
           659
            597
             728
              53   8
```

31 leaves 1 (which write) and bring down 0, etc.; they write the operation as here illustrated. This process is a saving in time and space, as also a drill in mental calculation.

Quotient of Powers of Numbers.—To divide two powers of a number, subtract the exponents.

$$Example: \frac{11^7}{11^3} \quad 11^{7-3} = 11^4$$

Characters of Divisibility.

By 2.—A number is divisible by 2 when it is an even number, that is to say when it ends with 0, 2, 4, 6 or 8, as 70, 836.

By 3.—A number is divisible by 3 when its residue is zero or is divisible by 3.

By 4.—A number is divisible by 4 when the number formed by the last two figures to the right is divisible by 4; 7528 is divisible by 4 because 28 is divisible by 4.

By 5.—A number is divisible by 5 when it ends with 0 or 5, as 75, 270.

By 6.—A number is divisible by 6 when it is divisible by 2 and 3 as 474, because when a number is divisible by several others it is divisible by their product.

By 8.—A number is divisible by 8 when the number formed by the last three figures to the right is divisible by 8; 37104 is divisible by 8 because 104 is divisible by 8.

By 9.—A number is divisible by 9 when its residue is 9 or 0.

By 10.—A number is divisible by 10 when the last figure to the right is 0.

By 100.—A number is divisible by 100 when the last two figures to the right are 00.

By 10^n. *Generally a number is divisible by 10^n when it ends with n zeros.*

By 11.—A number is divisible by 11 when the sum of the figures of even rank subtracted from the sum of the figures of uneven rank (increased by 11 if necessary) is 0 or divisible by 11, as 95832, 3304081.

By 12.—A number is divisible by 12 when it is divisible by 3 and 4 as 756.

By 15.—A number is divisible by 15 when it is divisible by 3 and 5 as 255.

Prime Number.—A prime number is one which is divisible only by itself and 1; as 1, 2, 3, 5, 7, etc.

Table of Prime Numbers.—To form a table of prime numbers, write the natural series of numbers 1, 2, 3, 4, 5, 6, 7, etc., and cross or expunge all the numbers which are not prime; that is to say, all multiples of prime numbers (except 1).

To expunge the multiples of 2 in the series of numbers, cross out all the second numbers beginning with 4.

To expunge the multiples of 3, cross out all the third numbers beginning with 9 (6 was crossed already as a multiple of 2).

To expunge the multiples of 5, cross out all the fifth numbers beginning with 25.

Continue in like manner by expunging the multiples of 7, 11, 13, etc.

1	~~2~~	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	11	~~12~~	13	~~14~~	~~15~~	*This Table may be*
~~16~~	17	~~18~~	19	~~20~~	~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	*continued as far as*
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~	41	~~42~~	43	~~44~~	~~45~~	*desired. It is known as*
~~46~~	47	~~48~~	~~49~~	~~50~~	~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~	*ERATOSTHENE'S SIEVE.*

Remark.—After crossing out the multiples of a prime number, it is proved that all the remaining numbers are prime up to the square of the next higher prime number. For example, after striking out the multiples of 2, we know that all the numbers not crossed out are prime up to 9, the square of 3, which is the next higher prime number; so that we are certain that 3, 5 and 7 are prime numbers.

Factoring.—Factoring is finding all the prime divisors of a number

Find the Prime Divisors of a Number.—Try the division of that number by the prime numbers smaller than half of itself beginning with the smallest (2). If successful, try the same divisor on the quotient and so on as many times as possible before passing to the next higher prime number. Continue until the last quotient is one.

Example: Find the prime divisors of 2520.

2520	is divisible	by	2	and	the quotient	is	1260;
1260	"	"	2	"	"	"	630;
630	"	"	2	"	"	"	315;
315	"	"	3	"	"	"	105;
105	"	"	3	"	"	"	35;
35	"	"	5	"	"	"	7;
7	"	"	7	"	"	"	1.

Arrangement of the operation.

2	2520
2	1260
2	630
3	315
3	105
5	35
7	7
	1

The prime divisors of 2520 are
- 2 three times or 2^3
- 3 twice " 3^2
- 5 once " 5
- 7 once " 7

and we have the relation: $2520 = 2^3 \times 3^2 \times 5 \times 7$.

Common Multiple (C. M.).—A common multiple of several numbers is a number exactly divisible by each one.

Abbreviation.—We represent a common multiple of several numbers a, b, c by the abbreviation C. M. (a, b, c).

Least Common Multiple (L. C. M.).—The least common multiple of several numbers is the smallest number exactly divisible by each one.

Examples: 168 = C. M. (7, 8); 56 = L. C. M. (7, 8).

Find the L. C. M. of Several Numbers.—Any C. M. of several numbers must be divisible by all the numbers; it must then contain all the prime factors of each one with an exponent at least equal to the greatest. Hence the rule: Form a product of all the different factors taking each with the greatest exponent.

Example: Find L. C. M. of 882, 588 and 210.

Arrangement of the Operation.

$$\begin{array}{r|r} 2 & 882 \\ 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array} \quad \begin{array}{r|r} 2 & 588 \\ 2 & 294 \\ 3 & 147 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array} \quad \begin{array}{r|r} 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ 7 & 7 \\ & 1 \end{array}$$

$$882 = 2\times 3^2\times 7^2$$
$$588 = 2^2\times 3\times 7^2$$
$$210 = 2\times 3\times 5\times 7$$

$$L.C.M.(882;588;210) = 2^2\times 3^2\times 5\times 7^2 = 8820.$$

Common Divisor (C. D.)—A common divisor of several numbers is a number which divides all of them exactly; as 6 = C. D. (36, 48, 72).

Greatest Common Divisor (G. C. D.).—The greatest common divisor of several numbers is the greatest number which divides all of them exactly; as 12 = G. C. D. (36, 48, 72).

Find the G. C. D. of Two Numbers.—Let 1260 and 924 be two numbers. Find the prime divisors of each one.

Notice that 2^2; 3 and 7 are common to both; their product $2^2\times 3\times 7$ is their G.C.D.

Arrangement of the operation.

$$\begin{array}{r|r} 2 & 1260 \\ 2 & 630 \\ 3 & 315 \\ 3 & 105 \\ 5 & 35 \\ 7 & 7 \\ & 1 \end{array} \quad \begin{array}{r|r} 2 & 924 \\ 2 & 462 \\ 3 & 231 \\ 7 & 77 \\ 11 & 11 \\ & 1 \end{array}$$

$$1260 = 2^2\times 3^2\times 5\times 7$$
$$924 = 2^2\times 3\times 7\times 11$$

$$G.C.D.(1260;\ 924) = 2^2\times 3\times 7 = 84$$

Find the G. C. D. of Several Numbers.—Find the prime divisors of the given numbers. Form a product of all the factors common to all the numbers, taking each factor with its smallest exponent.

Other Rule to Obtain the G. C. D. of Two Numbers.—Let us take the same numbers as given above: 1260 and 924.

Divide the greater number by the smaller. If there is no remainder the smaller number is the G. C. D. required. If there is a remainder, divide the smaller number by the first remainder, the first remainder by the second, the second by the third, and so continue until a division leaves no remainder The last divisor used, which is also the last remainder, is the G. C. D. required. In these successive divisions write the quotient above the divisors as in the example.

Arrangement of the Operation.

	3	1	2	1
84	352	336	924	1260
	251	252	672	924
	0	84	252	336

84 = G. C. D. (1260, 924).

Find the G. C. D. of Several Numbers.—Find the g. c. d. between the first two, then find the g. c. d. between the g. c. d. obtained and the third number, between this second g. c. d. and the fourth number and so continue until you have employed the last number; the last g. c. d. obtained is the G. C. D. of all the numbers.

Numbers Prime to Each Other.—When they have no other common divisor than 1.

FRACTIONS.

A **Fraction** is one or more equal portions of the unit.

How to Write a Fraction.—Write the numerator on top of the denominator and separate them with a horizontal line.

What the Denominator Represents.—The denominator represents into how many equal parts the unit is divided; it expresses the Name of that secondary unit.

What the Numerator Represents.—The numerator of a fraction represents how many equal parts the number contains. The numerator expresses Number.

How to Read a Fraction.—Read the numerator, then the denominator with the termination th or ths (except 1-2, 2-3, whose denominators are read half, thirds).

Terms of a Fraction.—The numerator and the denominator are called the Terms of the fraction.

Proper Fraction.—A proper fraction is one the numerator of which is less than the denominator. The fraction is less than one, as 1-2, 2-3, 3-7.

Improper Fraction.—An improper fraction is one the numerator of which is equal to or is greater than the denominator. The fraction is equal to One if the numerator is equal to the denominator, as 5-5; and it is greater than one if the numer. is greater than the denom. as 11-9.

Product of a Fraction by a Number.—We multiply a fraction by a number by either multiplying its numerator or dividing its denominator by that number; as

$$\frac{3}{5} \times 7 = \frac{3 \times 7}{5} \qquad \frac{21}{5}.$$

Quotient of a Fraction by a Number.—We divide a fraction by a number by either dividing its numerator or multiplying its denominator by that number; as

$$\frac{3}{5} \div 7 = \frac{3}{5 . 7} \qquad \frac{3}{35}.$$

Multiplying (or dividing) numerator and denominator by the same number does not change the value of a fraction; as

$$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}.$$

A fraction is **irreducible** when its terms are prime to each other; as 3-5.

Reductions of Fractions.—They are transformations of fractions which do not change their value.

There are four reductions of fractions:

First Reduction.—Reduce a whole or a mixed number to an improper fraction. Multiply the whole number by the denominator and add the numerator of the accompanying fraction if there is one. The result will be the numerator of the improper fraction.

Ex. I. Reduce 7 into fourths.

$$7 - \frac{7 \times 4}{4} = \frac{28}{4}.$$

Ex. II. Reduce 6 + 3-5 to an improper fraction.

$$6 + \frac{3}{5} \qquad \frac{6 \times 5}{5} + \frac{3}{5} \qquad \frac{30}{5} + \frac{3}{5} - \frac{33}{5}$$

Second Reduction.—Find the units contained in an improper fraction, or reduce an improper fraction to a whole or a mixed number. Divide the numerator by the denominator. The quotient will be the whole number or the integral part of the mixed number and the remainder (if there is one) will be the numerator of the accompanying fraction.

Ex. Reduce 731-12 into a mixed number.

$$\begin{array}{r|l|l} 12 & 731 & 60 \\ & \underline{72} & \\ & 11 & \end{array} \qquad \frac{731}{12} = 60 + \frac{11}{12}.$$

Third Reduction.—Reduce a fraction to its lowest terms. Divide both terms of the fraction by their G. C. D.

$$\text{Ex. } \frac{240}{360} = \frac{240 \div 120}{360 \div 120} = \frac{2}{3}$$

$$\text{G.C.D.}(240; 360) = 2^3 \times 3 \times 5 = 120.$$

$$\begin{array}{r|r} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array} \qquad \begin{array}{r|r} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

Fourth Reduction.—Reduce several fractions to equal fractions having the same denominator.

1st Case. To reduce two fractions to the same denomination, multiply the two terms of each by the other denominator.

Ex. 2-3 and 4-5.

$$\frac{2}{3} \quad \frac{2 \times 5}{3 \times 5} = \frac{10}{15},$$
$$\frac{4}{5} \quad \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

2d. or General Case.—Find the L. C. M. of all the denominators; Divide this L. C. M. successively by each denominator and multiply the corresponding numerator by the quotient.

$$\text{Ex. } \frac{2}{3}; \frac{4}{5}; \frac{5}{6}, \frac{7}{8}; \frac{5}{12} \qquad \text{L.C.M.}(3; 5; 6; 8; 12) = 120.$$

$$\frac{2}{3} \quad \frac{2 \times 40}{3 \times 40} \quad \frac{80}{120} \qquad 120 \div 3 = 40$$
$$\frac{4}{5} \quad \frac{4 \times 24}{5 \times 24} \quad \frac{96}{120} \qquad 120 \div 5 = 24$$
$$\frac{5}{6} = \frac{5 \times 20}{6 \times 20} \quad \frac{100}{120} \qquad 120 \div 6 = 20$$
$$\frac{7}{8} \quad \frac{7 \times 15}{8 \times 15} = \frac{105}{120} \qquad 120 \div 8 \quad 15$$
$$\frac{5}{12} = \frac{5 \times 10}{12 \times 10} = \frac{50}{120} \qquad 120 \div 12 = 10$$

The Least Common Denominator (L. C. D.) of several fractions is the L. C. M. of their denominators.

Addition of Fractions.—Fractions can be added or subtracted only when they have the same name which is when they have the same denominator.

If the given fractions have a common denominator (C. D.), add their numerators and give to their sum the common denominator (C. D.). If they have not, reduce them to a C. D. and thus revert to the preceeding case.

You may or not reduce the sum to a mixed number, according to the requirements of the particular problem you have to solve.

$$\text{Ex. } \frac{3}{4} + \frac{5}{8} + \frac{4}{7} + \frac{3}{10} = \frac{3 \times 70}{4 \times 70} + \frac{5 \times 35}{8 \times 35} + \frac{4 \times 40}{7 \times 40} + \frac{3 \times 28}{10 \times 28} = \frac{210}{280} + \frac{175}{280} + \frac{160}{280} + \frac{64}{280}$$
$$= \frac{210 + 175 + 160 + 84}{280} = \frac{629}{280} = 2 + \frac{69}{280}$$

In the case of mixed numbers, find first the sum of the fractions, then that of the whole numbers increasing it by the units which might be contained in the sum of the fractions.

Examples: 1° Add $\frac{3}{4}+\frac{5}{8}+\frac{4}{7}+\frac{3}{10}$; 2° Add $5\frac{1}{2}+\frac{3}{5}+14\frac{2}{7}+\frac{8}{11}$

	Numer's			Numer's
$\left.\begin{matrix}\frac{3}{4}\times\end{matrix}\right\}$ 70	210	$5+\left.\frac{1}{2}\times\right\}$	385	385
$\left.\frac{5}{8}\times\right\}$ 35	175	$\left.\frac{3}{5}\times\right\}$	154	− 462
$\left.\frac{4}{7}\times\right\}$ 40	160	$14+\left.\frac{2}{7}\times\right\}$	110	= 220
$\left.\frac{3}{10}\times\right\}$ 28	= 84	$\left.\frac{8}{11}\times\right\}$	70	560

C.D. = 280 | 629 | 2
560
69

Ans $2+\frac{69}{280}$

carried 2

C.D. = 770 | 1627 | 2
1540
87

Ans. $21+\frac{87}{770}$

Subtraction of Fractions.—Reduce the two fractions to a common denominator, then subtract the numerator of the subtrahend from that of the minuend; the difference will be the numerator of a fraction having the same denominator as the two given fractions.

In case of mixed numbers obtain first the difference of the fractions then that of the whole numbers.

If the fraction of the minuend is less than that of the subtrahend, increase it by 1 (by adding its denominator to its numerator) in order to render the subtraction possible, then add 1 to the subtrahend before subtracting the integers (whole numbers accompanying the fractions).

Examples: 1° $\frac{7}{8}$ $\frac{2}{3}$; 2° $23\frac{1}{3}$ $17\frac{3}{4}$.

$\left.\frac{7}{8}\times\right\}$ 3 21

$\left.\frac{2}{3}\times\right\}$ 8 = 16

Ans. $\frac{5}{24}$

$23+\left.\frac{1}{3}\times\right\}$ 4 4 ; 4+12 = 16

$17+\left.\frac{3}{4}\times\right\}$ 3 9
+ 1 7

Ans. $5\frac{7}{8}$

Multiplication of Fractions.—Multiply together the numerators of the fractions for the numerator of the required product. Reduce the fraction product to lower terms if possible.

Example $\frac{3}{4}\times\frac{7}{8}\times\frac{2}{7}\times\frac{5}{6}=\frac{3\times7\times2\times5}{4\times8\times7\times6}-\frac{5}{4\times8}\quad\frac{5}{32}$

In the case of mixed numbers reduce them to improper fractions; perform the multiplication as by above rule, reduce to lower terms if possible, then reduce back to a mixed number.

Example $5\frac{3}{4}\times6\frac{1}{3}\times2\frac{5}{6}\quad\frac{23}{4}\times\frac{19}{3}\times\frac{17}{6}=\frac{7429}{72}\quad103\frac{13}{72}$

Division of Fractions.—**Division of a fraction by an integer.** Either divide the numerator or, if not possible, multiply the denominator by the whole number.

$$Ex.\ I: \frac{15}{7} \div 3 = \frac{15 \div 3}{7} = \frac{5}{7} \qquad Ex.\ II: \frac{3}{4} \div 5 = \frac{3}{4 \times 5} = \frac{3}{20}$$

Division of a Whole Number by a Fraction.—Multiply the whole number by the fraction inverted.

$$Ex.\quad 7 \div \frac{2}{3} \qquad 7 \times \frac{3}{2} \qquad \frac{7 \times 3}{2} = \frac{21}{2} \qquad 10\frac{1}{2}$$

Division of a Fraction by another Fraction.—Multiply the fraction dividend by the fraction divisor inverted.

$$Ex.\quad \frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} \qquad \frac{3 \times 5}{4 \times 2} \qquad \frac{15}{8} = 1\frac{7}{8}$$

Division of Mixed Numbers.—Reduce them to improper fractions and perform the division as on simple fractions.

$$Ex.\quad 12\frac{3}{4} \div 3\frac{1}{8} = \frac{51}{4} \div \frac{25}{8} \qquad \frac{51}{4} \times \frac{8}{25} = \frac{51 \times 8}{4 \times 25} \qquad \frac{51 \times 2}{25} = \frac{102}{25} = 4\frac{2}{25}.$$

DECIMAL NUMBERS.

By a natural extension of the law of formation of numbers, it is natural to write the tenths of a number to the right of the units, the tenths of tenths or hundredths to the right of the tenths, the tenths of hundredths or thousandths to the right of the hundredths, etc.

Decimal Point.—In order to avoid confusion between the integral and the decimal portions of a number, it is agreed to separate them with a period called Decimal Point.

Ex. 3 and 7-10 = 3.7; 18 and 8-1000 = 18.008, etc.

Addition of Decimal Numbers.—Like the addition of whole numbers; the decimal point in the sum is placed under the decimal points of the numbers added.

$$\begin{array}{lr} \text{Ex.} & 2701.04 \\ & 602.904 \\ & 97.1 \\ \hline & 3401.044 \end{array}$$

Subtraction.—If the minuend has not as many decimals as the subtrahend, zeros can be supposed added to the right of it. Make the subtraction as for whole numbers and in the difference place a decimal point under those of the numbers subtracted.

$$\begin{array}{lr} \text{Ex.} & 271.06 \\ & 83.491 \\ \hline & 187.569 \end{array}$$

Multiplication.—Multiply the numbers as if they were whole numbers and in the product separate by a decimal point as many figures on the right as there are decimals in all the factors multiplied.

Ex. 64.03 × 2.6 × 9.801

64.03	4
2.6	8
38418	5
12806	
166.478	5
9.801	9
166478	9
1331824	
1498302	
1631.650878	·9

Division.—1. When the divisor is a whole number, divide the dividend as if it were a whole number, and in the quotient place a decimal point when the first decimal figure of the dividend is brought down.

2. When the divisor is a decimal number, add to the right of the dividend or of the divisor as many zeros as are necessary in order that both dividend and divisor shall have the same number of decimals. The decimal point in both is then suppressed and the case reverts to the division of two whole numbers.

Ex. 27.1 ÷ 3.037 = 27.100 ÷ 3.037 = 27100 ÷ 3037 which perform. This may also be written 27100.000 ÷ 3037 which is the first case.

Reduction of a Common or an Improper Fraction to a Decimal Number.—Divide the numerator of the given fraction by its denomi-

Ex. I: $\frac{7}{8}$ $7 \div 8$

```
8 | 70 | 0.875
    60
     40
      0
```

Ans. $\frac{7}{8} = 0.875$.

Ex. II: $\frac{5}{7} = 5 \div 7$

```
7 | 50 | 0.714
    49
     10
      7
     30
      2
```

The Fraction $\frac{2}{7}$ *means* $\frac{2}{7}$ *of* 0.001

Ans. $\frac{5}{7}$ $0.714\frac{2}{7}$.

Ex. III: $\frac{74}{3} = 74 \div 3$

```
3 | 74 | 24.66
    6
    14
    12
     20
     18
     20
     18
      2
```

Ans. $\frac{74}{3} = 24.66\frac{2}{3}$.

nator. If the denominator is not contained in the numerator, write 0 in the quotient, following it with the decimal point; then add zeros to the right of the dividend and continue the division until the desired approximation is obtained or until there is no remainder. If there is a remainder, this will be the numerator of a fraction having same denom. as the given one and it will have to be added to the quotient. That fraction will be a part of unit of the last decimal of the quotient.

Reduction of a Decimal Number to a Fraction.—If the decimal number is not accompanied by a fraction, take it without the decimal point as the numerator of a fraction whose denominator shall be 1 followed by as many zeros as there are decimals in the given number. If a fraction accompanies the given decimal number, suppress the decimal point, reduce the whole expression to an improper fraction adding to the right of the denominator as many zeros as there are decimals in the given number.

Ex. I $2.34 = \frac{234}{100} \quad \frac{117}{50} = 2\frac{17}{50}$

Ex. II : $15.17\frac{1}{7} \quad \frac{1517 \times 7 + 1}{7 \times 100} = \frac{10620}{700} \quad \frac{1062}{70} = \frac{531}{35} = 15\frac{6}{35}$

DENOMINATE NUMBERS.

Denominate Numbers belong to one of the following classes:

1º Extension { Length, Area, Volume
2º Capacity { Liquid, Dry
3º Weight { Troy, Avoirdupois
4º Time
5º Angle
6º Value

Extension.—A definite portion of space.

Length.—The unit of length is the yard (Yd.). The yard is divided into 3 feet and the foot into 12 inches.

The multiples of a yard are the rod (rd.), pole or perch which equals 5 and 1-2 yds.; the furlong (fur.) which equals 40 rds., or 220 yds.; the mile (mi.) which equals 8 furlongs, or 1760 yds.

Comparative Table of Length Measures:

Mi.	Fur.	Rds.	Yds.	Ft.	In.
1	8 -	320 =	1 760 =	5 280 =	63 360
	1	40 =	220	660	7 920
		1 =	5.5 =	16.5 =	198
			1 =	3 =	36
				1	12

The surveyor's chain (ch.) is 66 ft. long divided into 100 links (l.).

Comparative Table of Surveyor's Length Measures:

Mi.	ch.	rds.	l.	ft.
1 =	80 =	320 =	8 000 -	5 280
	1	4 =	100	66
		1	25 =	16.5
			1 =	0.66 = 7.92 in.

Area or Square Measure.—The unit of area is the square yard (sq. yd.). By squaring the above Rds., Yds., Ft. and In., remembering that an acre (A) is 10 × 66 × 66 ft. or 10 square chains (sq. ch.) and stating that furlong is not used in computing areas, we form the following:

Comparative Table of Areas or Square Measures:

Sq.Mi.	A.	Sq.Rds.	Sq.Yds.	Sq.Ft.	Sq. In.
1 =	640 =	102 400 =	3 097600 =	27 878 400 =	4 014 489 600
	1 =	160 =	4840 =	43 560 =	6 272 640
		1 =	30.25 =	272.25 =	39 204
			1	9	1 296
				1 =	144

1 square mile is a Section (Sec.) and 36 sections are a Township (twp. or tp.).

Comparative Table of Surveyor's Square Measures:

Twp.	Sec. or Sq. mi.	A.	sq.ch.	sq.ft.
1	36	23 040		
	1	640 =	80	
		1 =	10 =	43 560
			1	4 356

Volume.—The unit of vol. is the cubic yard (cu. yd.).

By cubing some units of the Table of Lengths we may form the following:

Comparative Table of Volumes or Cubic Measures:

cu. yd.	Cu ft.	cu. in.
1	27	46 656
	1 =	1 728

Wood in Pile.—1 cord (cd). = 128 cu. ft. It equals a pile 8 × 8 × 2 ft. or 8 × 4 × 4 ft.

Timber.—The unit of timber measurement is the foot **board measure** (B. M.) 1 ft. (B. M.) = 12" × 12" × 1" It is a square foot one inch thick.

Masonry.—It is now generally measured by the cu. yd. but sometimes by the **perch** (p.). 1 perch (p.) = 16.5 × 1.5 × 1 ft. It is 1 rd. long, 1 and 1-2 ft. wide and 1 ft. high. The perch varies in different places.

Capacity.—Term reserved for the measurement of liquids and dry substances.

for Liquid Measures the Unit is the Gallon (gal.)
for Dry Measures the Unit is the Bushel (bush.)

Liquid Measures.—1 gallon (gal.) = 231 cu. in. = 7" × 7" × 3".

1 cu. ft. of water weighs about 62.25 lbs.

Hogshead (Hgh.) and barrels (bbl.) often vary from the following.

Comparative Table of Liquid Measures:

Hogshead Hgh.	Barrel Bbl.	Gallon Gal.	Quart Qt.	Pint Pt.	Gill Gi.
1	2	63	252	504	2 016
	1	31.5	126	252	1 008
		1	4	8	32
			1	2	8
				1	4

Comparative Table of Dry Measures:

Bushel Bush.	Peck Pk	Quart Qt.	Pint Pt.
1	4	32	64
		8	16
		1	2

WEIGHT.

Troy Weight.—Unit of weight is the Troy pound (℔.).

Troy weights are used to weigh precious metals and for scientific purposes.

Carat has 2 meanings. 1st. For weighing diamonds; it weighs near 3.2 gr. Troy. 2d. It expresses the proportion of pure gold with its alloy in 24th. parts. Pure gold is called 24 car. gold; 16 carat gold is 16 parts pure gold to 8 alloy (16 + 8 = 24).

Comparative Table of Troy Weights·

Pound Lb.	Ounce oz.	Pennyweight pwt.	Grain gr.
1	12	240	5760
	1	20	240
		1	24

Avoirdupois Weight.—Used for coarser weighing.

Comparative Table of Avoirdupois Weights:

Ton T.	Hundredweight Cwt.	Pound lb.	Ounce Oz.	Dram dr.
1	20	2 000	32 000	512 000
	1	100	1 600	25 600
		1	16	256
			1	16.

A ton of 2,000 ℔s. is called **short ton.**

A ton of 2,240 ℔s. is called **long ton;** it contains also 20 cwts. of 112 ℔s. each.

TIME.

Comparative Table of Time Measures:

Century C.	Year yr.	Month mo.	Day d.	Hour hr.	Minute min.	Second sec.
1	100	1200				
	1	12				
		1	28, 29 Feb'y in Common year, Leap year; 30 in Ap. Jun. Sep. Nov.; 31 in Jan. Mar. May, July, Aug. Oct. Dec.			
			1	24	1440	86 400
				1	60	3 600
					1	60

ANGLE.

The angular measure of an arc is that of the angle formed by the radii which connect its extremities.

The angular space in a plane about a point of the plane, or the circumference of a circle having that point as a center is supposed divided into 360 equal angles or parts each of which is a degree (°). A degree is divided into 60 minutes (') and a minute into 60 seconds (").

Comparative Table of Angle (and Arc) Measures:

Circumference Cirf.	Quadrant quad.	Degree °	Minute '	Second "
1	4	360 =	21 600 =	1 296 000
	1	90 =	5 400 =	324 000
		1	60	3 600
			1	60

VALUE.

Comparative Table of Value Measures or Money:

Eagle E.	Dollar $	Dime d.	Cent ct.	Mill mi.
1 =	10	100 =	1 000	10 000
	1	10 =	100	1 000
		1 =	10	100
			1 =	10

The sign $ is the monogram of the first letters in "United States."

METRIC SYSTEM.

All the units are of decimal formation.

The Units of

Length	are	Meter (m.) = $\frac{1}{10\,000\,000}$ of $\frac{1}{4}$ the Earth Meridian.
Area of Land		Are (a.) = A square of 10 m. a side.
Capacity		Litre (l.) A cube of 0.1 m. a side.
Solidity		Stere (st.) = A cube of 1 m. a side (Used for measuring wood for fuel).
Weight		Gramme (gr.) = Weight of 1 cu. cm. distilled water at 4°C

In engineering, areas are reckoned in square meters (sq. m.) and volumes in cubic meters (cub. m.).

In itinerary distances the kilometer (k. m.) is taken as a unit.

In common weighing the kilogram (kg.) is taken as a unit.

Multiples.—The multiples are the names of the units prefixed with deca. for 10; hecto. for 100; kilo. for 1,000; myria for 10,000.

Sub-multiples are the same names prefixed with deci. for 10th; centi. for 100th; milli. for 1 000th.

...ble of Length Measures:

Myriameter	Kilometer		Hectometer	Decameter	Meter		Decimeter	Centimeter	Millimeter
Mym.	Km.		Hm.	Dm.	M.		dm.	cm.	mm.
0	0	,	0	0	0	.	0	0	0

Table of Area Measures:

Mym.²	Km.²	Hm.²	Dm.²	M.²		dm.²	cm.²	mm.²
00 ,	00 ,	00 ,	00 ,	00	.	00 ,	00 ,	00

Table of Volume Measures:

		M.³		dm.³	cm.³	mm.³
000 ,	000 ,	000	.	000 ,	000 ,	000

Table of Agrarian Measures:

Hectare Ha.	Are A.	Centiare ca.	
00 ,	00 .	00	1 ca. = 1 M.²

Table of Capacity Measures:

Kiloliter	Hectoliter	Decaliter	Liter		deciliter	centiliter	milliliter
Kl	Hl	Dl.	L		dl.	cl.	ml.
0 ,	0	0	0	.	0	0	0

Table of Solidity Measures: (Wood for Fuel)

		Decastere	Stere	decistere
		Dst.	St	dst.
0	0 ,	0	0 0	0

Table of Weight Measures:

Ton	Quintal	Kilogram	Hectogram	Decagram	Gram		decigram	centigram	milligram
T	Q	Kg.	Hg.	Dg.	G.		dg.	cg.	mg.
0	0	0 ,	0	0	0	.	0	0	0

The reduction of any unit to a multiple or sub-multiple is effected by multiplying or dividing it by 10; 100; 1000; etc., except for areas where they are to be multiplied or divided by 100; 10,000, etc., and for volumes where the progression is by 1,000; 1,000,000, etc.

U. S. Equivalents of Metric Units.

1 Meter (M.)	=	39.37 ± inches	1.093 ± yds.
1 Sq. M.	=		1.196 ± sq. yds.
1 Cu. M.		35.3166 ± cu. ft.	
1 Are (A)			= 119.603 ± sq. yds.; 1 Ha. = 2.471 ± Acres.
1 Liter (L)	=	0.2642 gals.	1.0567 ± qts.
1 Stere (St.)		35.3166 ± cu. ft.	0.2759 ± cord
1 Gram (G.)	=	0.03527 ± oz. Avoirdupois	= 15.432 ± gr. Troy

INVOLUTION.

Involution is proceeded with by multiplication, taking the given number as many times as a factor as is indicated by the exponent.

EVOLUTION.

Evolution is the finding of one of the equal factors the product of which is the given number. The given number is placed under the

radical sign as $\sqrt{}$ and between the opening of the V a small figure called $\sqrt[]{81}$ index is written indicating what root is to be taken or how many equal factors had to be multiplied one by the other to produce the number under the radical sign. When no index is written, 2 is to be understood.

$\sqrt{81}$, square root of 81 indicates that a certain number was taken twice as a factor to produce 81. That number is 9; ($9^2 = 9 \times 9 = 81$) and we write $\sqrt{81} = 9$

$\sqrt[3]{343}$, cube root of 343 indicates that a certain number was taken 3 times as a factor to produce 343. That number is 7; ($7^3 = 7 \times 7 \times 7 = 343$) and we write $\sqrt[3]{343} = 7$

Square of a Sum of two Numbers: $\overline{65}^2 = 65 \times 65 = (60+5) \times (60+5) = 60 \times 60 + 60 \times 5 + 60 \times 5 + 5 \times 5 = \overline{60}^2 + 2(60 \times 5) + 5^2$.

Rule: *The square of a sum of 2 numbers equals the square of the first, plus twice the first by the second, plus the square of the second.*

This may also be stated thus: The square of a number containing tens and units is composed of the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

Cube of a Sum of Two Numbers. $\overline{53} = (50+3) \times (50+3) \times (50+3) =$
$= (50+3)^2 \times (50+3) = (\overline{50}^2 + 2 \times 50 \times 3 + 3^2) \times (50+3) = \overline{50}^3 + 3 \times \overline{50}^2 \times 3 + 3 \times 50 \times 3^2 + 3^3$.

Rule. The cube of a sum of 2 numbers equals the cube of the first plus 3 times the square of the first by the second, plus 3 times the first by the square of the second, plus the cube of the second. This may also be stated thus: The cube of a number containing tens and units is composed of the cube of the tens, plus 3 times the square of the tens by the units, plus 3 times the tens by the square of the units, plus the cube of the units.

	INVOLUTION.									EVOLUTION.
Table of Squares	*Squares:*	1	4	9	16	25	36	49	64	81 *Numbers.*
and Square Roots.	*Numbers:*	1	2	3	4	5	6	7	8	9 *Square Roots.*

Square Root of a Number less than $\overline{10}^2 = 100$. *It will be found in the above Table*

Examples: 1° $\sqrt{64} = 8$; 2° $\sqrt{73} = 8$ within a unit.

Square Root of a Number greater than $\overline{10}^2 = 100$.

Example. $\sqrt{5\ 29\ 92\ 04}$

Arrangement of the Operation:

```
Number  5 29 92 04  | 2302  Root
        4           | 2
        ----        |
        12 9        | 43
        12 9        |  3
        -------     | ----
         0 92 04    | 4602
           92 04    |    2
           -----    | ----
               0
```

SQUARE ROOT.

It is the finding of one of two equal factors the product of which is a given number.

Rule. Separate the given number into sets of two figures beginning with the units. Take the square root of the greatest square contained in the last set to the left (there may be only one figure in that set), it will be the first figure to the left of the root required; subtract the square of that figure from the left hand set, and to the right of the remainder bring down the next set of two figures from which separate the last one to the right; divide the left hand portion by the double of the first figure found, the quotient is the second figure of the root or is too large. Verify by writing it to the right of the double of the first figure and multiplying the no. thus formed by the fig. on trial; if the product can be subtracted from the remainder followed by the second set, the fig. is correct, if not, diminish it until a subtraction is possible. By the side of the second remainder bring down the 3d set of figs. and separate the last figure as before. Divide the left hand portion by the double of the no. formed by the two figs. obtained at the root; the quotient is verified as the first. The operation is so continued until all the sets of two figs. have been used, and their number is the same as the number of figs. in the root.

If an approximation greater than a unit is desired in the root, place a decimal point after the root already obtained and continue by adding sets of two zeros at the right of the successive remainders for each of which a decimal will be obtained in the root.

Abbreviation.—When we have more than half the number of figures desired in the square root of a number, the other figures may be had by dividing the remainder by double the root obtained.

Square Root of a Fraction.—The square root of a fraction equals the quotient of the square roots of its two terms.

$$\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}}.$$

When the denominator is not a perfect square, it is always advantageous to render that denominator rational.

Rationalizing the Denominator of a Fraction.—When the denominator is not rational, multiply both terms of the fraction by the denominator.

$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \quad \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{2 \times 3}}{\sqrt{3^2}} \quad \frac{\sqrt{6}}{3}$ *an expression which is more easily calculated than* $\frac{\sqrt{2}}{\sqrt{3}}$.

CUBE ROOT.

It is the finding of one of 3 equal factors the product of which is a given number.

	INVOLUTION									EVOLUTION
Table of Cubes	Cubes:	1	8	27	64	125	216	343	512	729 Numbers.
and Cube Roots:	Numbers:	1	2	3	4	5	6	7	8	9 Cube Roots.

Cube Root of a Number less than $\overline{10}^3 = 1000$

Example $\sqrt[3]{177\ 697\ 529\ 389}$

Arrangement of the Operation:

$N = 177\ 697\ 529\ 389$	$\underset{5\ 6\ 2\ 3}{T\ h\ t\ u}$ = Root		
$T^3 = 125$	5	900	3 360
		36	4
52 697	2500	84 36	944 164
50 616	3	36	4
2 081 529	$3T^2 = 7500$	940 800	$947\ 53200 = 3(Tht)^2$
1 888 328	$3Th = 900$	3360	$33\ 720 = 3(Tht)u$
193 201 389	$h^2 = 36$	4	$4 = u^2$
189 573 848	8436	944 164	947 86924
	6	2	2
3 627 541	50 616	1 888 328	189 573 848

Rule.—Separate the given number into sets of 3 figures beginning with the units. Take from the Table of Cubes the cube root of the greatest cube contained in the last set to the left (which may have only 1 or 2 figures), it will be the first figure to the left of the root. Subtract the cube of that first figure from the left hand set and to the right of the remainder bring down the next set of 3 figures from which separate the last two. Divide the left hand portion of this or any subsequent remainder by 3 times the square of the number formed by the root already obtained to have the next fig. of the root, and subtract from said remainder the product by said next figure, of the sum formed by adding together 3 times the square of the root obtained, three times the product of the root obtained by the next figure and the square of the next figure. If the product is greater than the remainder, reduce it until a subtraction is possible. The operation is so continued until all the sets of 3 figures have been used and their number is the same as the number of figs. in the root.

If an approximation greater than a unit is desired in a cube root, place a decimal point after the root already obtained and continue by adding sets of 3 zeros at the right of the successive remainders for each of which a decimal will be had in the root.

Abbreviation.—When we have more than half the number of figures desired in the cube root of a number, the other figures may be had by dividing the remainder by 3 times the square of the number by the portion of the root already obtained.

Cube Root of a Fraction.—The cube root of a fraction equals the quotient of the cube roots of its two terms. Rationalize the denominator if necessary.

$$\sqrt[3]{\frac{17}{48}} = \frac{\sqrt[3]{17}}{\sqrt[3]{48}} \qquad \frac{\sqrt[3]{17} \times \sqrt[3]{48^2}}{\sqrt[3]{48} \times \sqrt[3]{48^2}} = \frac{\sqrt[3]{17 \times 48^2}}{48}$$

RATIO AND PROPORTION.

Arithmetical Ratio of Two Quantities—It is generally defined as the number measuring one quantity when the other quantity is taken as the unit of measurement; or again as the quotient of the numbers which express the measures of the two quantities when referred to the same unit.

A ratio is written as follows: Suppose that A and B measured with a third quantity contain respectively 7 and 11 units, the ratio of A to B is represented by 7 : 11, or better by $\frac{7}{11}$, and the ratio of B to A by 11:7, or better by $\frac{11}{7}$.

Inverse Ratio.—These ratios are inverse of each other and their product is 1:

$$\frac{7}{11} \times \frac{11}{7} = 1$$

The terms of a ratio may be multiplied or divided by the same number.

$$\frac{7}{11} = \frac{7\times3}{11\times3} = \frac{7\div2}{11\div2}$$

Proportion.—The equality of two ratios as:

7:11 :: 14:22, or better by $\frac{7}{11} = \frac{14}{22}$, and is read 7 is to 11 as 14 is to 22, or 7 on 11 equals 14 on 22.

The first term of each ratio is an **antecedent.**

The second term of each ratio is a **consequent.**

The first and last terms are also called **extremes.**

The second and third terms are also called **means.**

In a proportion we can multiply (or divide) by the same number either the antecedents or the consequents.

If we have $\frac{7}{11} = \frac{14}{22}$ *we shall have also* $\frac{7\times2}{11} = \frac{14\times2}{22}$ *and* $\frac{7\div3}{11} = \frac{14\div3}{22}$.

The product of the extremes is equal to the product of the means.

$$\frac{7}{11} = \frac{14}{22} \qquad \frac{7\times22}{11\times22} = \frac{14\times11}{22\times11} \qquad 7\times22 = 14\times11$$

The order of the antecedents and consequents may be changed provided that in each transformation the product of the extremes shall equal the product of the means

$\frac{7}{11}=\frac{14}{22}$; $\frac{7}{14}=\frac{11}{22}$; $\frac{22}{11}=\frac{14}{7}$; $\frac{11}{7}=\frac{22}{14}$; $\frac{14}{22}=\frac{7}{11}$; $\frac{11}{22}=\frac{7}{14}$, $\frac{14}{7}=\frac{22}{11}$, $\frac{22}{14}=\frac{11}{7}$.

Again we have: $\frac{7}{11}+1=\frac{14}{22}+1$, $\frac{7+11}{11}=\frac{14+22}{22}$ *or* $\frac{7+11}{14+22}=\frac{11}{22}=\frac{7}{14}$.

Also: $\frac{7}{11}-1=\frac{14}{22}-1$, $\frac{7-11}{11}$ $\frac{14-22}{22}$ *or* $\frac{7-11}{14-22}=\frac{11}{22}=\frac{7}{14}$ *which may be expressed thus.*

The sum or the difference of the first two terms of a proportion is to the sum or the diff. of the last two terms as the first is to the third or as the second is to the fourth

Hence also: $\frac{7+11}{14+22}=\frac{7-11}{14-22}$ *or* $\frac{7+11}{7-11}=\frac{14+22}{14-22}$ *or* $\frac{7+11}{11-7}=\frac{14+22}{22-14}$ *which may be expressed thus .*

The sum of the first two terms of a proportion is to their difference as the sum of the last two terms is to their difference.

TECHNICAL REQUIREMENTS

Instruments.—The axeman wields an ax in preliminary work over a wooded country; often a hatchet is sufficient. He drives or erects poles; he drives stakes and plugs, and should be able to cut them himself. He drives tacks into plugs. He helps set stone monuments or to uncover existing ones and assists in chaining, in holding the rod, in carrying the transit or the level, in cleaning these instruments, in packing them in their respective boxes. Hence he should know what are an ax, a hatchet, a pick, a shovel, a cold chisel, a stake, a plug, a tack, a pole, a plumb-bob, a chain, a rod, a transit and a level.

DEFINITIONS OF SOME INSTRUMENTS.

The Ax.

..**Definition.**—An ax is a cutting tool or instrument formed with two parts: a metallic head and a wooden handle. The head is generally steel.

Description.—There are three portions in the head (or poll) of an ax. 1st. The slightly curved cutting edge: 2d. The re-enforced body through which an oblong socket or eye is left to receive the end of the handle, and 3d. the back or heel which is blunt like a hammer and is used as such. The wooden handle (or helve) is slightly curved in length and is composed also of three portions: 1st. The tool end is oblong to conform with the eye or socket of the head and into this opening it is driven with great force when the wood is very dry; this is done in order to secure great adhesion. 2d. The body or central portion with rounded edges for better grasping the tool and 3rd. the free end which is made a little thicker than the body in order to act as a stop to the slipping of the tool through the hands.

The Hatchet.—A hatchet is a small ax with a short straight handle. One side of the head has often a slit used to pull up nails.

The student is earnestly advised to write similar definitions of the well known tools, a pick, a shovel, a cold chisel, which he may have to use. The above definition and description of an ax are given rather as a guide.

Stake.—A stake is a piece of wood generally 1" × 2" × 18" (the sign ' denotes feet and the sign " denotes inches), sharpened at one end for driving it into the ground. One of the faces at the other end must be made smooth in order to permit of writing on it. Any kind of wood at hand may be used, but old chestnut rails can easily be cut into very good stakes. The length of the stakes varies with the practice of different departments. In brushy country, longer sticks may be used with a paper or card stuck in a slit on the top.

Plug or Hub.—A plug (also called hub) is a piece of wood generally 2" × 2" × 18", sharpened at one end like a stake.

Tack.—A tack is a short nail with a relatively broad head. Some heads are rounded, others are conical, others have a dent in the center. Brass tacks are preferable.

Pole.—A pole is a long and slender piece of wood (generally a straight limb of a tree) with a sharpened end for driving it into the ground. Poles are often fitted with a white or red rag or flag at the upper end.

Plumb-bob.—A plumb-bob is an instrument used to determine the vertical of a place. It is a heavy body freely suspended from a cord or string; it is generally top-shaped and a little elongated. It is made of iron or brass with a steel point; a cap with a central hole is screwed on the neck and through this hole a string is previously passed and knotted on the inside or secured to a concealed reel. The arrangement insures the center of gravity of the bob and its point being on a line with the string, that is to say in the vertical, when the instrument is freely suspended.

How Should Stakes and Poles be driven.—Poles and stakes must be placed in a vertical position and the rod must also be held vertically.

No explanation is thought to be necessary for the use of tools such as an ax, a hatchet, a pick, a shovel, etc.

HOW TO USE THE INSTRUMENTS.

Poles. How to set them.—When running long preliminary lines the object of which is to study the ground in order to select the best place for a location, principally when such lines run through a wooded country, distant points are often marked with long poles surmounted with a bright flag when they would not otherwise be conspicuous and easily discernible. The lower end of a pole is sharpened to secure

a better hold in the ground. To set a pole in position a hole must be dug, with pick and shovel, deep enough to insure the pole against being blown down by the wind or upset by accidental shocks. When it is not possible to dig a hole deep enough to insure the stability of the pole or when the height and therefore the usefulness of the pole would be diminished by burying too much of it, or when rock is struck near the surface, stones and boulders are placed at the foot of the pole and all around it in as secure a way as possible, or it may be found necessary to steady the pole with at least three wooden braces one end of which is sharpened and driven obliquely into the ground and the other is nailed to the pole.

Surveying Pole.—A surveying pole is a wooden rod 5 to 10 ft. long, painted alternately white and red and with a sharp ferrule at the lower end. The upper end may or not carry a flag. These poles are intended for temporary use only.

How to Drive a Stake.—The stake should be held, over the point to be marked, in a vertical position with one hand and driven with a hammer (it may be the heel of the hatchet) held in the other hand, striking gentle blows at first, and increasing the effort until the top of the stake is within 6" or 8" (practice varies in different departments) from the ground. In that operation care must be taken to keep the smooth face of the stake toward the **B. S.** (**B. S.** is the abbreviation for back-sight).

Level Stakes.—Stakes on which levels are to be taken are driven flush with the ground and on line; they are referenced by stakes placed one or two ft. to one side.

Back Sight.—The **B. S.** is the direction from which the line that is being staked was started, or again where the chainmen come from, or again where the last center stake was set.

Fore Sight.—The **F. S.** is the direction toward which the line that is being staked goes, or again where the chainmen go to, or again where the next center stake will be set.

Center Stake.—Any stake set on the line which is being run is a center stake.

Offset Stakes.—In soft marshy ground an offset is made and a line parallel with the principal alignment is run that will clear the bad ground, and on this parallel, stakes are set which are called offset stakes.

Precautions Necessary to Drive a Stake Properly.—It must not deviate too much from the vertical direction by the point passing between pieces of rock. A slight variation may be corrected by striking gently the sides of the stake so as to bring the top to the right point. When a deviation begins to take place, it is better to leave the stake as it is if the point has penetrated the ground sufficiently deep to insure security of position. If rock is struck near the ground, it may be uncovered, a mark such as a cross made on it with the hatchet or with a small cold chisel, then covered over with stones and a refer-

ence stake driven as near the point as possible. The distance from the point to the stake is then marked on the smooth face of the stake which must face the point.

How to Drive a Plug.—A plug is driven with the same care required for driving stakes; it is driven deeper or until the top is nearly flush with the ground, say not more than 1" from it.

How to Care for His Instruments.—The axeman should keep his tools and the instruments or implements in his charge clean and in their proper place.

THE AXEMAN'S WORK IN THE FIELD.

What He Carries.—He carries an ax, a bundle of stakes and plugs, marking crayon, tacks and some nails.

Reconnaissance.—On preliminary work or reconnaissance he accompanies the engineer or chief of party, who selects the points through which the lines are to run, and he marks these points with poles or stakes. With poles if the points are far apart or in a wooded country, with stakes if they are closer together and in relatively open ground. He clears the ground between these points cutting brush, underbrush and sometimes even limbs of trees so that the points may be easily visible to the transitman and the leveler.

Caution.—The cleared space about transit stations must be ample, the line free from underbrush, but the axeman must not injure trees or shrubs unnecessarily.

Location.—On location work, that is to say when the line is finally determined and marked on the ground, he accompanies the transitman and the leveler and drives stakes and plugs as directed.

Where Stakes are Set.—A stake is driven at every point on the line where an elevation is taken, generally at each station, that is at the end of every chain (100 ft.), even though no elevation were taken at some of these points.

Plus Stake.—Any stake set between stations, for instance at Sta. 32 + 18.3, is called a **plus stake,** and the point where it is set a **plus point.**

How He Acts When Setting Stakes.—When the setting of stakes is directed by the transitman, he should stand to one side and be attentive to signals. When he is through he must move quickly and leave room for the chainmen.

Elevation.—Datum—An **elevation** is the height of a point above or below another known point named **datum.**

Negative Elevations.—The elevations of points below the datum are negative. For instance E.—82.1 indicates that the point considered is 82.1 ft. below the datum.

Marking a Point Where No Stake Can Be Driven.—When a stake cannot be driven at a point which is to be recorded, as on a hard surface roadbed, on a sidewalk, etc., a mark such as a cross X, an arrow point, or ← crow-foot may be made with either a sharp tool, red paint, a colored pencil, or even chalk, and markings which should be on the stake are written at the point of the arrow or on a near-by sidewalk, tree, house, lamp post, etc., or on a reference stake if practicable.

Reference Stake or Guard.—A reference stake is one driven near the place or point where a stake or plug should be and at right angle to the alignment; its smooth face is turned towards the point and on it are written the station of the point with the distance right (**R**) or left (**L**) that the reference stake is from the point.

Where Plugs Are Set.—A plug is set at every permanent point, that is to say at points which are of importance and must remain on the ground, such as a transit point.

A point of curve (where a curve begins) **P. C.**, a point of tangent (where a tangent begins), **P. T.** In leveling, **T. P.** indicates a turning point which is the point where the rod is placed when two readings, a back-sight (**B. S.**) and a fore-sight (**F. S.**) are taken on it.

Transit Point.—A transit point (**T. P.**) is generally the end of an alignment and the beginning of another, or the point of intersection of two lines. A transit point may be on a long straight line. Many engineers prefer the abbreviation **P. I.**, point of intersection, instead of **T. P.**, transit point, because of the similarity between **T. P.** and **P. T.**

Marking the exact point where a Plug is driven.—A tack or nail is driven into the head of a plug to mark the exact point to be recorded by the plug.

Referencing a Plug.—Near each plug reference stakes are set with the markings of the plug on them, such as **P. I.** 9 + 40.62 25 **R** or 25 **L.**

Four reference stakes are necessary to properly reference a point where a plug cannot be placed. The intersection of the diagonals of the quadrilateral which they form is the point. (Fig. 1)

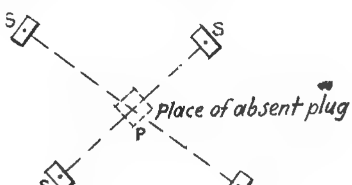


Fig. 1

Alignment.—An **alignment** is the direction of a straight line marked with plugs and stakes driven into the ground.

He helps find Points and Line.—The axeman helps the transitman and leveler in finding the alignments, the **T. P.**'s and the stakes, because he is more than they familiar with the ground, having gone over it in the preliminary work.

Protects Transitman and Leveler.—The axeman may have to protect the transitman and the leveler from the rays of the sun which might prevent them from correctly sighting or reading; this he may do by interposing his hands or his hat between the sun and the instrument. Likewise in very windy weather, to prevent the shaking of the transit or level, he may stand on the windy side extending his coat as a protection.

Protects Chainmen.—In windy weather the chain may sag too much under the pressure of the wind, the axeman may then place his hands or fingers against it with arms extended near the middle of the chain.

Bench Mark.—A bench-mark is a small level surface prepared on a rock, a stump or other durable material on which to rest the rod, and the elevation of which is carefully taken: it serves to correct lines of levels or to start new ones: its abbreviation is **B. M.**

Preparing Bench Marks.—The axeman will have to cut bench marks (**B. M.**) where directed, as on a rock, on a wall, on the stump of a tree. On rock he may have to use a cold chisel in preparing a level bed for the rod. The elevation is painted near by.

On the stump of a tree he uses the hatchet in preparing such a bed, then he cuts away a little of the stump above and below that bed and finally blazes a portion of the trunk for a place on which to write or paint the elevation.

Distance between Bench Marks.—Bench marks should not be further apart than half a mile, their elevation should be carefully checked with a second line of levels and the errors should not be more than 0.01 ft. or 0.015 ft.

Another duty of the axeman is helping to set monuments or uncovering existing monuments set under the surface of the ground.

Monument.—A monument is a block of stone generally square on top permanently set in the ground, or a bolt sealed in the rock, which marks a corner as of a street, a property, the intersection of two principal alignments, or simply a point of an important line such as the center line of a long structure.

Setting a Monument.—With pick and shovel he digs a hole a little larger than the monument to be set and to a depth determined by the engineer; the bottom is carefully prepared for the bed of the stone. A few flat stones or spawls will prop up the monument and make it steady, or a bed may be made with concrete. When in position the

remaining space is filled with earth and stones uniformly packed on all sides or with concrete, care being taken not to disturb the position of the stone by unequal settling or by shocks. When ready, a cross is cut on the top face or a copper plug is sealed in it with a marking nick at the center of it to indicate the exact point of the ground. If the monument is to be buried, some large stone must first be placed on top to protect it from the blows of the pick in case it should be necessary to uncover it later.

Copper Bolts.—On rock, monuments are sometimes replaced by copper bolts sealed and marked at the center.

Monuments indicate Direction—Bench Marks indicate Elevation— Monuments refer to points and lines or indicate direction; bench-marks indicate elevation.

Uncovering a Monument.—When it becomes necessary to uncover a monument, great care must be taken so as not to injure its top with the pick or to shake it from its position.

Cross-section Stakes.—During construction the axeman sets stakes where cross-sections are taken, one on the center line on the **B. S.** face of which are written the station, as 12 + 18.3, and sometimes the depth of cut (excavation) as **C.** 5.4 or + 5.4 (which means that the elevation of the ground is equal to the elevation of the grade + 5.4 ft.), or the height of fill (embankment), as **F.** 13.8 or —13.8 (which means that the elevation of the ground is equal to the elevation of the grade —13.8 ft.).

Markings on Cross-section Stakes.—The figures written on a stake are called **markings.** In the example just given, the station number 12 + 18 3 might be called the name of the stake; it indicates its position which is 12 chains (the engineer's chain is 100 ft. long) plus 18 and 3 tenths feet, or 1218.3 ft. distant from the beginning of the line. The figures **C.** 5.4 or + 5.4 indicate that the ground is 5.4 ft. higher than grade and that there will be a cut or excavation 5.4 ft. deep at that point; the figures **F.** 13.8 or —13.8 indicate that the ground is 13.8 ft. lower than grade and that there is a fill or embankment 13.8 ft high at that point

Slope Stakes.—Other stakes are placed on cross-sections; at the point where an excavation is to begin or at the point where an embankment will meet the ground. These are called slope stakes and are marked **S. S.** If such slope stakes are placed at these very points, they are apt to be disturbed by careless workmen, so that they should be referenced with other stakes placed at convenient points on the same cross section line and bearing a marking such **R. S. S. 4 L** which means Reference Slope stake 4 ft. to the left, or **R. S. S. 3 R,** which means Reference Slope stake 3 ft. to the right. Some engineers simply have their slope stakes set at a uniform distance right or left from the point where they should be driven. Many of these stakes however will have to be reset from time to time.

Acting as Flagman, Poleman.—The axeman may be required to act as a **flagman** or **poleman** in establishing a line.

In the capacity of a flagman he establishes transit or plug points under the direction of the engineer or chief of party. These points should be so selected as to be in positions relatively clear from undercbrush as along a ridge They should be visible in both directions, backsight and foresight, and their distance should be easily chainable.

He sets a plug as directed, drives a tack into its head and sets the pole with the point of the ferrule on the tack, standing squarely behind it and holding it vertical by balancing it with the tips of the fingers of both hands.

When obstructions are on the line, he leaves a sight mark on them, such as a piece of paper nailed to a fence (top rail), an indentation or a crayon mark on a projecting rock, etc.

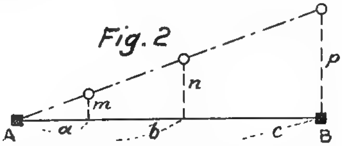
Ranging in a Line.—It consists 1° in producing an alignment until a good transit point is obtained, and 2° in interpolating points in an alignment between two transit points (or angle points).

Prolongation of an Alignment.—Standing a foot or two back of the rear pole, the engineer or transitman directs his sight tangent to one side of the two poles, motioning to the poleman, who stands further than the second pole, for the right place where to set a third pole on a line with the first two. This third pole must be covered by the sights tangent to the right and to the left sides of the first two poles. When the third pole is in position, the first pole is removed and a stake or plug is substituted. This first pole is then carried further than the one last set and is placed on line by means of the second and third poles in the same manner that the third pole was set.

Interpolating Poles.—This is done to facilitate the chaining of a long line. When one point is visible from the other, the operation is easy enough. A pole is set at each transit point, being held vertically using a plumb-bob if necessary. The transitman steps back one or two ft. from the rear pole and sights tangentially along the line until the front pole is hidden from view; he then directs, by motions of the arm, the placing of intermediate poles in the true line of his visual ray. If he has been sighting along the right sides of the poles, he now sights along the left sides in which direction the poles should also be covered; if not, a correction is made in the ranging. **Stakes are always substituted for poles removed.**

If the head transit point is not visible (Fig. 2), a random line may be run with a succession of poles in the supposed direction. The deviation is measured at the end of the line and the position of the ranging poles changed in proportion to their distance from the rear point. A second trial may be necessary.

Fig. 2



$$m = \frac{ap}{c} \qquad n = \frac{bp}{c}$$

Intersection of two Alignments (Fig. 3).—A pole is placed at **E** in the alignment **CD.** The engineer standing back of **A** sights **AB** while the poleman moves back from **DE** keeping on a line with them until he is on the alignment **AB** when the engineer motions him to stop.

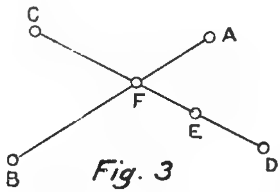


Fig. 3

Giving Foresight or Backsight to the Transitman.—By holding a pole vertical with the ferrule on the tack.

He may on a long line cut some straight sticks and range them with a piece of paper sticking in a slit on top of them.

Pacing.—He is sometimes ordered to place a stake or pole a certain distance, more or less, away from a point. He should know the length of his pace at different gaits, and he may acquire that knowledge by pacing first slowly, then on a common walk, on a trot, on the run, a line of a measured length and several times, taking a written record of each performance and averaging the results.

THE AXEMAN'S WORK IN THE OFFICE.

The axeman will see that enough stakes and plugs are on hand and ready for at least one day's work; that his ax or hatchet are sharp and all his tools clean; that there is a supply of tacks for the next day and that his tools or instruments are in their proper place.

He may advance his work by numbering stakes with consecutive stations for the work of the following day.

When it is raining steadily or when it is too cold to be in the field, he will have to assist other men in checking their notes, their work or their calculations, in comparing descriptions, tracings or maps; he may have also to make tracings or plot cross-sections and do other work not properly within the sphere of his duty but the performance of which may accelerate the work of the engineer and will in all cases give him some of the experience he should always be anxious to get.

In any spare time study for the next examination.

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The Assistant Engineer

BY

Prof. JEAN P. GENTHON
Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK II

THE CHAINMAN

WRITTEN FOR

THE CHIEF

Journal of the Civil Service

PUBLISHED BY
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THE CHAINMAN.

Book II.

PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with that most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3°. The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examination in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON,

July 1, 1907. New York.

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1°. Aptitude for mathematics.
2°. Habit of observation.
3°. Good memory.
4° System.
5°. Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION.—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials, or new appliances and processes are used for old or new purposes. Notice the several kinds of labor, and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results: principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figure and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations, in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around you will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public

works have a Chief Engineer who prepares the work and directs its construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

			Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist.
Chief Engineer.			Inspectors.	
	Division Engineers.		Transitmen.	Chainmen. Axemen.
		Assistant Engineers	Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen.

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.
Boston, Mass.
Buffalo, N. Y.
Charleston, S. C.
Chattanooga, Tenn.
Chicago, Ill.
Cincinnati, Ohio.
Cleveland, Ohio.
Detroit, Mich.
Galveston, Tex.
Grand Rapids, Mich.
Jacksonville, Fla.
Little Rock, Ark.
Louisville, Ky.
Memphis, Tenn.
Milwaukee, Wis.
Mobile, Ala.
Nashville, Tenn.
New London, Conn.
New Orleans, La.
Newport, R. I.
New York, N. Y.
Norfolk, Va.
Philadelphia, Pa.
Pittsburg, Pa.
Portland, Me.
Portland, Oreg.
Rock Island, Ill.
St. Louis, Mo.
St. Paul, Minn.
San Fancisco, Cal.
Savannah, Ga.
Seattle, Wash.
Sioux City, Iowa.
Tampa, Fla.
Vicksburg, Miss.
Washington, D. C.
Wheeling, W. Va.
Wilmington, Del.
Wilmington, N. C.
YellowstonePark,Wyo.

where examinations may be taken except that of Assistant Engineer which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken in

Albany.
Amsterdam.
Auburn.
Binghampton.
Buffalo.
Elmira.
Hornelsville.
Ithaca.
Jamestown.
Kingston.
Lockport.
Malone.
Newburg.
New York.
Ogdensburg.
Olean.
Plattsburg.
Poughkeepsie.
Rochester.
Utica.
Syracuse.
Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewers or construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman, topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps.

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service appear regularly in "THE CHIEF."

BOOK II.

THE CHAINMAN

Chainman.—One who measures lines on the ground with a chain or a tape. Two chainmen are needed to measure a line.

Head Chainman.—The one who goes ahead and is directed.

What He Carries.—The head chainman usually carries a flag pole.

Rear Chainman.—The one who follows and directs.

What He Carries.—The rear chainman usually carries the chain or tape; chaining pins, pocket foot-rule, some nails and note book.

Who His Superiors Are.—The chainman is under the orders of the chief of party or the transitman.

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service—Panama Canal.

Tittle: Chain man.

Salary: $60 per month.

Written Examination.

	Subjects.	Relative weights.
1.	Spelling (20 words of average difficulty in common use)	15
2.	Arithmetic (simple tests in addition, subtraction, multiplication and division of whole numbers, and in common and decimal fractions, and U. S. money)	20
3.	Letter-writing (a letter of not less than 125 words on some subject of general interest. Competitors will be permitted to select one of two subjects given)	20
4.	Penmanship (the handwriting of the competitor in the subject of copying from plain copy will be considered with special reference to the elements of legibility, rapidity, neatness, general appearance, etc.)	15
5.	Copying from plain copy (a simple test in copying accurately a few printed lines in the competitor's handwriting)	15
6.	Training and experience	15
	Total	100

Applicants must have had not less than one year's practical experience in similar work, provided that two years' study in a school of civil engineering will be accepted as equivalent for this experience.

One day will be required for this examination.

New York State and County Civil Service.

Age Limit: Not less than 18 years old.

Salary: $2.50 to $3.00 per day when employed.

Written Examination.

Subjects.	Relative weights.
1. Arithmetic and use of chain	8
2. Experience and education	2
Total	10

SPECIMEN QUESTIONS.

1. A wall 1,690 ft. long is to be built in 30 days, and it is found that 7 men in 14 days have completed only 490 ft. How many additional men must be employed for the remainder of the time so that the wall may be completed in the required time?
2. Find the cu. yds. of masonry in a wall 88 and 1-2 ft. long, 10 ft. high, 1 ft. 6 in. thick at the top, 3 ft. 6 in. thick at the bottom, with sides sloping uniformly from the top to the bottom.
3. Two adjoining lots of city land 132 ft. deep contain respectively 31 sq. rods, 112 and 1-2 sq. ft. and 35 sq. rods, 36 sq. ft. These two lots together are to be redivided into 5 equal building lots. What will be the width in ft. of each lot?
4. If the rainfall on a certain day was 1-4 in., how many gallons of water fell on a rectangular piece of land 22 rods, 9 ft. wide and a quarter-mile long? (One gallon contains 231 cu. in.).
5. What is the length of an arc of 30° in a circle whose diam. is 18 ft.? (Give answer to 3 decimal places The circumference is 3.1416 times the diam.).
6. (a) If you were sent to make a chain survey of a piece of rough meadow land, what tools or instruments would you need to take with you? (b) Explain carefully and in detail how you would begin the work and how proceed with the measurement of the first course? (c) How would you keep the measurement always in a straight line with the points from which and to which you are chaining?
7. Suppose that you have arrived at a corner or angle in the meadow and have with you neither transit nor surveyor's compass, how would you measure the angle in the line you are chaining so that a map could be made from the measurements and notes you are taking?

8. (a) How would you proceed if your line crosses a broad gully very steep on one side? Explain carefully and in detail. (b) Suppose that your line passes over a hillock that shuts out from view the rod toward which you are chaining; how would you overcome the difficulty? (c) How would you proceed if a large tree stood in the way of the line you were chaining?

9. (a) What would you do if a hundred-foot length, in your measurement, should end in the middle of a brook that flows swiftly over a smooth stone bottom? (b) What would you do if your line crossed a bed of quicksand or a deep marsh more than a chain length across? Explain carefully.

10. (a) How would you lay out a line at right angle to the course you have just measured? (b) Having finished your survey, how would you fold your chain?

11. If your hundred-foot chain is 5-8 in. too short, what is the true length of a line that you have measured and recorded as 2,416.5 ft. in length?

12. Have you had any experience as chainman or assistant in a survey? If so describe in detail, giving time, place and length of service. What experience or other education have you had that you consider qualifies you for duties of chainman?

New York City Municipal Civil Service.

Position eliminated Oct., 1907; duties assigned to Rodman. (See Book III.—The Rodman).

Note: Axeman must serve one year before promotion to Rodman.

Scope of Examination.

(Old Schedule.)

1°. That he is within the limits of age prescribed for the situation of chainman.

2°. That he is free from any physical defect or disease likely to interfere with the proper discharge of his duties.

3°. That his character is such as to qualify him for such employment; and

4°. That he possesses the required knowledge and ability to enter upon the discharge of the duties of a chainman.

The fourth article of the certificate shall be determined by a written competitive examination which shall have reference to the special qualifications, expert or otherwise, required for the position of chainman, and shall be practical in its character.

Written Examination.

Subjects.	Relative weights.
1. Handwriting (as shown in examination papers)	1
2. Arithmetic, viz.: Addition, subtraction, multiplication and division	1
3. Questions relating to the technical knowledge required for the position of chainman	6
4. Experience tending to qualify him for that position	2
Total	10

QUESTIONS GIVEN AT CIVIL SERVICE EXAMINATIONS.

MATHEMATICS.

Give all the figuring on the ruled sheets.

1. What is the area of a trapezoid of which the two parallel sides are 127' 8 and 1-3" and 55' 6", and the distance between them is 42' 3"

2. Add 19'; 6 and 1-2"; 13', 3 and 1-4"; 7' 2 and 3-4"; 19.75'; 27.667'; 3.5"; 66" and 196" and express the result in (a) feet and inches; (b) feet and decimals of a foot and (c) inches.

3. A pipe 4' inside diameter, running full, discharges into a canal 5 feet wide with vertical sides; how deep will the water be in the canal?

4. A rectangular field contains 1 and 1-4 acres and is 3 and 1-8 times as long as it is wide; what are the dimensions of the field?

5. Three measurements added together give 2,400 feet; the longest is 1,100 feet more than the shortest and the other is 1,100 ft.; how much is the longest and how much is the shortest?

6. Reduce the following to thousandths of a foot, 7 feet 9 inches, 6 and 7-10 feet, 42 inches, 12 and 607 thousandths feet and 17 and 3-8 inches.

7. Four men can shovel 9 yards of gravel per hour; how long will it take 50 men to shovel 4,500 yards of gravel working 8 hours a day?

8. What is the contents in cubic yards of a retaining wall which has one face vertical and is 123 feet long; one end is 18 feet high, base 10 feet, top 2 feet, and the other end is 12 feet 6 inches high, base 7 feet 6 inches, and top 2 feet. (The prismoidal formula may be used, but is not required.)

9. If a rod has the target at 8.23 feet and is 4 inches out of plumb, how much of an error in thousandths is made in the elevation?

10. Find the square of 20 39-200. Find the square root of 5053.172 correct to four places of decimals.

11. A street a quarter of a mile long has on each side a sidewalk 7 and 1-2 feet wide; what will it cost to pave this sidewalk (both sides) with stones each 2 ft. 9 in. long by 1 ft. wide, and costing 75 cents?

12. Two vessels sail at the same time from the same place, one sailing due east and the other due north, at the rate of 6 miles and 8 miles an hour respectively. How far apart will they be at the end of 12 hours?

13. A man paid $2,262 for a field in the shape of a trapezoid. The parallel sides were 119 yards and 200 yards, respectively. The distance between them 110 yards. What did the land cost him by the acre?

14. How many square yards in a graveled walk 6 ft. wide, running around a circular fish pond whose diameter is 70 yards?

15. (a) What is the difference between the square of 1 2 of 32 and 1-2 of the square of 32? (b) Extract the square root of 25.0 and 3-4 correct to four places of decimals.

16. The perimeter of a rectangular field is 140 rods, from one corner to the centre of the field is 25 rods; find dimensions and area.

17. On opposite sides of a stream 40 feet wide stand two trees, 83 feet and 57 feet in height; what is the distance between the tops of the trees?

18. A city lot costs $2,250 at 40 cents a square foot; its parallel sides were 39 feet and 51 feet respectively; what was the length of the lot?

19. A cylindrical column of granite is 13 00 feet in circumference and its volume is 818,125 cubic feet. What is the height of the column?

TECHNICAL.

1. How do the duties of a Rodman differ in surveys with (a) a level; (b) transit; (c) transit with stadia?

2. What are contour lines? How are they obtained?

3. (a) A cubic foot of water contains how many gallons?
   (b) Weighs how much?

4. How is the velocity of flow of a stream measured?

5. (a) What is the difference in decimals of a foot between 9.66 inches and .572 feet; (b) Express thirty-six thousandths of a foot in decimal figures (c) Turn 94.86 inches into feet and decimals of a foot and express the result in words.

6. Assume that a New York rod can be clearly seen at a proper distance. What must the rodman do to assure a correct reading?

7. The distances between sights on a bench run are as follows: B. S. 150'; F. S. 200'; B. S. 600'; F. S. 300'; B. S. 150'; F. S. 300'; B. S. 150'; F. S. 300'; B. S. 50'; F. S. 300'. Will there be any error from curvature? Give your reasons.

8. Describe how by the use of a chain alone you can measure an angle in the field so it can be mapped.

9. A target set on a rod at 5.00' is .3" out of plumb. How much difference will it make in the elevation read as compared with the correct elevation?

10. Explain how each of the following points necessary to be observed in accurate measurement with a tape are attained; level, tension, temperature, alignment.

11. How would you establish and mark a first class bench on the root of a tree?

12. In making a long measurement what is the best method of keeping accurate tally of the number of chain or tape lengths?

13. Explain the principle of the vernier and illustrate by a sketch showing a reading of 2.424 feet.

14. Describe two methods of turning a right angle offset by use of a chain alone.

15. In accurately measuring a base line for triangulation, what special precautions are taken?

16. State what points a rodman should keep a record of in the readings, and give a sample of such notes.

17. State all the difficulties a rodman has to contend against in trying to do good work in the city.

18. (a) In what way does the sun affect the work? (b) How does wind affect the work of leveling? (c) How may frost, ice, snow, etc., affect it?

19. Describe the characteristics of a thoroughly good turning point; also of a poor one.

20. In what ways are errors caused by a rodman being careless of not understanding his business fully?

21. Does it make any difference in the progress of work as to how the rodman locates turning points? And, if so, state how.

22. Sketch a vernier on a rod and explain how it is that thousandths of a foot can be read by it.

23. What fraction of an inch is one-thousandth of a foot equal to?

24. Describe the sights used in running transit lines for back sighting to.

25. Name five things requiring close attention in making accurate measurements with tape or chain.

26. How does the heat of the sun affect a tape, and how is this compensated for in the use of an engineer's tape?

27. Describe clearly and fully the duties of a rear chainman in measuring up a hill.

28. How can the rear chainman hold the tape steady over a point?

29. (a) Describe the duties of a forward chainman; (b) How is the record of chain-lengths kept?

30. State all the difficulties that tend to introduce error in the work of chaining.

31. Suppose you are measuring a practically level line a mile long, state actually all you would do and especially how the tally is kept.

32. In measuring uphill, how does the rear chainman hold steadily at the right point? Does he ever touch the stake, and if so, why?

33. Where the surface is very steep, what is done in chaining, and what precautions should be taken to prevent wrong tally being made?

34. Suppose a survey has been made of a lot of which all the corners were definitely fixed before the survey, and the chain is found to be long, how should the notes be corrected?

35. State all the ways in which errors may be introduced or caused in chaining?

36. Suppose that a bank of a river is to be located from field notes, how would the work be done by the chainman?

37. How would you measure past a tree on a line, and produce the line in practically the same direction without an instrument?

38. Suppose a piece of ground to have two sides parallel but of unequal lengths (in other words, trapezoidal in form), how would you measure it and compute the area?

39. (a) Explain the construction of a vernier on a leveling rod. (b) For what purpose is it desirable to use the vernier, or do you consider it essential to always read to thousandths of a foot?

40. Suppose the rod to be extended to ten feet, figure how much out of plumb must it be to make error of one thousandth in the reading?

41. State all the means made use of by rodmen to keep the rod plumb.

42. State all the means made use of by rodmen who make errors in reading the rod.

43. State all the ways in which errors are made in connection with turning points, from every cause whatever.

44. State all the causes of error in any way connected with the target aside from its reading.

45. State the consideration that should always govern a rodman in locating turning-points

46. Describe briefly and state the difference between the rods used in transit, level and stadia work.

47. (a) To which of the above kinds of work does the rule "foresights and backsights must be equal" apply? (b) What is the reason of above named work?

48. Why should the rod be held plumb? State the different reasons for each kind of above named work.

49. Assume an inch divided into quarters; describe and show by sketch, a vernier which will divide it into twentieths.

50. State the several ways in which a good rodman can facilitate the work of leveling.

51. (a) In leveling, why should a rod be waved to and fro? (b) Would it do to wave it from side to side provided it was always in a plane perpendicular to the line of sight? Give reasons.

52. What part of an inch is a thousandth of a foot?

53. If a level is out of adjustment and reads an error of .03 at 100 feet, what will the error be at 400 feet distance?

54. (a) How can you lay off a right angle with a chain? (b) How with a piece of cord?

55. Describe fully the work of the two chainmen in chaining for levels up a steep hillside.

56. In the use of a tape, what four important points must be observed to get accurate results? State in their order of importance.

57. (a) Why is the Gunters' chain used for farm surveys? (b) Why the steel tape for city work?

58. In leveling, what errors which affect the accuracy of results, are due to a rodman's lack of care?

59. State every way in which the reading of a rod should be checked.

60. If the wind was blowing hard, state every precaution you would exercise in measuring a line where considerable accuracy was desired.

SCIENTIFIC REQUIREMENTS.

What preceedes (See Book I.): Algebra, including Quadratics; Plane Geometry.

ALGEBRA.

Algebra.—Algebra generalizes questions relating to numbers.

Known Quantities.—In a problem, known quantities are represented by the first letters of the alphabet **a**, **b**, **c**, etc., but very often by the first letter of their names, such as **h** for height, **l** for length, **w** for weight, etc.

Unknown Quantities.—Unknown quantities are represented by the last letters of the alphabet.

SIGNS :	$+$ *denotes addition*,	$\sqrt{}$ *denotes a root*;
	$-$ " *subtraction*,	$=$ " *equality*;
	$\times$ " *multiplication*;	$>$ GREATER THAN } *denote inequality*;
	$\div$ " *division*,	$<$ SMALLER THAN }
	( ); [ ]; $\overline{}$ *denote aggregation*	

The **multiplication sign** $\times$ is often replaced by a point . ; it is even agreed that several letters placed one after another without a sign between them are to be multiplied.

The **division sign** $\div$ is more generally replaced by the horizontal line of a fraction. The numerator of the fraction is the dividend and the denominator is the quotient.

Parenthesis indicate that the quantities placed within are to be considered as an aggregate or a single quantity.

Vinculum is used over quantities to aggregate them in the same manner as the parenthesis.

Brackets are used when some quantities within are already aggregated by a parenthesis or a vinculum

Algebraic Expression.—Combination of letters and numbers upon which operations indicated by signs are to be performed.

Formula.—Algebraic expression indicating the series of operations to be performed on the quantities of a problem in order to solve it.

Rational Expression.—Contains no radical sign, as

$$\frac{7(x+3)(2a+b)}{5c}$$

Irrational Expression.—Contains radical signs, as

$$a + b\sqrt{5c}$$

Entire Expression.—Contains neither denominator nor radical, as

$$ax^2 + bx + c$$

Fractionary Expression.—Contains a denominator, as

$$\frac{a^2+b^2}{c}$$

Monomial.—Contains only one letter or a single product, as

$$b \;,\; 3cd^2m^3$$

Binomial.—Contains two monomials separated by the sign + or —

$$a^2-b^2 \quad . \quad 8ab^2x-7bc^2y$$

are binomials, and the monomials composing it are called its **terms.**

Trinomial.—Contains 3 monomials or terms separated by the sign + or —, as

$$ax^2+bx+c$$

Polynomial.—Contains several terms. Binomials and trinomials are polynomials with special names.

Coefficient.—A number, sometimes an algebraic expression which, written to the left of another, indicates that this other quantity is to be multiplied by the first, as

$8a^3b^2c$ *in which* 8 *is coefficient of* a^3b^2c ,
$(a^2-b^2)x^3$ *in which* a^2-b^2 *is coefficient of* x^3

Degree of a Monomial Entire.—Sum of the exponents of its literal factors.

$3a^4b^2c$ *is of the* 7^{th} *degree* ; $9a$ *is of the first degree.*

Degree of a Polynomial.—Is the degree of that term which has the highest degree.

$a^9+b^8-c^4a+10a^3b$ *is of the* 9^{th} *degree on account of* a^9

Homogeneous Polynomial.—One in which all the terms are of the same degree, as

$$ax^2+bxy+cy^2$$

Numerical Value of an algebraic quantity is that final number which is obtained when, after replacing all the letters by their respective values in a particular problem, the operations indicated by the signs have been performed.

Similar Terms.—Those which have the same letters affected with the same exponents whatever their signs and coefficients.

Reduction of Similar Terms.—Add together the coefficients of the terms preceeded by + and together the coefficients of the terms pre-

ceeded by — ; subtract the smaller sum from the greater, give the sign of the greater and write the similar term to the right of the difference.

Ex. I $15a^3b^2c - 7a^3b^2c = (15-7)a^3b^2c = 8a^3b^2c$
Ex. II. $mab^2x - nab^2x = (m-n)ab^2x$.

Positive Term.—Or additive term, one preceeded by the sign +.

Remark.—The sign + is suppressed before a single positive term, or before the first of a series of terms.

Negative Term.—Or subtractive term, one preceeded by the sign —.

Algebraic Operations.—They consist in transforming algebraic quantities into simpler but equivalent ones.

ADDITION.

Rule.—Write all the quantities to be added one after the other preceeding each with its own sign. If any reduction is possible, perform it afterward.

Example: Add $7a^3+3a^2b+3ab^2$ with $3a^2b-ab^2-4a^3$ with $2a^3+a^2b+7ab^2-b^3$

$$7a^3+3a^2b+3ab^2+3a^2b-ab^2-4a^3+2a^3+a^2b+7ab^2-b^3$$
$$=(7-4+2)a^3+(3+3+1)a^2b+(3-1+7)ab^2-b^3 = 5a^3+7a^2b+9ab^2-b^3$$

General Principles.—1. We can take in any order the parts of a sum.

2. A polynomial has the same value whatever be the order of its terms; so we can invert that order provided we preserve the signs of the terms.

3. To add a sum it is sufficient to add in any order all the parts of that sum.

4. To add to a quantity the difference of two others, add the first and subtract the second.

SUBTRACTION.

Rule.—Write after the minuend or quantity to be subtracted from all the terms of the subtrahend or quantity to be subtracted changing the signs of the latter + into — and — into +. If any reduction is possible, perform it afterward.

Ex From a^2+b-cd^3 subtract a^2-cd^3

$$(a^2+b-cd^3)-(a^2-cd^3) = a^2+b-cd^3-a^2+cd^3 = b.$$

MULTIPLICATION.

Product of Monomials.—Rule. The product of monomials is the

product of their coefficients followed by the common letters with the sum of their exponents followed in turn by the letters that are different with their respective exponents.

$$\text{Ex. } 5a^3b^2c \times 13ab^3cd^2 = 5\times 13\, a^{3+1}b^{2+3}c^{1+1}d^2 = 65a^4b^5c^2d^2$$

Product of a Polynomial by a Monomial.—Rule. Multiply each term of the polynomial by the monomial multiplier, preserving the signs of the multiplicand if the multiplier is positive and changing them if it is negative.

Ex. I · $(x^2+bx-c)\times 4ay^2 = 4ax^2y^2+4abxy^2-4acy^2$.
Ex. II: $(a^2+b^2-c^2)\times(-2ac) = -2a^3c-2ab^2c+2ac^3 = 2ac^3-2a^3c-2ab^2c$.

Factoring or Separating a Common Factor.—When a factor is common to several terms of a polynomial, it may be suppressed in them provided the resulting polynomial be multiplied by the common factor.

Ex.: $20a^3b^3c^3d-15a^2b^2c^3d^2+10a^4b^3c^2dx+7m$
We see that $5a^2b^2c^2d$ *is common to the first three terms*
We may write the given quantity $=5a^2b^2c^2d(4abc-3cd+2a^2bx)+7m$.

In Ex. 2 above, the factor **2ac** is common to all the terms of the product and may be separated by the rule, which gives:

$$2ac(c^2-a^2-b^2),$$

and if we compare this result with the given data of the problem, namely

$$(-2ac)(a^2+b^2-c^2)$$

we see that **we can change the signs of any two factors in a product.**

Product of Polynomials.—Rule. Multiply all the terms of the multiplicand by each term of the multiplier; each partial product will have the sign + if the factors have the same sign, and the sign — if they have different signs.

This is often given as a **Rule of Signs:** In multiplication + by + gives +, — by — gives +, + by — gives —, and — by + gives —.

When the product is obtained, reduce the similar terms.

Ex.: $(a^2b+2ab^2-c^3)\times(a^2b-2ab^2+c^3)$

Operation ·

$$\begin{array}{lllll}
a^2b & +2ab^2 & -c^3 & & \\
a^2b & -2ab^2 & +c^3 & & \\
\hline
a^4b^2 & +2a^3b^3 & -a^2bc^3 & & \\
 & -2a^3b^3 & & -4a^2b^4 & +2ab^2c^3 \\
 & & +a^2bc^3 & & +2ab^2c^3 \quad -c^6 \\
\hline
a^4b^2 & & & -4a^2b^4 & +4ab^2c^3 \quad c^6
\end{array}$$

Arranged Polynomials.—When a polynomial contains a letter with different exponents, it is always advantageous to change the order of the terms in such a manner that the exponents of that letter shall gradually increase or diminish from the first to the last.

In the multiplication above, both factors are arranged according to the decreasing powers of **a**; they happen to be also arranged according to the increasing powers of **b**.

Square of a Sum.—

$$(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$$

The square of a sum of 2 quantities equals the square of the first plus twice the first by the second plus the square of the second.

Square of a Difference.—

$$(a-b)^2=(a-b)(a-b)= a^2-2ab+b^2$$

The square of a difference of 2 quantities equals the square of the first minus twice the first by the second plus the square of the second.

Product of a Sum by a Difference.—

$$(a+b)(a-b) = a^2-b^2$$

The product of the sum of 2 quantities by their difference equals the difference of their squares.

DIVISION.

Quotient of Monomials.—Rule. Obtain the quotient of the coefficients; follow it with the common letters with the difference of their exponents and the letters that are in the dividend and not in the divisor with their respective exponents.

$$\text{Ex.}\ \frac{36a^8b^4c^3d}{12a^4b^3c^2} = \frac{36}{12}a^{8-4}b^{4-3}c^{3-2}d \qquad 3a^4bcd$$

When, according to the above Rule, we divide a^m by a^n, we have $\frac{a^m}{a^n} = a^{m-n}$

There may be 3 cases:

1°- $m > n$ as $\frac{a^7}{a^3} = a^4$

2°- $m = n$ as $\frac{a^6}{a^6} = a^0$ which expression has no meaning

3°- $m < n$ as $\frac{a^3}{a^5} = a^{-2}$ which expression also has no meaning.

SYMBOL $a^0=1$ Referring to the expression $\frac{a^m}{a^m} = 1 = a^{m-m} = a^0$, it is agreed that $a^0 = 1$

SYMBOL $a^{-m}=\frac{1}{a^m}$. Referring to the expression $\frac{a^3}{a^5} = \frac{1}{a^2} = a^{3-5} = a^{-2}$, it is agreed that $a^{-2} = \frac{1}{a^2}$ and generally that $a^{-m} = \frac{1}{a^m}$

Quotient of a Polynomial by a Monomial.—Rule. Divide each term

of the dividend by the monomial preserving the signs of the dividend if the monomial is positive and changing them if it is negative. This operation is the same as factoring.

Ex. I: $\dfrac{18a^4b^3c^2d - 12a^3bc^3d^2 + 6a^2b^2c^2}{6a^2bc^2} = \dfrac{18a^4b^3c^2d}{6a^2bc^2} - \dfrac{12a^3bc^3d^2}{6a^2bc^2} + \dfrac{6a^2b^2c^2}{6a^2bc^2}$

Ex. II. $\dfrac{3a^2b^2 - 6a^3b - 9ab^3}{-3ab} = 3a^2b^2d - 2acd^2 + b$

$= -ab + 2a^2 + 3b^2$

$= 2a^2 - ab + 3b^2$

Quotient of 2 Polynomials.—Rule. Arrange them both according to the decreasing powers of a letter (the same in both). Divide the first term of the dividend by the first term of the divisor for the first term of the quotient. Multiply the whole divisor by that first term and subtract the product from the dividend. Divide the first term of the remainder by the first term of the divisor for the second term of the quotient and so continue.

The rule is applicable whether the coefficients of the several terms are numerical or literal, monomials or polynomials.

Ex. . $\dfrac{a^3 - b^3}{a - b}$

$$\begin{array}{rl|l}
\text{Dividend} = & a^3 - b^3 & a - b = \text{Divisor} \\
& \underline{-a^3 + a^2b} & a^2 + ab + b^2 \quad \text{Quotient} \\
1^{st}\text{ Remainder} = & a^2b - b^3 & \\
& \underline{-a^2b + ab^2} & \\
2^{nd}\text{ Remainder} = & ab^2 - b^3 & \\
& \underline{-ab^2 + b^3} & \\
& 0 &
\end{array}$$

Theorem I A polynomial entire in x is divisible by $x - a$ if it becomes nul *(equal to zero)* when we substitute in it a for x

Theorem II. *As we can write* $x + a$ $x - (-a)$ *we may say that* A polynomial entire in x is divisible by $x + a$ if it becomes nul when we substitute in it $-a$ for x.

Corollaries : 1° $x^m - a^m$ is always divisible by $x - a$.
Changing x *into* a *it becomes* $a^m - a^m = 0$

2° $x^m + a^m$ is never divisible by $x - a$.
Changing x *into* a *it becomes* $a^m + a^m = 2a^m$ *and not* 0

3° $x^m - a^m$ is divisible by $x + a$ when m is even.
Changing x *into* $-a$ *we have* $(-a^m) - a^m$. *In order that* $(-a^m)$ *be positive* m *must be even*.

4° $x^m + a^m$ is divisible by $x + a$ when m is odd.
Changing x *into* $-a$ *we have* $(-a^m) + a^m$ *In order that* $(-a^m)$ *be negative* m *must be odd*.

FRACTIONS.

Properties.—We can multiply by the same quantity the two terms of a fraction.

We can divide by the same quantity the two terms of a fraction.

Reducing a Fraction.—Suppress all the factors common to the two terms. It is easy to see the common factors when the terms are monomials.

Ex.: $\frac{36a^3b^5cdm^2}{28ab^4c^5mk}$. The factors common are $4ab^4cm$

$$\frac{36a^3b^5cdm^2}{28ab^4c^5mk} = \frac{4ab^4cm(9a^2bdm)}{4ab^4cm(7c^4k)} = \frac{9a^2bdm}{7c^4k}$$

When the terms are polynomials we see also their common monomial factors.

$$\text{Ex.: } \frac{12a^4b^3 - 8a^3b^2}{16a^5b - 20a^2b^4} = \frac{4a^3b^2(3ab-2)}{4a^2b(4a^3-5b^3)} = \frac{ab(3ab-2)}{4a^3-5b^3}.$$

As for the common polynomial factors, we may sometimes discover them by the application of some theorem or by a judicious remark.

Ex. I - $\frac{a^4-2a^3+4a^2-7a+4}{a^2+5a-6}$. We note that both num. and denom. become zero if we change a into 1; therefore they are both divisible by $a-1$ and we have:

$$\frac{a^4-2a^3+4a^2-7a+4}{a^2+5a-6} = \frac{(a-1)(a^3-a^2+3a-4)}{(a-1)(a+6)} = \frac{a^3-a^2+3a-4}{a+6}.$$

$$\text{Ex. II - } \frac{8a^2c^2d^3 - 72b^2c^2d^3}{6ac^3d^2 - 18bc^3d^3} = \frac{8c^2d^3(a^2-9b^2)}{6c^3d^2(a-3b)} = \frac{4d(a+3b)(a-3b)}{3c(a-3b)} = \frac{4d(a+3b)}{3c}$$

Reduction of Fractions to a Common Denominator.—Multiply the two terms of each by the product of all the other denominators.

$$\text{Ex. } \frac{a}{b}, \frac{c}{d}, \frac{e}{f} \text{ become } \frac{adf}{bdf}, \frac{cbf}{bdf} \text{ and } \frac{ebd}{bdf}$$

We can often obtain a C. D. simpler than the product of all the denominators. Select any expression divisible by all the denominators and multiply each numerator by the quotient obtained in dividing that C. D. by the denominator of the corresponding fraction.

Ex. I - $\frac{A}{12a^3b^2c}, \frac{B}{16a^2b^4}, \frac{C}{18abc^3}$ The C.D. $= 144a^3b^4c^3$ and we have

$$\frac{A \times 12b^2c^2}{144a^3b^4c^3}, \quad \frac{B \times 9ac^3}{144a^3b^4c^3}, \quad \frac{C \times 8a^2b^3}{144a^3b^4c^3}$$

Ex. II - $\frac{2a}{3b^2}, \frac{a+b}{2b(a-b)}, \frac{a-b}{4a(a+b)}, \frac{a^3+2b^3}{9a^2(a^2-b^2)}$. The C.D. $=36a^2b^2(a^2-b^2)$ and we have

$$\frac{24a^3(a^2-b^2)}{36a^2b^2(a^2-b^2)}, \frac{18a^2b(a+b)^2}{36a^2b^2(a^2-b^2)}, \frac{9ab^2(a-b)^2}{36a^2b^2(a^2-b^2)}, \frac{4b^2(a^3+2b^3)}{36a^2b^2(a^2-b^2)}$$

Addition and Subtraction of Fractions.—Rule. Add or subtract their numerators after they are reduced to a common denominator.

Multiplication of Fractions.—Rule. Divide the product of the numerators by the product of the denominators.

$$\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{a \times c \times e}{b \times d \times f} = \frac{ace}{bdf}$$

If the numerators become equal to each other and the denominators become equal to each other, such a product becomes

$$\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \left(\frac{a}{b}\right)^3 = \frac{a \times a \times a}{b \times b \times b} = \frac{a^3}{b^3}.$$

Power of a Fraction.—Rule. Raise the two terms to that power.

Division of Fractions.—Rule. Multiply the fraction dividend by the fraction divisor inverted as

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Symbol $\frac{m}{0}$. This form is the ultimate value of a fraction $\frac{m}{n}$ in which, the numerator remaining the same, the denominator diminishes gradually until it become *zero*. The successive quotients (*or values of* $\frac{m}{n}$) increase more and more. We say that the value of the fraction tends toward infinity (∞) and we write $\frac{m}{0} = \infty$

Symbol $\frac{0}{0}$. It represents *indetermination*; but this indetermination may be apparent only and may come from the presence, in a fractionary expression, of a factor common to both terms and which becomes 0 in a particular case.
By expunging that common factor we may find the true value of the expression.

RADICALS.

Root.—A root is indicated by the sign

$\sqrt{}$ *called radical* *In the* $\sqrt{}$ *of the radical a small figure, called* index *states what root is to be taken*.

When no index appears a square root is always understood, thus

$\sqrt{3a}$ indicates the *square root* of $3a$ or one of the two equal factors the product of which is $3a$;
$\sqrt[5]{m}$ indicates the *fifth root* of m or one of the five equal factors the product of which is m
The whole expressions $\sqrt{3a}$, $\sqrt[5]{m}$ are also called *radicals* for brevity sake

Double Roots.—

In $\sqrt[m]{A}$, when A is positive and m even, $\sqrt[m]{A}$ has two equal values but of contrary signs Thus $\sqrt{4} = \pm 2$ because $(+2) \times (+2) = +4$ and $(-2) \times (-2) = +4$.

In the same expression, when A is negative and m odd, there is a negative value for the root. Thus $\sqrt[3]{-27} = -3$ because $(-3)^3 = (-3)^2 \times (-3) = 9(-3) = -27$.

1st Principle.—We can induce a coefficient of a radical under the radical by raising it to the power indicated by the index, thus:

$$a\sqrt[n]{b} \quad \sqrt[n]{a^n b}\text{, and conversely:}$$

We can separate a quantity from a radical by extracting from it the root indicated by the index

2nd Principle.—We can multiply both index and exponent of a radical by the same number, thus:

$$\sqrt[m]{a^n} = \sqrt[mp]{a^{np}}\text{, and conversely:}$$

We can divide both index and exponent of a radical by the same number.

Reduction of Radicals to a Common Index.—Rule. Multiply index and exponent of each by the product of all the other indices.

Ex.: $\sqrt{a^3}$; $\sqrt[3]{b^4}$; $\sqrt[5]{b^2}$ are equivalent to $\sqrt[2\times3\times5]{a^{3\times3\times5}}$; $\sqrt[2\times3\times5]{b^{4\times2\times5}}$; $\sqrt[2\times3\times5]{b^{2\times2\times3}}$ or to $\sqrt[30]{a^{45}}$; $\sqrt[30]{b^{40}}$; $\sqrt[30]{b^{12}}$

Radicals can be reduced to the least common index.

Addition and Subtraction of Radicals.—Rule: Write them as usual, one after the other, separating them with the sign + or the sign — .

Reduction of Radicals.—They can only be reduced when they have same index and are similar besides.

Product of Radicals.—Rule: Reduce them to a common index; multiply together the quantities under the radical and give to their product the common index:

$$\text{Ex.}\ \sqrt{\frac{a}{b^2}} \times \sqrt[3]{\frac{a^2}{b}} = \sqrt[6]{\frac{a^3}{b^6}} \times \sqrt[6]{\frac{a^4}{b^2}} = \sqrt[6]{\frac{a^3}{b^6} \times \frac{a^4}{b^2}} = \sqrt[6]{\frac{a^7}{b^8}}\ .$$

Quotient of Radicals.—Rule: Reduce them to a common index; divide the quantities under the radical and give to their quotient the common index.

$$\text{Ex.}\ \frac{27\sqrt[3]{a^4b^2}}{36\sqrt{a^3b}} = \frac{27\sqrt[6]{a^8b^4}}{36\sqrt[6]{a^9b^3}} = \frac{27}{36}\sqrt[6]{\frac{a^8b^4}{a^9b^3}} = \frac{3}{4}\sqrt[6]{\frac{b}{a}}$$

Power of a Radical.—Rule: Raise to that power the quantity under the radical, preserving the index.

$$\text{Ex.:}\ \left(\sqrt[3]{ab^2}\right)^2 = \sqrt[3]{(ab^2)^2} = \sqrt[3]{a^2b^4} = b\sqrt[3]{a^2b}$$

Root of a Radical.—Rule: Multiply the index of the radical by the index of the root.

$$\text{Ex.}\ \sqrt[3]{\sqrt{a^3b^2c}} = \sqrt[3\times2]{a^3b^2c} = \sqrt[6]{a^3b^2c}$$

Fractionary Exponents.—Since we can divide both index and exponent by the same number,

if we have $\sqrt[3]{a^7}$ *we may write it* $a^{\frac{7}{3}}$

provided we make the following agreement.

$$\text{Symbol} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}.$$

$$\text{Symbol} \quad a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{\sqrt[n]{a^m}}.$$

Rationalizing the Denominator of a Fraction.—A very useful operation.

$$\text{Ex. I} - \frac{m}{\sqrt{a}} = \frac{m\sqrt{a}}{\sqrt{a}\sqrt{a}} = \frac{m\sqrt{a}}{(\sqrt{a})^2} = \frac{m\sqrt{a}}{a}$$

$$\text{Ex. II} - \frac{m}{\sqrt{a}+\sqrt{b}} = \frac{m(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{m(\sqrt{a}-\sqrt{b})}{a-b}.$$

EQUATIONS OF THE 1ST. DEGREE.

Theorem I.—We can add (or subtract) the same quantity to (or from) both members of an equation.

$$a + x = b + 3x \;;\quad a + x - x = b + 3x - x \;;\quad a = b + 2x \;.$$

Theorem II.—We can multiply (or divide) by the same quantity (not zero) both members of an equation.

$$74x = 18ab^2 \;;\quad \frac{74x}{74} = \frac{18ab^2}{74} \;;\quad x = \frac{9}{37}ab^2.$$

Theorem III.—In raising quantities to the square, cube, or any power, we generally introduce foreign solutions. When this has been done to both members of an equation, we must see that the solutions obtained agree with the given equation and discard those which do not.

Solution of an Equation of the First Degree in x.—

Ex.: $3x - \frac{4}{3} - \frac{x}{4} = \frac{5x}{21} + 2x + 13$ RULE: 1° *Reduce both members to the*

$\frac{252x - 112 - 21x}{84} = \frac{20x + 168x + 1092}{84}$ *same denominator (84) and*

$252x - 112 - 21x = 20x + 168x + 1092$ *suppress the common denom.*

2°- *Displace to one member of the equation the terms containing the unknown and to the other member the known terms,*

$252x - 21x - 20x - 168x - 1092 + 112$. *changing the signs of the terms so displaced.*

$43x \quad 1204$ 3°- *Perform any possible reduction.*

$x \quad \frac{1204}{43} = 28$ 4°- *Divide both member by the coefficient of the unknown.*

If the coefficients are literal they are treated in the same way.

Equation of a Problem.—It is a relation of equality between the known and unknown quantities; between the data given and the results required. The conditions imposed are so variable for each class of problems that no general rule can be given. The skill of the operator consists in finding the formulas of those quantities that are to be equal.

System of Equations of the First degree in x, y, z, etc.—

A system of equations is two or more equations which become identities for the same values of the unknown quantities which they contain.

Theorem I.—We can substitute to one of the equations of a system the equation obtained by adding the others (or only some of them) member to member.

For instance

The System $\left\{\begin{array}{l} A \quad B \\ C = D \\ E = F \end{array}\right.$ *is equivalent to the System* $\left\{\begin{array}{lcl} A+C+E & = & B+D+F \\ C & & D \\ E & & F \end{array}\right.$

Theorem II.—When one equation of a system is solved with regard to one unknown quantity, we can replace in the other equations that unknown quantity by its value, thus reducing the system to another having one equation and one unknown less.

Eliminating an Unknown Quantity.—To eliminate an unknown x for example, from a system of equations, consists in replacing the given system by a new one having one less equation and which shall not contain x.

System of Two Equations.—

General form : $\left\{\begin{array}{l} ax + by = c \\ a'x + b'y = c' \end{array}\right.$ (a' is read : a prime)

1st Method - *Substitution* Take the value of x : $x = \frac{c-by}{a}$ from the first equation and replace x by that value in the second equation

which becomes $a'\frac{c-by}{a} + b'y = c'$ Solve this for y.

$$y = \frac{ac'-a'c}{ab'-a'b}$$ which becomes known.

To obtain x, replace y by its value in the equation $x = \frac{c-by}{a}$.

$$x = \frac{c - b\frac{ac'-a'c}{ab'-a'b}}{a} = \frac{b'c-bc'}{ab'-a'b}$$

2nd Method - *Comparison*. In the same general form as above, take the values of x from the two equations

$x = \frac{c-by}{a}$ $x = \frac{c'-b'y}{a'}$ and equate these values:

$\frac{c-by}{a} = \frac{c'-b'y}{a'}$. Solve this equation for y.

In like manner you may take the values of y in the two equations, equate them and solve for x.

3rd Method – *Addition and subtraction.*

$$\begin{cases} (1)\quad ax + by = c \\ (2)\quad a'x + b'y = c' \end{cases}$$ Multiply by a' the 2 members of (1) and by a the 2 members of (2)

$$\begin{cases} (3)\quad aa'x + a'by = a'c \\ (4)\quad aa'x + ab'y = ac' \end{cases}$$ Subtract (4) from (3), there remains

$$a'by - ab'y = a'c - ac'$$ Factor y

$$y(a'b - ab') = a'c - ac'$$ Divide by the coefficient of y

$$y = \frac{a'c - ac'}{a'b - ab}$$

x in turn would be obtained by multiplying (1) by b' and (2) by b, subtracting the results, factoring x and dividing both terms by its coefficient.

These principal methods are sufficient in all cases They are not exclusive, however, which means that should we begin by employing a method, we may conclude by using another if we find it more convenient.

System of 3 Equations.—

$$\begin{cases} (1)\quad a^3 + a^2x + ay + z = 0 \\ (2)\quad b^3 + b^2x + by + z = 0 \\ (3)\quad c^3 + c^2x + cy + z = 0 \end{cases}$$ Subtracting (1) from (2) and (3) we have

$$\begin{cases} (4)\quad b^3 - a^3 + (b^2 - a^2)x + (b-a)y = 0 \\ (5)\quad c^3 - a^3 + (c^2 - a^2)x + (c-a)y = 0 \end{cases}$$ or, dividing (4) by $b-a$ and (5) by $c-a$

$$\begin{cases} (6)\quad b^2 + a^2 + ab + (b+a)x + y = 0 \\ (7)\quad c^2 + a^2 + ac + (c+a)x + y = 0 \end{cases}$$ Subtracting (6) from (7) we have

(8) $c^2 - b^2 + a(c-b) + (c-b)x = 0$ Dividing by $c-b$

(9) $c + b + a + x = 0$ from which $x = -a - b - c = -(a+b+c)$.

Substituting the value of x in (6) we would obtain $y = ab + ac + bc$
Substituting the values of x and y in (1) we would obtain $z = -abc$.

Equations of the Second Degree in x or Quadratics.—

An equation *complete* of the second degree contains terms in x^2, terms in x and known terms, and may be represented by the general form

$$ax^2 + bx + c = 0$$

in which a, b, c may be monomials or polynomials; b and c may be positive or negative, but a is positive *(which may always be brought about by transposition)*; b may be 0 or c may be 0.

An equation of the second degree may therefore assume one of the following 3 forms:

(1) $ax^2 + bx = 0$ when $c = 0$,
(2) $ax^2 + c = 0$ when $b = 0$;
(3) $ax^2 + bx + c = 0$ which is the general form:

1st Form: $ax^2+bx=0$ This may be written $x(ax+b)=0$. In order to satisfy this last equation, one of the factors must be nul. Hence we have two solutions: $x=0$ and $ax+b=0$ or $x=-\frac{b}{a}$

2nd Form: $ax^2+c=0$; $x^2=-\frac{c}{a}$; $x=\pm\sqrt{-\frac{c}{a}}$; $x'=\sqrt{-\frac{c}{a}}$ and $x''=-\sqrt{-\frac{c}{a}}$.

General Form or Complete Quadratics: $ax^2+bx+c=0$

This may be transformed into an equation of the form

$$(mx+n)^2=p$$

which can be easily solved.

In order to do this, multiply it by $4a$ which is not nul

$4a^2x^2+4abx+4ac=0$ or $4a^2x^2+4abx=-4ac$

Add b^2 to both members of this last equation

$4a^2x^2+4abx+b^2=b^2-4ac$ which may be written in succession:

$$(2ax+b)^2=b^2-4ac$$

$$2ax+b=\pm\sqrt{b^2-4ac}$$

$$2ax=-b\pm\sqrt{b^2-4ac}$$

Formula of solution: $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad (A)$$

The roots will be: $$\begin{cases} x'=\dfrac{-b+\sqrt{b^2-4ac}}{2a} \\ x''=\dfrac{-b-\sqrt{b^2-4ac}}{2a} \end{cases}$$

We may now give the following rule of solution of any quadratics reduced to the general form of a trinomial, which will dispense from going through the intermediate calculations.

Rule I. *The values of x are in the form of a fraction the numerator of which is the coefficient b of the term in x taken with a contrary sign, plus or minus the square root of the difference obtained by subtracting from the square b^2 of that coefficient 4 times the product $4ac$ of the coefficient a of the term in x^2 by the third or known term c, and the denominator of which is $2a$, twice the coefficient a of the term in x^2.*

Remark: a being positive, $-4ac$ has always a sign contrary to c.

Modification of Formula (A).—

Modification of the Formula of solution (A) when b is even.

Suppose $b=2b'$. (*For instance* $18=2\times 9$)

Substitute $2b'$ for b in (A); it becomes.

$$x=\frac{-2b'\pm\sqrt{4b'^2-4ac}}{2a}=\frac{-2b'\pm\sqrt{4(b'^2-ac)}}{2a}$$

$$x=\frac{-2b'\pm 2\sqrt{b'^2-ac}}{2a} \quad \text{and finally}$$

$$x=\frac{-b'\pm\sqrt{b'^2-ac}}{a} \qquad (B)$$

in which b' is $\frac{1}{2}$ of the coefficient of the term in x. In this case the above Rule is modified as follows:

Rule II. *The values of x are in the form of a fraction the numerator of which is the semi-coefficient b' of the term in x taken with a contrary sign, plus or minus the square root of the difference obtained by subtracting from the square b'^2 of that semi-coefficient the product ac of the coefficient a of the term in x^2 by the third or known term c, and the denominator of which is a, the coefficient of the term in x^2.*

Ex. I. Solve $\frac{1}{a} + \frac{1}{a+x} + \frac{1}{a+2x} = 0$

Reduce to the same denominator and cancel it:

$$a^2 + 3ax + 2x^2 + a^2 + 2ax + a^2 + ax = 0$$

Reduce similar terms and arrange ·

$2x^2 + 6ax + 3a^2 \quad 0$ which is of the general form.

The coefficient $6a$ of x being even, apply Formula (B)

$$x \quad \frac{-3a \pm \sqrt{9a^2 - 6a^2}}{2} = \frac{-3a \pm \sqrt{3a^2}}{2} = \frac{-3a \pm a\sqrt{3}}{2} = \frac{a(-3 \pm \sqrt{3})}{2}.$$

$$\begin{cases} x' = \dfrac{a(-3+\sqrt{3})}{2} & \dfrac{a}{2}(-3+\sqrt{3}) \\ x'' \quad \dfrac{a(-3-\sqrt{3})}{2} & -\dfrac{a}{2}(3+\sqrt{3}) \end{cases}$$

Ex. II Solve $(1+x+x^2)^{\frac{1}{2}} = a - (1-x+x^2)^{\frac{1}{2}}$

Square the two members $1+x+x^2 = a^2 - 2a(1-x+x^2)^{\frac{1}{2}} + 1 - x + x^2$

which reduces to $2a(1-x+x^2)^{\frac{1}{2}} = a^2 - 2x$

Square again: $4a^2 - 4a^2x + 4a^2x^2 = a^4 - 4a^2x + 4x^2$

which reduces to $4a^2x^2 - 4x^2 = a^4 - 4a^2$

$$x^2(4a^2-4) = a^2(a^2-4)$$

$$x^2 = \frac{a^2(a^2-4)}{4(a^2-1)}$$

$$x \quad = \pm\frac{a}{2}\sqrt{\frac{(a+2)(a-2)}{(a+1)(a-1)}}.$$

SOLUTION OF PROBLEMS.

Rule.—Represent the known quantities or data by **a. b, c,** etc., and the unknown or required quantities by **x, y, z.** etc. Study the conditions imposed by the question upon these several quantities and express them by equations. Solve these equations and you will obtain formulas for the solution of all problems similar to the given one.

Problem.—An Assistant Engineer, Chief of Party, acting as topographer, sends by a chainman an order to the transitman who is 2 miles ahead on the road. After delivering the order the chainman returns and meets the topographer at the very spot that was occupied by the transitman at the moment when the chainman started on his journey. How far did the chainman have to travel, supposing that he did not stop and maintained a uniform rate of speed; supposing also that the parties in the field work with a uniform rate of progress?

Draw a diagram similar to the figure annexed (see fig. 4)

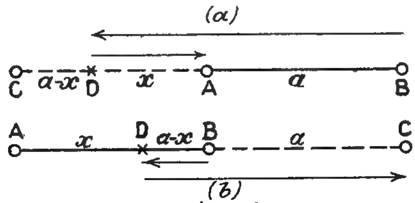


Fig. 4

Referring to the diagram, let B be the position occupied by the Assist^t Engineer when he sends the Chainman to the Transitman then at A, a distance of 2 miles *(make $2 = a$)*. The party working ahead in the direction of the arrow, the chainman will meet the transitman at a certain point D beyond A; he then returns and meets the topographer at A. During that time the transitman has moved up to C, 2 miles *(a)* beyond A.

Make $AD = x$; then $DC = a - x$.

Distances travelled by chainman: $a + x$ when going and x when ret'g
Distances gone over by transitman during the same time: x and $a - x$:

Let v be the velocity of the chainman and v' that of the transitman; let also t be the time required by both to reach the point D.

We have the equations: $\begin{cases} (1)\ vt = a + x & \text{for the chainman ;} \\ (2)\ v't \quad x & \text{" " transitman.} \end{cases}$

Dividing (1) *by* (2): $\quad (3)\ \dfrac{v}{v'} = \dfrac{a+x}{x}$

Let now t' be the time required by the chainman to return to A and by the transitman to proceed to C (a miles away from A).

We have the equations: $\begin{cases} (4)\ vt' = x & \text{for the chainman ;} \\ (5)\ v't' = a - x & \text{" " transitman.} \end{cases}$

Dividing (4) *by* (5): $\quad (6)\ \dfrac{v}{v'} = \dfrac{x}{a-x}$

Equating (3) *and* (6) *the first members of which are equal,*

we have · $\quad \dfrac{a+x}{x} = \dfrac{x}{a-x}$ *then successively*

$$a^2 + ax - ax - x^2 = x^2$$

$$x = \pm \frac{a}{\sqrt{2}}$$

$$x = \pm \frac{a\sqrt{2}}{2} \quad \text{solution of the equation}$$

and $\quad X = a + 2x = a \pm a\sqrt{2} = a(1 \pm \sqrt{2})$ formula of the problem

then making $a = 2$ *we have* $X = 2(1 \pm \sqrt{2})$ $\quad$ solution of the problem.

The negative value $a(1-\sqrt{2})$ would be the distance travelled by the chainman leaving the engineer at B (*where he acts as transitman*), and carrying the order to the topographer then at A. The progress of the party would be in the opposite direction as shown by the second diagram (*fig 4 b*).

We give this example to illustrate a manner of using artifices in the treatment of problems and in the solution of equations.

GEOMETRY.

Axiom.—Self evident truth.

Theorem.—Enunciation of a proven truth.

Postulatum.—Enunciation of an theorem admitted to be true without proof.

Problem.—Question to be solved.

Solution of a Problem.—Indication of the several steps to be taken to solve it, or method to be followed.

Proposition.—Enunciation of an Axiom, a Theorem or a Problem.

Hypothesis.—A supposition

Corollary.—A consequence.

Scholium.—A remark.

Three Parts of a Theorem or Problem.—1°. The **proposition** which is the enunciation of the truth to be proven, or statement of the question to be solved—2°. The **hypothesis** or supposition—or the solution in the case of a problem—3°. The **Consequence** or conclusion which is the statement of the proposition into a theorem—or that the result last obtained in the problem is what was required.

Reciprocal Propositions.—Two propositions one having the consequence of the other as an hypothesis and its hypothesis as a conclusion. The reciprocal of a true proposition is not always true.

Solid or Body.—Finite portion of indefinite space. Its constituting materials or elements are not considered in geometry.

Extent of a Solid.—The finite portion of space which it occupies

Surface of a Solid.—The limit which separates it from surrounding space. The surface of a solid is not always continuous but is generally formed with different distinct contiguous portions called faces and sometimes also surfaces.

Line.—Intersection of surfaces.

Point.—Intersection of lines.

Point.—A point may also be conceived as an infinitesimal definite

portion of space, or a position.

Line.—A line may be conceived as generated by the motion of a point through space.

Surface.—Some surfaces may be conceived as generated by the motion of a line through space.

Solid.—Some solids may be conceived as generated by the motion of a surface through space.

Geometry.—It may be defined the science of extent.

Straight Line (Fig. 5).—Shortest distance between two points **A—B.**

Fig. 5

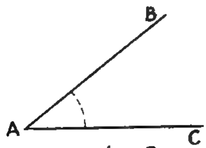


Fig. 6

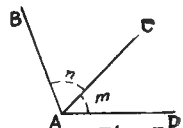


Fig. 7

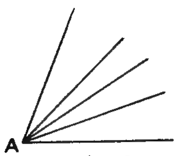


Fig. 8

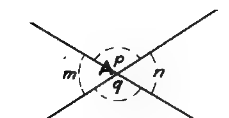


Fig. 9

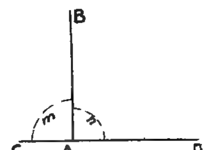


Fig. 10

Plane.—Surface which contains entirely the straight line connecting any two of its points.

A smooth table gives the idea of a plane.

Angle (Fig. 6).—Figure formed by two st. lines **AB—AC** which meet at **A** when only their spreading **m** is concerned.

Sides of an Angle.—The straight lines **AB—AC** which form it.

Vertex of an Angle.—The point **A** of intersection of the sides.

Adjacent Angles (Fig. 7).—They have the same vertex **A** and a common side **AC.**

Beam (Fig. 8).—Several lines issuing from the same point **A.** A series of lines about a point.

Angles opposite through the Vertex (Fig. 9).—Angles not adjacent formed by 2 intersecting lines,

Perpendicular (Fig. 10).—A line **BA** which meets another line **CD** and makes with it equal adjacent angles: **m=n.**

Oblique (Fig. 11).—A line **BA** which meets another line **CD** and makes with it unequal adjacent angles:

$$m > n$$

Right Angle.—One the sides of which are perpendicular to each other as **m** and **n** in Fig. 10.

Acute Angle.—One smaller than a right angle, as **n** in Fig. 11.

Obtuse Angle.—One greater than a right angle, as **m** in Fig. 11.

Bisectrix of an Angle (Fig. 12).—A line **AD** which divides an angle into two equal adjacent angles: **m=n.**

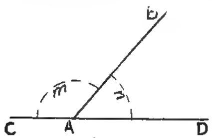


Fig. 11

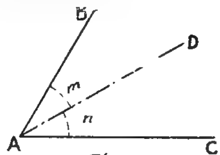


Fig. 12

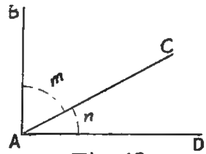


Fig. 13

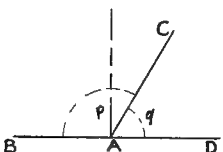


Fig. 14

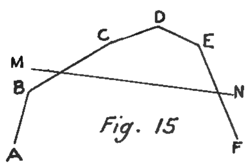


Fig. 15

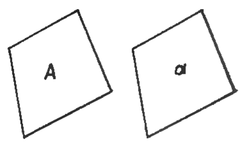


Fig. 16

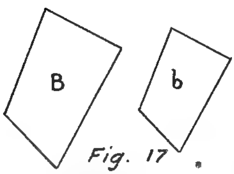


Fig. 17

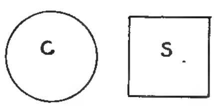


Fig. 18

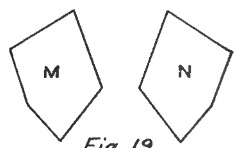


Fig. 19

Measurement of an Angle.—An angle is measured by means of another angle. A right angle is supposed divided into 90 equal adjacent angles, each of which is called an angle of one degree, which is written 1° The angle of 1° is sub-divided into 60 equal adjacent angles each of which is an angle of one minute, which is written 1'. The angle of 1' is sub-divided in turn into 60 equal adjacent angles each of which is an angle of one second, which is written 1" An angle of 12 degrees 27 minutes 50 seconds is written 12° 27' 50".

Complementary Angles (Fig. 13).—Their sum is 90° or one right angle: m+n=90°.

Supplementary Angles (Fig. 14).—Their sum is 180° or two right angles: p+q=180°.

Polygonal Line or Broken Line.—A line formed with several straight lines.

Curved Line or Curve.—A line neither straight nor composed of straight elements.

Convex Line (Fig. 15).—A plane line which can be cut only in two points by a straight line.

Geometrical Figure.—Representation of a point, line, surface or solid.

Plane Figure.—One that can be drawn in a plane.

Plane Geometry.—Treats of plane figures.

Defined Figure.—One of which we know the form or shape, the dimensions or size and the locus or position.

Equal Figures (Fig. 16).—They have same shape and size. They are superposable: **A=a.**

Similar Figures (Fig. 17)—They have same shape and different size, as **B** and **b.**

Equivalent Figures (Fig. 18).—They have different shape and same size or dimension: length for lines; area for surfaces and volume for solids.

Symmetrical Figures (Fig. 19).—They have equal elements arranged in opposite order, as **M** and **N.**

Geometrical Locus.—A figure all the points of which have alone a common property.

Axioms.—We can draw one and only one st. line between two points.

Any st. line is shorter than any other line having the same extremities.

Two equal figs. can coincide by superposition.

Two figs. which coincide by superposition are equal.

Equal angles have equal complements and supplements.

Angles having equal complements or supplements are equal.

Theorems.—One and only one perpendicular can be erected on a line at a given point of the same.

One and only one perpendicular can be drawn to a line through a given point without the line. That perpendicular measures the distace from the point to the line.

All right angles are equal.

Two adjacent angles, the exterior sides of which form a st. line are supplementary—and conversely.

The sum of all the angles formed by any number of lines drawn from a given point in a plane is 4 right angles or 360°.

Angles opposite through the vertex are equal.

Any convex polygonal line is smaller than any enveloping line having the same extremities.

The bisectrices of adjacent supplementary angles are perpendicular to each other.

$$
\begin{aligned}
&\angle a \text{ being one angle} \\
\text{and } &\underline{2b \quad \text{" the other,}} \\
&2a + 2b = 2(a+b) = 180^\circ, \\
\text{and } &a + b = 90^\circ .
\end{aligned}
$$

Triangle (Fig. 20).—A triangle is a plane figure **ABC** bounded by three st. lines **AB, AC** and **BC** or **c, b** and **d.**

Six Elements of a Triangle.—A triangle has six elements: : 3 sides, **a, b** and **c** and 3 angles, **A, B** and **C,** which may also be read **BAC, ABC** and **ACB.** (The letter at the vertex is read second.)

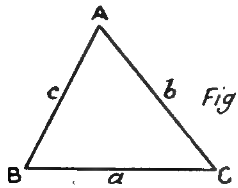


Fig. 20

Theorems.—Any side of a triangle is smaller than the sum of the other two sides:

$$a < b + c \;;\; b < a + c \text{ and } c < a + b$$

In a beam intersecting a line, a perpendicular is shorter than any oblique (Fig. 21):

$$OA < OB \;;\; OA < OC \;;\; OA < OD \;;\; OA < OE .$$

Two obliques are equal if they meet the line at equal distances from the foot of the perpendicular, and conversely. If **AB=AC,** then **OB=OC.**

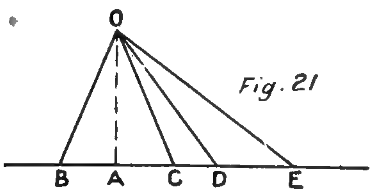


Fig. 21

The longer the distance the point of intersection is from the foot of the perpendicular, the longer the oblique is, and conversely.

$$\text{If } AE > AD , \text{ then } OE > OD .$$

The shortest distance from a point to a line is the perpendicular.

Only two (2) equal lines can be drawn from a point to a line.

All the points of a perp. erected on the middle of a line (Fig. 22) are equally distant from the extremities of the line: **NB= NC**; **MB =MC.**

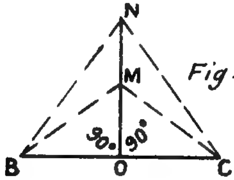


Fig. 22

All the points equally distant from the extremities of a line are on the perp. erected on the middle of said line, which last two theorems establish the following:

The geometrical locus of all the points of a plane equally distant from two points in that plane is the perp. erected on the middle of the line connecting the two points.

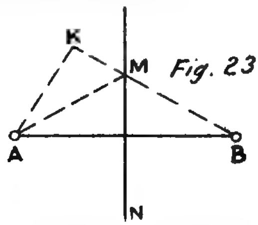


Fig. 23

All the pts. without the perp. erected on the middle of a line (Fig. 23) are unequally dist. from the extremities of the line.

$$KA < KM + MA$$
$$KA < KM + MB$$
$$KA < KB.$$

Perimeter of a Triangle.—The sum of its 3 sides. The perimeter is generally represented by 2 **p**, the sides by **a, b, c,** the angles opposite to the sides by **A, B, C.**

We have $\quad 2p = a + b + c$

and $\quad p = \dfrac{a+b+c}{2}$.

Right angled Triangle (Fig. 24).—One that contains a right angle **A.**

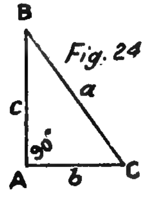


Fig. 24

Hypothenuse.—The side **a** opposite the right angle of a right angled triangle.

Acute angled Triangle (Fig. 25).—One **ABC** the three angles **A-B-C** of which are acute.

Obtuse angled Triangle (Fig. 26).—One which contains an angle greater than a right angle.

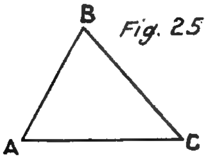


Fig. 25

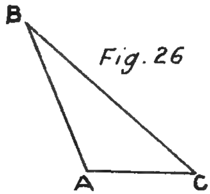


Fig. 26

Fig. 27

Isosceles Triangle (Fig. 27).—One which contains two equal sides The 3rd. side is the base.

Equilateral Triangle (Fig. 28).—One the three sides of which are equal.

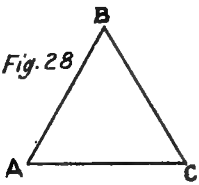


Fig. 28

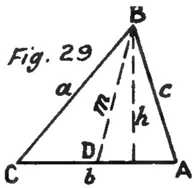


Fig. 29

Base of a Triangle (Fig. 29).—The side **b** on which the triangle seems to be resting.

Vertex of a Triangle.—Point of intersection **B** of the other 2 sides **a-c.**

Height of a Triangle.—The perp. **h** drawn from the vertex **B** to the base **b.**

Median.—A line **m** drawn from a vertex **B** to the middle of the opposite side **b.**

In any triangle there may be 3 different altitudes if we consider in rotation each side as the base; 3 medians and 3 bisectrices.

Cases of Equality of Triangles.—(Fig. 30).

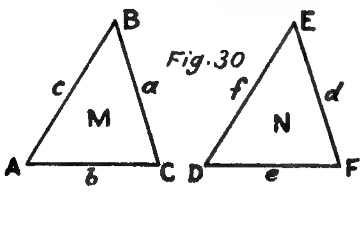


Fig. 30

1°. When they have one equal side **b=e** adjacent to 2 equal angles **A=D; C=F.** If the equal angles are reversely placed the triangles are symmetrical.

2° When they have an equal angle **A=D** formed with 2 equal sides **b=e; c=f.** If the equal sides are reversely placed the triangles are symmetrical.

3° When their 3 sides are equal **a=d; c=f; b=e.** If the equal sides are reversely placed the triangles are symmetrical.

Equality of Right Angled Triangles (Fig. 31).—1°. When the hypothenuse and one side are equal **a=d; b=e.** They may be symmetrical.

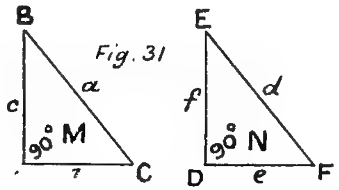


Fig. 31

2°. When the hypothenuse and one angle are equal **a=d; B=E.** They may be symmetrical.

Theorems.—In two triangles (Fig. 32) with two sides respectively equal **a=d; c=f**, but forming unequal angles **E > B** the third sides are unequal and the one opposite the greater angle is the greater side **e > b** and conversely.

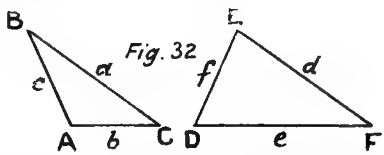


Fig. 32

In two triangles with two sides respectively equal **a=d; c=f**, but having the 3rd. sides unequal **e > b** the angles which the equal sides form are unequal and the greater is opposite the greater 3rd. side; **E > B.**

In an isosceles triangle (Fig. 33) the angles opposite the equal sides are equal **B=C**; conversely.

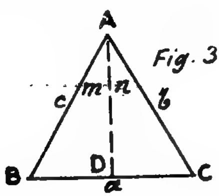


Fig. 33

When 2 angles of a triangle are equal the sides opposite the equal angles are equal $b=c$ and the triangle is isosceles.

In an isosceles triangle the line AD joining the vertex to the middle of the base BC bisects the angle: $m=n$.

In an isosceles triangle the perp. drawn from the vertex bisects the angle: $m=n$ and the base: $BD=DC$.

In an isosceles triangle the perp. erected on the middle of the base passes through the vertex and bisects the angle: $m=n$.

All the angles of an equilateral triangle (Fig. 34) are equal and that fig. is equiangular; $A=B=C$.

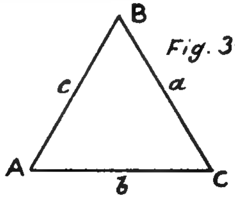


Fig. 34

In any triangle (Fig. 35) the greatest side is opposite the greatest angle and conversely;

if $a > b > c$,
then $A > B > C$

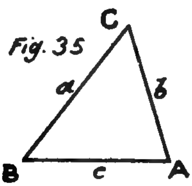


Fig. 35

Any point of the bisectrix of an angle (Fig. 36) is equidistant from the sides $p=q$; conversely.

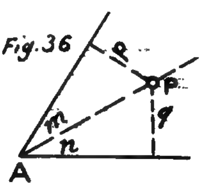


Fig. 36

Any point equally distant from the sides of an angle belongs to the bisectrix; therefore:

The locus of the points of a plane equally distant from the sides of an angle in that plane is the bisectrix of the angle.

Parallels (Fig. 37).—Lines in a plane **A—B—C—D** which are everywhere the same dist. apart. They will therefore never meet however far they may be supposed extended.

Fig. 37

Postulatum.—Through a point without a line only one parallel to that line can be drawn.

Theorems.—Two lines perp. to a 3rd. line (Fig. 38) are parallel.

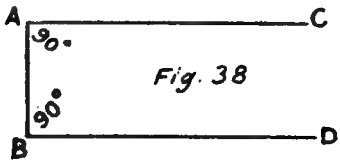


Fig. 38

Any perp. to a line (Fig. 39) is perp. to all the parallels to the line.

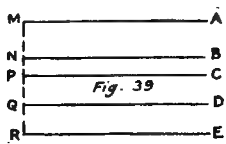


Fig. 39

The geometrical locus of all the points of a plane equally dist. from a str. line is 2 parallels, one on each side of the str. line.

Secant (Fig. 40).—A line **A** cutting one or more others.

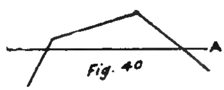


Fig. 40

Angles formed by two parallels and a Secant (Fig. 41).—There are 8 angles, 4 at each point of intersection.

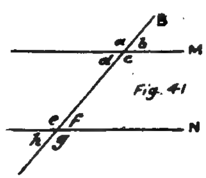


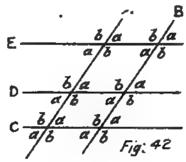
Fig. 41

Corresponding Angles.—Are on the same side of the secant and of the parallels—4 sets: **a=e; d=h; c=g; b=f.**

Alternate Internal Angles.—Within the parallels and one on each side of the secant—2 sets; **d=f; c=e.**

Alternate External Angles.—Without the parallels and one on each side of the secant—2 sets; **a**=**g**; **b**=**h**.

Theorems.—In a set of parallels (Fig. 42) the corresponding angles are equal.



The alternate internal angles are equal.

The alternate external angles are equal.

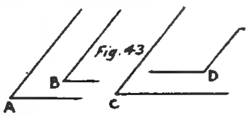
Internal angles on the same side of the secant are supplementary.

External angles on the same side of the secant are supplementary.

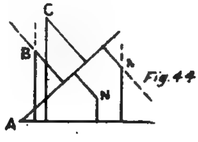
The converses of the above 5 theorems are true.

In sets of intersecting parallels the segments (portions) of parallels comprised between parallels are equal.

Two angles the sides of which are parallel (Fig. 43) are equal **A**=**B** or supplementary; **C**+**D**=180°.



The angle formed by two prep. issuing from one point to the 2 sides of an angle (Fig. 44) are equal **B**=**C**; **M**=**N** or supplementary **C**+**M**=180° **B**+**N**=180°.



Polygon—Plane figure bounded by straight lines.

Triangle.—Polygon of 3 sides.

Quadrilateral.—Polygon of 4 sides.

Pentagon.—Polygon of 5 sides.

Hexagon.—Polygon of 6 sides.

Octagon.—Polygon of 8 sides.

Decagon.—Polygon of 10 sides.

Triangle:
- Right angled. *One right angle.*
- Equilateral. *Three equal sides.*
- Isosceles. *Two equal sides.*
- Scalene. *Three unequal sides.*

Quadrilateral:
- Parallelogram. *Opposite sides parallel.*
  - Square. *4 sides and 4 angles equal.*
  - Rectangle. *4 angles equal.*
  - Rhomboid. *4 sides equal.*
- Trapezoid. *Two opposite sides parallel and unequal.*
- Trapezium. *4 sides and angles unequal.*

Inscribed Polygon.—One which has all its vertices on a certain figure.

Circumscribed Figure.—One which has an inscribed one within itself.

Equiangular Polygon.—Has all its angles equals; $A=B=C=D=\ldots=M$.

Equilateral Polygon.—Has all its sides equal; $a=b=c=\ldots=m$.

Regular Polygon.—Is both equilateral and equiangular. $A=B=C=\ldots=M$; $a=b=c=\ldots=m$.

Diagonal.—A line connecting 2 vertices not adjacent; AC, AD, etc.

Number of Diagonals.—A vertex may be connected to all the vertices except itself and the 2 adjacent ones, and $n-3$ is the number that can issue from it. There will be as many issuing from the other vertices so that $\frac{1}{2}n(n-3)$ is the total number of diagonals in a polygon. We divided by 2 because in the above analysis every diagonal was counted twice an issuing from either of its extremities.

External Angle.—The angle formed by one side with the prolongation of the next one.—Each external angle is the supplement of the adjacent angle in the polygon.

Theorems.—The sum of the 3 angles of a triangle (Fig. 45) is equal to 2 right angles, or 180°; $A+B+C=180°$.

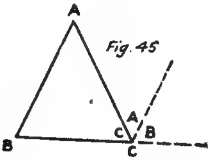


Fig. 45

Any external angle of a triangle equals the sum of the other two opposite angles as shown in the fig.

If 2 triangles have 2 angles equal each to each, the 3rd. angles also are equal;

$$\begin{array}{l} A = A' \\ B = B' \\ \hline A+B = A'+B' \end{array} \quad \text{but} \quad \begin{cases} A+B+C = 180° \\ A'+B'+C' = 180° \end{cases} \quad \begin{cases} C = 180° - (A+B) \\ C' = 180° - (A'+B') \\ C = C' \end{cases}$$

In a right angled triangle (Fig. 46) the acute angles are complementary.

$$\begin{array}{rl} A+B+C= & 180^\circ \\ A\quad = & 90^\circ \\ \hline B+C= & 90^\circ \end{array}$$

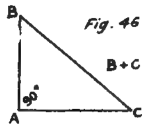


Fig. 46

The sum of all the angles of a polygon equals as many times 2 right angles as there are sides less two; because it may be decomposed by diagonals into that number of triangles.

$\therefore S = 180^\circ(n-2)$. If the polygon is regular, each angle $A = \frac{180(n-2)}{n}$.

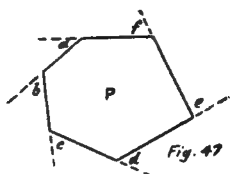


Fig. 47

The sum of the external angles of a polygon (Fig. 47) equals 4 right angles.

If S_1 is the sum of all the external angles, we have $S+S_1 = 180n$; $S = 180(n-2) = 180n - 360$, so that $S_1 = 180n - (180n - 360) = 360^\circ$.

The opposite sides (and angles) of a parallelogram (Fig. 48) are equal, a=c and b=d and conversely.

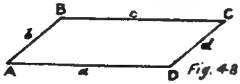


Fig. 48

The adjacent angles of a parallelogram are supplementary. We have

$A+B+C+D=360^\circ$; but $A=C$ and $B=D$;

$2A\ \ 2B=360^\circ$ and $A+B=180^\circ$.

$2B+2C=360^\circ$ and $B+C=180^\circ$.

$2A+2D=360^\circ$ and $A+D=180^\circ$.

$2B+2D=360^\circ$ and $B+D=180^\circ$.

A quadrilateral having two opposite sides parallel and equal is a parallelogram.

The diagonals of a parallelogram (Fig. 49) bisect each other **AO=OC**; **BO=OD**, and conversely: whence it follows that.

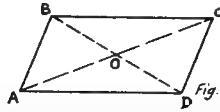


Fig. 49

The diagonals of a square (Fig. 50) bisect each other **AO=OC**; **BO=OD**, are perpendicular to each other and are equal **AC=BD**; those of a rectangle are equal; those of a rhomb are perpend. to each other.

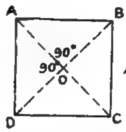


Fig. 50

Circumference (Fig. 51).—A line in a plane all the points of which are equally distant from a given point **O** in the plane. It may also be defined as the locus of the points of a plane equally distant from a point O of that plane.

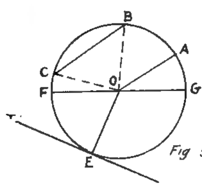


Fig 51

Arc.—A portion **BC** of a circle.

Circle.—That portion of a plane bounded by a circumference— For the sake of brevity the term circle is often used for cirf. when no mistake can result.

Center.—The given point **O** from which those of the circumference are equally distant.—The term center is also used for middle point of any line.

Radius.—The constant distance **r** from any pt. of the cirf. **A** to the center **O.** Therefore all the radii are equal.

Diameter.—A line **FG** passing through the center and terminating on both sides to the circumference; it equals 2 radii· **d=2r**. Therefore all diameters are equal.

Chord.—A line **BC** connecting any two pts. **B—C** of a cirf.

A line joining the extremities of an arc.

A diam. is a chord passing through the center.

Tangent.—A line **T** having only one pt. **E** in common with the cirf. It is also defined: The limit of the successive positions assumed by a secant slowly revolving about one of its points of intersection until the second point of intersection coincides with the first. —It may also be considered as the limit of the successive positions assumed by a secant moving away from the center while remaining parallel to its former position until the 2 pts. of intersection coincide.

Point of Contact.—The pt. **E** common to an arc and its tangent.

Normal.—A perpend. **OE** to a tangent at the pt. of contact.

Concentric Circumferences (Fig. 52).—Those **A—B** having same center **O.**

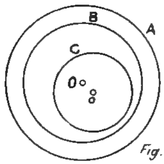


Fig. 52

Excentric Cirfs.—Those **A—C** having different centers **O—o.**

Central Angle (Fig. 53).—An angle **a** formed by any two radii **ON—OP**—The vertex is at the center **O.**

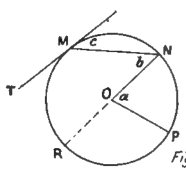


Fig 53

Inscribed Angle.—An angle **MNO** having its vertex **N** on the cirf. and the sides of which are chords; **NM—NR.**

Segment Angle.—An angle **NMT** having its vertex **M** on the cirf. and the sides of which are a chord **MN** and a tangent **MT.**

Theorems.—The cirf. is a convex line.

No three points of a cirf. are on a straight line; therefore a cirf. is a curve.

A diam. **AB** (Fig. 54) divides the cirf. into 2 equal parts; **ACB= ADB.**

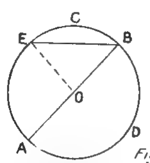


Fig 54

In a circle any chord is smaller than the diam.

$BE < BO + EO$; $BE < 2r$!

The shortest dist. from a point **A** to a cirf. (Fig. 55) is that portion **AB** of the radius **OB** passing through that point **A** contained betw. the point **A** and the cirf.—If the point be exterior, **M** it is that portion **MN** of the prolongation of the radius between the point and the cirf. From which it follows that (dist.=**d**).

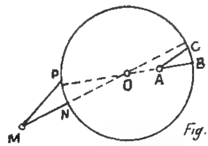


Fig. 55

The locus of all the points equidistant from a cirf. is a system of two concentric cirf. with radii such as

$$R = r + d \quad \textit{and} \quad R' = r - d$$

In a circle (or in equal circles) (Fig. 56).

Equal central angles intercept equal arcs and conversely;

$$\text{if } m = n \text{, then } AMB = CND;$$
$$\text{if } m > p \text{, then } AMB > BRC.$$

A greater central angle intercepts a greater arc and conversely;

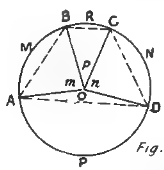


Fig. 56

Equal arcs have equal chords; if **AMB=CND,** then **AB=CD.** A greater arc has a greater chord and conversely;

$$\text{if } AMB > BRC \text{, then } AB > BC.$$

A radius perpend. to a chord (Fig. 57) bisects the chord and the arc; **AD=DB**; **AC=CB.**

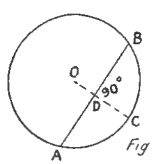


Fig 57

A perpend. bisecting a chord bisects the arc and passes through the center; if **AD=DB** then **AC=CB.**

The line connecting the middle of the arc and the middle of the chord is perp. to the chord and passes through the center. It follows that—

The locus of the centers of all the arcs which can pass through 2 points is the perp. erected on the middle of the line connecting the 2 points.

Through 3 given points not in a line (Fig. 58) one circumference and one only can be drawn. It follows that:

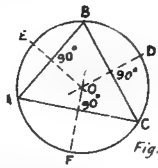


Fig. 58

The perpendiculars bisecting the 3 sides of a triangle pass through the same point which is the center of the circumscribed circle.

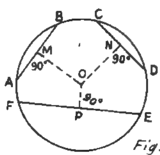


Fig. 59

In the same or in equal circles (Fig. 59) equal chords are equally distant from the center and conversely; if $AB = CD$, then $OM = ON$.

A greater chord is nearer the center;

if $FE > AB$, then $OF < OM$

and conversely. The diam. is the greatest chord.

A line perp. to the extremity of a radius (Fig. 60) is tangent to the cirf., and conversely.

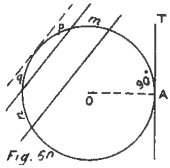


Fig. 60

Two parallels intercept equal arcs: $m = n$.

Two cirf. (Fig. 61) having a common point without the line of cen-

ters, have another such point symmetrical to the first with regard to that line; if M is common so is N.

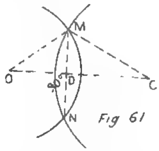


Fig 61

Measurement of Angles.—Ratio of an angle to the angle unit. $\frac{A}{a}$

The unit a is generally an angle on 1°, sometimes a right angle.

Measurement of Arcs.—About the center of a circle 4 right angles only can be drawn which divide the cirf. into 4 equal parts called quadrants; or 360 equal central angles of 1° each can be drawn which divide the cirf. into 360 equal parts. If an arc contains n of these equal parts we say that its measures is n°. This is not a length measure, it is to be understood as the measure of the central angle of that arc. So we say that the measure of the cirf. is 360° because the central angle becomes 4 right angles.—All the circumferences whatever their radii have the same measure of 360°.

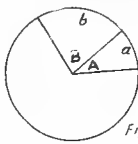


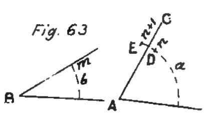
Fig. 62

Theorems.—Any two angles (Fig. 62) are to each other as the arcs intercepted by their sides if drawn with equal radii from their vertices as centers.

The ordinary demonstration consists in supposing a common measure between the arcs a and b which are then in the ratio $\frac{n}{m}$ $\frac{a}{b} = \frac{n}{m}$. By connecting the points of division with the center, n and m equal central angles are respectively formed in A and B, so that $\frac{A}{B} = \frac{n}{m}$; therefore $\frac{A}{B} = \frac{a}{b}$.

But the case of the arcs having no common measure is differently treated by authors. One theory is that there exists between the arcs an infinitely small common measure.

Fig. 63



We give here a demonstration applicable to all cases. (Fig. 63).

Divide arc b into any number of equal parts (which is always theoretically possible) and lay one of them on the arc a as many times as possible. Two cases may arise: 1°, one of these parts is exactly contained in arc a then the small arc thus exactly contained in b and a is a common measure of the two and the case is treated as above.

2° The point C will fall between the n^{th} and the $(n+1)^{th}$ division so that (1) $\frac{n}{m} < \frac{a}{b} < \frac{n+1}{m}$. Notice that the denominators are equal and the numerators are increasing.

Now suppose all the points of division joined to the vertices A and B. Angle A will contain more than n equal angles and less than $n+1$, and B contains m equal angles, so that

(2) $\frac{n}{m} < \frac{A}{B} < \frac{n+1}{m}$. The extreme fractions $\frac{n}{m}$ and $\frac{n+1}{m}$ differ by $\frac{1}{m}$ which quantity tends toward zero when m increases indefinitely. The denominator is always equal to b; the numerator n increases while being always less than a and the numerator $n+1$ diminishes while being always greater than a. Hence $\frac{a}{b}$ and $\frac{A}{B}$ are the limits of the converging fractions $\frac{n}{m}$ and $\frac{n+1}{m}$; they are therefore equal $\frac{a}{b}$ $\frac{A}{B}$ or $\frac{A}{B} = \frac{a}{b}$.

The measure of an inscribed angle (Fig. 64) is one half that of the arc it comprises m=½AMB; very generally expressed as: The measure of an inscribed angle is one half the arc it comprises.

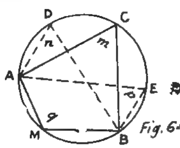


Fig. 64

All angles inscribed in the same segment are equal; m=n.

An angle inscribed in a semi-circle (Fig. 65) is a right angle m=90°.

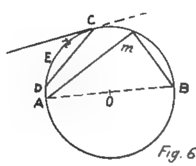


Fig. 65

Angles inscribed in opposite segments (segments on each side of the same chord) are supplementary; p+q=180°.

The measure of a segment angle is one half that of the arc included between its sides. n=½DEC.

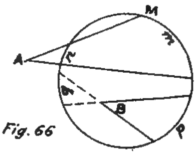


Fig. 66

The measure of an angle whose vertex is within the cirf., but not at the center (Fig. 66) is half the sum of the arcs comprised one between its sides and the other between their prolongations.

$$B = \frac{p+q}{2}.$$

The measure of an angle the sides of which are 2 secants is the semi-difference of the arcs they comprise;

$$A = \frac{m-n}{2}.$$

The opposite angles of an inscribed quadrilateral (Fig. 67) are supplementary and conversely; $A+C=180°$; $B+D=180°$.

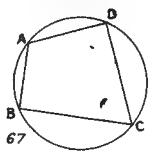


Fig 67

When a cirf. is divided into any number of equal parts (Fig. 68), the polygon **ABCDE** formed by joining in sequence the points of division is an inscribed regular polygon, and

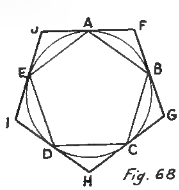


Fig. 68

The polygon **FGHIJ** formed by tangents at each point of division is a circumscribed regular polygon.—Conversely.

A circle can be circumscribed to or inscribed in, a regular polygon.

Center of a Regular Polygon (Fig. 69).—Common center O of the inscribed and circumscribed circles.

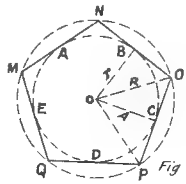


Fig 69

Radius of a Regular Polygon.—Radius R of the circumscribed circle, or distance from its center to any one vertex.

Apothem of a Regular Polygon.—Radius r of the inscribed circle, or perpendicular from its center to any one side.

Remarks.—The radii divide a regular polygon into equal isosceles triangles.

The radii bisect the angles of the polygon and the angles formed by the apothems.

The apothems bisect the angles formed by the radii.

Proportional Lines (Fig. 70).—Lines which, compared two and two give equal ratios; or again: Lines the measures of which are proportional.

Fig. 70

If a, b, c and d are the measures of A, B, C and D, then $\frac{A}{B} = \frac{C}{D}$ if $\frac{a}{b} = \frac{c}{d}$.

Theorems.—In a triangle (Fig. 71), a parallel to one of the sides divides the other two sides proportionally;

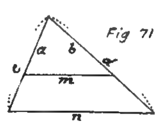


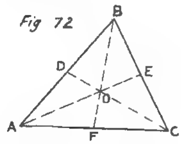
Fig 71

$$\frac{a}{c} = \frac{b}{d} = \frac{m}{n} \text{ and conversely.}$$

A line connecting the middle of two sides of a triangle is parallel to the third side and equals one half of it;

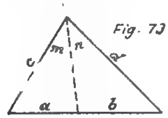
$$\text{if } a = \frac{c}{2} \text{ then } m = \frac{n}{2}$$

The medians of a triangle (Fig. 72) intersect at a common point which is 2-3 of each from its corresponding vertex;



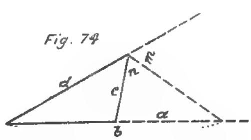
$OB=\frac{2}{3}BF$; $AO=\frac{2}{3}AE$; $CO=\frac{2}{3}CD$.

A bisectrix of a triangle (Fig. 73) divides the opposite side in portions proportional to the other two sides;



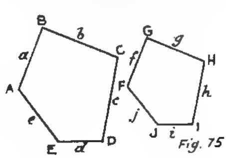
$\frac{a}{b}=\frac{c}{d}$ and conversely.

The bisectrix of an exterior angle of a triangle (Fig. 74) determines on the opposite side produced a point such that its distances from the extremities of that side are proportional to the other two sides;



$\therefore$ if $m=n$, then $\frac{a}{b}=\frac{c}{d}$.

Similar Polygons (Fig. 75).—Their angles are equal and similarly placed **A=F**; **B=G**, etc., and the homologous sides are proportional;



$\frac{a}{f}=\frac{b}{g}=$ etc.

Homologous sides are adjacent to the equal angles.

Ratio of Similitude.—Is the constant ratio of the homologous sides;

$$\frac{a}{f} = r$$

Theorems.—Two triangles (Fig. 76) are similar:

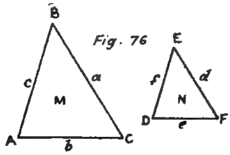


Fig. 76

1°. When they have two equal angles.

2°. When they have one equal angle between proportional sides.

3°. When their three sides are proportional.

Two triangles whose sides are perpendicular (or parallel) to each other (Fig. 77) are similar;

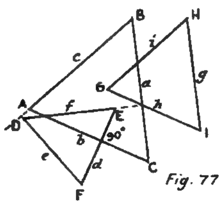


Fig. 77

$$A = D = G \;;\; B = E = H \;;\; C = F = I \text{ and } \frac{a}{d} = \frac{b}{e} = \frac{c}{f} \;;\; \frac{a}{g} = \frac{b}{h} = \frac{c}{i}.$$

Two regular polygons having the same number of sides are proportional and the ratio of their sides equals that of their radii and of their apothems.

$$\frac{a}{m} = \frac{b}{n} = \ldots\ldots = \frac{r}{r'} = \frac{R}{R'}.$$

Value of an Angle of a regular Polygon of n sides.—The sum of all the angles

is $(n-2)180°$ and each angle $A = \frac{(n-2)180°}{n}$.

Theorems.—Any two circles are similar figures.

Two similar polygons can be divided in the same number of similar triangles similarly placed—and conversely.

In two similar polygons all the homologous lines are in the ratio of similitude. This is also applicable to their perimeters.

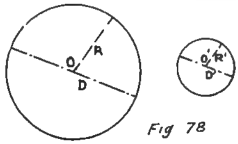


Fig 78

Any two circumferences (Fig. 78) are to each other as their radii.

$$\frac{C}{C'} = \frac{R}{R'} = \frac{2R}{2R'} = \frac{D}{D'}$$ from which $\frac{C}{C'} = \frac{D}{D'}$ and $\frac{C}{D} = \frac{C'}{D'}$, which shows that

The Number π.—The ratio of a circumference to its diameter is constant.

This ratio is represented by π and $\pi = 3.14159265+$, but is generally taken as $\pi = 3.1416$.

From $\frac{C}{D}$ $\frac{C}{2R} = \pi$ we derive $C = 2\pi R$ or $C = \pi D$.

Length of an Arc of n Degrees.—

An arc of one degree $= \frac{2\pi R}{360°} = \frac{\pi R}{180°}$ so that $l = \frac{\pi R n°}{180°}$; if reduced to minutes, $l = \frac{\pi R n'}{10\,800}$; if reduced to seconds, $l = \frac{\pi R n''}{648\,000}$.

If we call N the ratio of the arc to the circumference or $\frac{N}{360°}$, then $l = 2\pi R N$ and if we suppose, as is often done, $R = 1$,

$$l = 2\pi N.$$

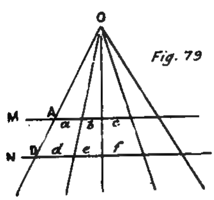


Fig. 79

Theorem.—If a beam is intersected by parallels (Fig. 79), these are cut proportionally.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}.$$

Sum, Difference, Product, Quotient or Ratio of Lines.—Are the sum, the difference, the product, the quotient of the numbers which express their lengths.

Square of a Line.—Square of the number measuring the line.

The known formulas
$$(a+b)^2 = a^2 + 2ab + b^2;$$
$$(a-b)^2 = a^2 - 2ab + b^2.$$
$$(a+b)(a-b) = a^2 - b^2.$$

in which **a** and **b** represent lines may be expressed thus:

1°. The square of the sum of two lines (Fig. 80) equals the sum of the squares of the two lines, plus twice their product:

$$AB - (a+b)^2 = a^2 + b^2 + ab + ab = a^2 + 2ab + b^2.$$

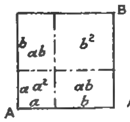


Fig 80

2°. The square of the difference of two lines (Fig. 81) equals the sum of the squares of the two lines minus twice their product:

$$AB = (a-b)^2 = a^2 + b^2 - ab - ab = a^2 - 2ab + b^2$$

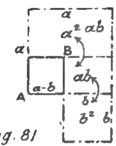


Fig. 81

3°. The product of the sum of two lines by their difference (Fig. 82) equals the difference of their squares.

$$AB = (a+b)(a-b) = m+n = m+n+p-(n+o) = m+n+p-n-o$$
$$AB = (m+p)-o = a^2 - b^2$$

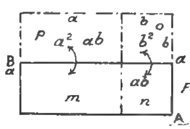


Fig. 82

In any right angled triangle, (Fig. 83).

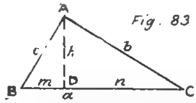


Fig. 83

1°. Each side b is a mean proportional between its projection n on the hypothenuse and the hypothenuse itself;

$$b^2 = an \; ; \; c^2 = am.$$

2°. The perpendicular is a mean proportional between the segments into which it divides the hypothenuse;

$$h^2 = mn.$$

3°. The square of the hypothenuse equals the sum of the squares of the other two sides:

$$a^2 = b^2 + c^2$$

In any triangle, the square of a side a opposite an obtuse (or acute) angle (Fig. 84), equals the sum of the squares of the other 2

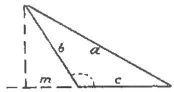
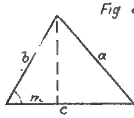


Fig 84



sides b and c plus (or minus) twice the product of one of them c by the projection m upon it of the first side:

$$a^2 = b^2 + c^2 + 2nb \; ;$$
$$(a^2 = b^2 + c^2 - 2nb)$$

$$\begin{cases} a^2 = b^2 + c^2 + 2mc \; , \\ (a^2 = b^2 + c^2 - 2mc) \; ; \end{cases}$$

In a circle (Fig. 85).—

1°. A perpendicular m to the diameter 2R is a mean proportional between the segments a and 2R—a of the diameter:

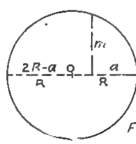


Fig 85

$$m^2 = a(2R-a).$$

2°. The product of the segments a, b of a chord intersected by another chord (Fig. 86) equals the product of the segments c, d of the other chord: ab=cd.

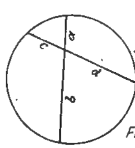


Fig 86

3°. When two secants S and s issue from the same point A (Fig. 87), the products of each by its external portion M—m are equal: SM=sm.

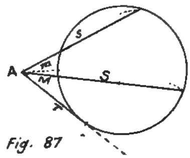


Fig. 87

4°. When a secant S and a tangent T issue from the same point A, the tangent is a mean proportional between the secant S and its external portion M:

$$T^2 = SM.$$

The side a of the regular hexagon (Fig 88) equals the radius: a=R.

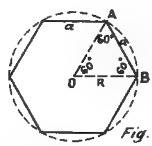


Fig. 88

The side a of the regular decagon equals the greater segment of the radius R divided into mean and extreme ratio:

$$a = \frac{R}{2}(\sqrt{5}-1).$$

Knowing the radius R and apothem A of a regular polygon (Fig. 89), we can calculate the radius r and apothem a of another regular polygon isometrical with the first (having same perimeter) and with a number of sides double.

We have : $a = \frac{R+A}{2}$ (1) and $r = \sqrt{aR}$ (2).
$a > A$ because $2a = R + A > 2A$ and $r < R$ because $r^2 = aR < R^2$;

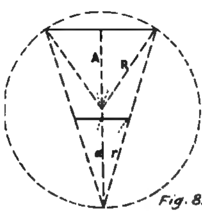


Fig. 89

Therefore the new apothems increase and the new radii diminish while these last are always greater than their corresponding apothems. These quantities therefore tend towards a limit which is the radius of the isometrical circle.

Calculation of π (Schwab's Method).—Starting from a square the side of which is one and the perimeter 4, we can calculate its apothem and radius and from these calculate the apothem and radius of the octagon of same perimeter 4, then continuing the same process, successively the apothems and radii of polygons of 16, 32, 64, etc., sides, all of perimeter 4. The 12th. polygon would have 8192 sides and its apothem and radius calculated to 7 places of decimals would be

$$A_{12} = 0.636\,6196 \quad \text{and} \quad R_{12} = 0.636\,6196$$

either of which can be taken for an approximate value of the radius of circumference 4:

$$\text{from } C = 2\pi R \text{ we get } \pi = \frac{C}{2R} = \frac{4}{2\times 0.636\,6196} = \frac{2}{0.636\,6196}, \text{ finally}$$

$$\pi = 3.141593\ldots$$

AREAS.

Unit of Area.—The unit of area is the square constructed on the unit of length.

Equal Surfaces.—They have same form and will coincide if placed one on top of the other.

Equivalent Surfaces.—Have different form but equal area.

Area of Square (Fig. 90).—Square of the side s:

$$A = s^2$$

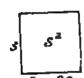


Fig. 90

Area of Rectangle (Fig. 91).—Product of the two sides **a**, and **b**;

$$A = ab.$$

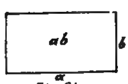


Fig. 91

Area of Parallelogram (Fig. 92).—Product of base **b** by height **h**,

$$A = bh.$$

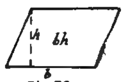


Fig. 92

Area of Triangle (Fig. 93).—

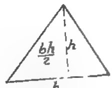
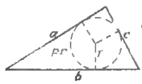


Fig. 93

1°. Semi-product of base b by height **h**:

$$A = \frac{bh}{2}$$

Area of Triangle (Fig. 94).—



a+b+c = 2p

Fig. 94

2° Product of the semi-perimeter **p** by radius **r** of inscribed circle:

$A = \frac{1}{2}ra + \frac{1}{2}rb + \frac{1}{2}rc = \frac{1}{2}r(a+b+c)$; $A = pr$.

3°. Square root of the product of the semi-perimeter **p** by the differences between that semi-perimeter and the three sides **a, b, c.**:

$$A = \sqrt{p(p-a)(p-b)(p-c)}.$$

Area of Circumscribed Polygon.—
Semi-product of perimeter 2p by radius r of inscribed circle.

$$A = pr.$$

Area of Trapezoid (Fig. 95).—Semi-product of sum of bases **a** and **b** by height **h**:

$$A = \frac{(a+b)h}{2}$$

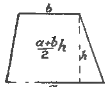


Fig. 95

Area of Regular Polygon (Fig. 96).—Semi-product of perimeter **2p** by apothem **a**:

$$A = pa$$

ns=2p

Fig. 96

Area of Circle (Fig. 97).—Semi-product of circumference by Radius:

$$\frac{C \times R}{2} = \frac{2\pi R \times R}{2};\quad A = \frac{CR}{2} = \pi R^2$$

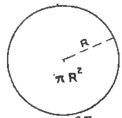


Fig. 97

Area of Sector (Fig. 98).—Semi-product of arc

$$l = \frac{\pi R n}{180} \text{ by radius } R: \begin{cases} A = \frac{lR}{2} \\ A = \frac{\pi R^2 n}{360}. \end{cases}$$

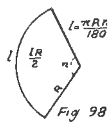


Fig 98

Area of Segment (Fig. 99).—Difference between sector **OACB** and triangle **OAB**.

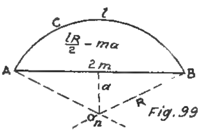


Fig. 99

TECHNICAL RECUIREMENTS OR SURVEYING.

INSTRUMENTS, THEIR USE AND CARE.

Chain.—A number of equal lengths of wire called links, bent at each end to form an eye and united by rings. The end links are shorter, fitted with a handle oval in shape and their length, added to that of the handle, equals the length of the other links.

There are two kinds of chains.

Gunter's Chain.—From the name of its inventor is also called **surveyor's chain.** Its length is 66 ft. and 80 chain lengths measure a mile. **66×80=5280.**—It is divided into 100 links (one link therefore =7.92 inches). Every tenth link carries a brass tag with 1, 2, 3 or 4 dents according to its distance from the end of the chain. The center has a circular tag.

Engineer's Chain.—It is either 50 ft. or 100 ft. long. The links are a foot long and every tenth foot is marked like the Gunter's chain.

Steel Tape.—A steel ribbon 1-2 to 1-8 inch wide with a fixed or detachable handle or ring forming or not part of the length which is 50 ft., 100 ft., sometimes several hundred feet. The wide kinds are divided by etchings, into feet, tenths and hundredths; or feet, inches and eighths, on one or the two faces. The narrow kinds are generally graduated every 25 ft.

Steel Band.—A little wider than the tape. The graduation in feet is by brass rivets. Every 5th foot is marked by a sleeve. The end foot is divided into tenths.

Often, when the handle is not a part of the length, the ends or 0 and 100 ft. graduations are marked by brass shoulders set in the same direction. The end graduations are sometimes indicated by a circular hole just large enough for the insertion of a chaining pin.

Other Form of Handle.—Another form of handle is semi-cylindrical and hollow with an interior diameter equal to that of a chaining pin.

A Cause of Error in Chaining, Due to Handles Forming Part of the Length (Fig. 100).—When using such a chain or tape, three methods are followed:

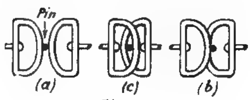


Fig. 100

1°. The head chainman sets the pin against the exterior of the handle and the rear chainman holds the exterior of the handle against the pin (a). In this case, the thickness of the pin is not measured and the chaining will be short.

2°. The head man sets the pin against the exterior of the handle and the rear man holds the interior of the handle against the pin (b). In this case the thickness of the handle is measured twice and the chaining will be long.

3°. Both chainmen hold the handle or set the pin so that the pin is against the inside face of the handle (c) In this case the thickness of the pin and of the handles are measured twice and the chaining will be longer yet.

In accurate chaining, allowance should be made for that cause of error.

The second method is generally followed.

Chaining Pins.—Generally 11 in a set so that when one is in the ground, the total carried by both chainman is always 10. They are made of steel wire (W. G. 6 to 11) sharpened at the end to facilitate their setting, the other end bent into an eye to facilitate transportation and handling; their length is 12 to 14 inches. They are carried in a ring of spring steel wire.

How to Chain a Line—Detecting Kinks.—The line must have been previously ranged with poles. At the starting point, the head chainman holding one handle in his hand throws the chain back, sets a pin at the beginning of the line and walks briskly ahead along the line while the rear chainman gets hold of the chain and lets it slip between his hands to detect any kink which he straightens.

Calling Halt.—The rear man calls "halt" a little before the chain is fully extended, at which moment the head man turns, shakes the chain to make sure it is straight and pulls it gently, holding it in a horizontal direction, either to the right or the left according to the signs of the rear man. When the direction is correct, the rear man slips the handle over the pin with one hand and steadies the pin with the other while the head man sets a pin against the handle which he holds, at the command "stick" from the rear man.

Pulling Up the Rear Pin.—His answer "stuck" is a signal to the rear man to pull the pin at his end of the line. Both then proceed along the line marking every chain length in the same way, carrying the chain, not dragging it.

A Tally.—After setting his ten pins the head man calls "tally"; the rear man walks towards him counting his pins (there must be ten); he returns them to the head man who puts them on the ring, counting them in turn. The rear man makes an entry in his book for 10 chains and resumes his position.

Reading the Chain.—When the end of the line is reached, the rear man steps up and reads the feet and fractions, which, added to the number of chains is the length of the line, and this he enters in his field book. (The fractions of a foot are read either with the end foot of the chain or a pocket rule when the chain is divided only in feet.)

Entering Distance and Station in Field Book.—Suppose 1739.84 ft. have been measured on the first course from the zero plug. The point reached will be known as Station 17+39.84. A second course 920.61 ft. long will be entered as such and the station of the second point (or angle) will be (1739.84+920.61)=sta. 26+60.45, and so on.

Why the Chaining is Made Horizontally.—1°. Because in land surveying it is admitted that a field on a slope does not produce more and therefore is not worth more than a level field of same nature the area of which equals the area of the horizontal projection of the first.

Productive Base.—This horizontal projection is called the productive base of the field.

2°. In engineering surveying, the constant changes of slopes, in all directions, constitute warped surfaces which it would be impossible to plot on paper; hence it is agreed to measure the lines horizontally as the projections of the real lines on the ground which can always be found in length and slope from the plot, by their measured projections and the differences in elevation of their several points.

How the Chainman Can Steady the Chain.—By resting the elbow of the arm holding the chain against the knee.

Measuring a Line on Sloping Ground.—It is easier to measure down hill than up hill because the rear chainman can steady himself while giving direction. The head chainman transfers the end of the chain to the ground by means of a plumb line. If the slope is too great, the line is measured in sections of 50 ft., 25 ft. or even less. Care must then be taken not to loose the pins or count them as chain lengths. When measuring up-hill, it is the rear chainman who uses the plumb line to transfer the point of the ground to the end of the chain.

Use of a Loaded Pin.—It is dropped from the end of a chain held a considerable distance from the ground, should a pole of a plumb bob not be available. The point where it strikes is noted and an ordinary chaining pin is substituted for it on the right spot.

When the Line is Measured Along the Ground (Fig. 101).—When the slope is great and regular and when the wind is very strong. In

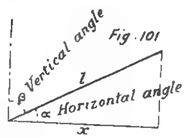


Fig. 101

these cases the angle of the slope is carefully measured and the horizontal distance calculated and recorded as

$x = l \cos \alpha$ if the horizontal angle is measured;
$x = l \sin \beta$ „ „ vertical „ „ .

Chaining Pins Often Dispensed With.—A station stake is set at the end of every chain length (100 ft.). It is driven to within 4 or 5 inches from the ground. The end of the chain is marked in pencil by a line perpendicular to the direction of the line which is being measured. and the exact direction, given by the transitman, is obtained by means of a plumb-line held by the axeman or the chainman who moves it along the pencil mark until the signal "down" is given by the transitman. At that point a brass tack is driven into the head of the stake.

How to Ascertain the Exact Length of a Chain or Tape.—In important work it is necessary to know exactly the length of the chain or tape used.

A guaranteed chain or tape is certified to be of such a length, at such a temperature and with such a pull. (The Bureau of Weights and Measures, U. S. Coast and Geodetic Survey, Washington, D. C., will, for a small fee, compare a tape with the Government standard and issue such a certificate). For example: "U. S. W. & M. 312" is 99.9972 ft. at 62° F. and a pull of 12 lbs.

Necessary Corrections.—1° For expansion, 2° for pull, 3° for sag.

For Expansion: $e = \frac{l-l'}{l(t-t')}$ in which e = coefficient of expansion per degree F.;
l = length of line at temperature t;
l' = " " " " " t';
both observations being made with same pull.

For Pull: $\sigma = \frac{l-l'}{l(p-p')}$ in which σ = coefficient of stretch or pull;
l = length of line with pull p;
l' = " " " " " p';
both observations being made at same temperature

For Sag: $d = \frac{1}{24}\left(\frac{wl}{p}\right)^2$ in which d = difference between measured length l and true length L,
or $d = l - L$;
w = weight of chain per ft.;
p = pull.

Several observations should be taken over a line a little less than 100 ft. long, and with different pulls, correction being made for temperature before applying formula.

Establishing a Standard.—By stretching a guaranteed chain horizontally along a wall, a curb or other permanent structure and making corrections for expansion, pull and sag, an exact length of 100 ft. may be laid and permanently marked; this will be a standard.

Standard Comparisons.—Before starting on a survey, compare the chain with the Standard. After the day's work repeat the operation.

Some Sources of Errors in Measuring Lines.—

1.—The chain or tape not held with a uniform pull.

2.—Plumbing done without care.

3.—Alignment imperfectly established.

4.—Effect of wind not compensated.

5.—Variations in temperature not observed.

6.—Chain or tape not standardized.

Erecting a Perpendicular at a Point of a Line.—

1°. The 3-4-5 **method** (Fig. 102).

Lengths in the proportion of 3, 4, 5, all included in the expressions $3m$, $4m$, $5m$, form a right-angled Δ because $\overline{3m}^2 + \overline{4m}^2 = \overline{5m}^2$; whether $m \gtreqless 1$.

Take on the line a length of 9 ft. (for instance), from given point **A**. From **A** draw arc **m** with a radius equal to 12 ft. From **B** intersect arc **m** with arc **n** of 15 ft. radius. Stake line **AC** the perpend.

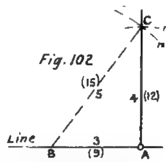


Fig. 102

If a chain is used, it can be fixed to a pin **A**, at zero and 36 ft., held or pinned at **B** on ring marking 9 ft. then tightened at **C** on ring marking 24 ft.

2°. **Isosceles Triangle Method** (Rig. 103).—Set **AB=AC.** Fix chain at **B** on zero and at **C** at an even figure, say 26. Draw the chain tight, holding it in the middle (13) of that distance. **D** will be the point.

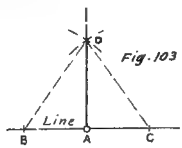


Fig. 103

Or draw from **B** and **C** arcs of equal radii intersecting at **D.** Stake **AD.**

When obstables prevent that construction, the following may be used:

3°. **Semi-circle Method** (Fig. 104).—Assume a convenient point **O.** Chain **OA** and lay-off **OB=OA.** Range **C** on **BO** and take **OC=OB.** Stake **AC.**

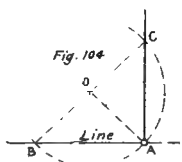


Fig. 104

Or draw from **O** circle **BAC** and range on it **C** on **BO.**

Let Fall a Perpendicular from a Point to a Line.—1°. (Fig. 105).—**Reverse** the **3rd. Method** above given. Draw any line **AB** and bisect **it at O.** Lay off **OC=OA.** Stake **CA.**

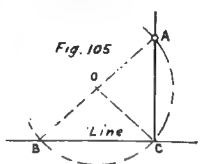
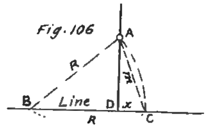


Fig. 105

2°. **By Calculation** (Fig. 106).—Run any line **AB**—Lay off **BC**=**BA**=**R**. Chain **AC**=**m**.

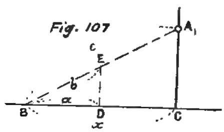
Then $x = \frac{m^2}{2R}$ which lay off, getting D

Stake **DA**.



3°. **By Similar Triangles** (Fig. 107).—Erect on line a perpendicular **DE** shorter than **AC**. Range **B** on **EA** and **DC**. Measure **BD**=**a**; **BE**=**b** and **BA**=**c**.

Then $x = \frac{ac}{b}$ which lay off, getting C.



Stake **CA**.

Case of an Obstruction Between Line and Point (Fig. 108).—Erect a perpendicular **BC** and produce it by an equal length **BD**=**BC**.—Range **E** on **CA** and line. Range **F** on **AD** and line. Range **G** on **CF** and **DE**. Range **H** on **GA** and line. Stake **GHA** as far as needed.

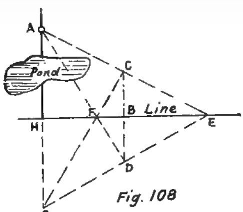


Fig. 108

Run a Parallel to an Alignment through a Given Point (Fig. 109).—1°. Let fall a perpendicular **AB** from **A** to line and chain it. At a point **C** as far distant from **B** as convenient erect a perpendicular **CD** to line making it equal to **BA**. Stake **AD** the parallel.

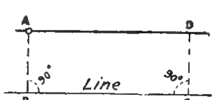


Fig. 109

2°. (Fig. 110). From A as a center describe arc **m**. From **B** as a center describe arc **n**. Chain chord **AC** and lay it off from **B** to **D**. Stake **AD**.

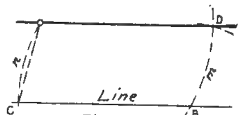


Fig. 110

3°. (Fig. 111).—Let fall a perpendicular **AB** to line. Erect perpendicular **AC** to **AB** and stake it.

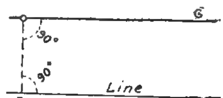


Fig. 111

4°. (Fig. 112).—Run any line **AB** and bisect it at **C**. Run any line through **C** and take **CE=CD**. Stake **AE**.

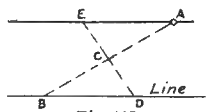


Fig. 112

5°. (Fig. 113).—Assume **B**. Take **BC=BD**. Range **E** on **AC**. Range **F** on **EB** and **AD**. Range **G** on **FS** and **ED**. Stake **AG**.

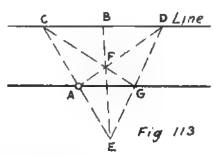


Fig 113

6°. (Fig. 114).—Assume **B**. Run **BAC** chaining it **BA=a**; **BC=b**. Run **BD** anyway convenient and chain it **BD=c**.

$$\text{Then } x = \frac{ac}{b}$$

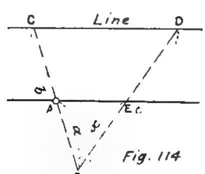


Fig. 114

Stake **AE**.

Case of an Obstruction Between Line and Point (Fig. 115).—

B and **C** are supposed poled. Range **D** on **AC**—Assume **E** convenient. Run **AF** parallel to **EC**—Range **F** on **ED.** Run **FG** parallel to **EB**—Range **G** on **BD.** Stake **AG.**

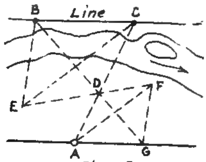


Fig 115

Measuring an Angle with Chain.—1°. (Fig. 116) (a).—At **A** (convenient), erect perpendicular **AB**=**c.**

$$\left.\begin{array}{llll} & \text{Chain} & c \text{ and} & b \\ \text{or} & \text{"} & c \quad \text{"} & a \\ \text{"} & \text{"} & b & a \end{array}\right\} \text{then} \left\{\begin{array}{l} tg\, C = \frac{c}{b} \\ \sin C = \frac{c}{a} \\ \cos C = \frac{b}{a} \end{array}\right.$$

2°. ((Fig. 116) (b).—Take **AB**=**AC**—chain a and **b.**

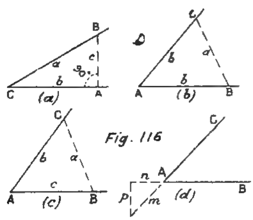


Fig. 116

$$\sin \frac{A}{2} = \frac{a}{2b}\text{, from which } \frac{A}{2} \text{ and } A.$$

3°. Or by the Table of chords. Take **b**=1 chain (100 ft.); then will **a** be the length of the chord for the corresponding angle. This angle will be found opposite the length in the Table of chords.

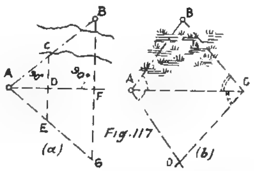
4°. (Fig. 116) (c).—Assume **B** and **C** as convenient—Chain **a, b** and **c.**

$$\sin A = \frac{2}{bc}\sqrt{p(p-a)(p-b)(p-c)} \quad \text{in which } p = \frac{a+b+c}{2}.$$

5°. (Fig. 116) (d).—**Case when the interior of the angle is inaccessible.** Measure any distances **m** and **n** on the prolongation of the sides; also measure **p.** Then **A** can be determined.

Distance of Two Points, One Inaccessible—1°. (Fig. 117) (a).—Run **AF** convenient and select **D.** Erect **DC** perpendicular to **AF**—

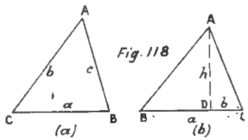
Range **C** on **AB** and perpendicular. Take **DE=DC.** Let fall **BF** perpendicular to **AF.** Range **G** on **AE** and perpendicular. Chain **AG.**



2°. (Fig. 117) (b).—Run **AC** convenient.—Measure angles at **A** and **C** in triangle **ABC** and make angles equal to them on opposite side of line **AC** and from same vertices **A** and **C.** Stake lines **AD** and **CD.** Determine their intersection **D** and chain **AD.**

Chain Survey.—The corners should be well defined; by poles if possible.

Triangular Field—1°. (Fig. 118) (a).—Chain the 3 sides. Angles obtained by formula given above.



Area given by formula $S = \sqrt{p(p-a)(p-b)(p-c)}$

2°. (Fig. 118) (b).—Run perpendicular **AD.** Chain **a, b** and **h.**

$$tg\, C = \frac{h}{b}\ ;\ tg\, B = \frac{h}{a-b}\ ;\ A\quad 180° - (B+C)\ ;\ Area = S = \frac{ah}{2}.$$

Also by means of 2 sides and the angle opposite one of them.

Also by means of 2 sides and the included angle.

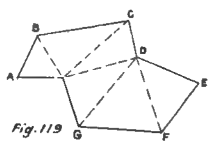


Fig. 119

Polygonal Field (Fig. 119).—Decompose it into triangles either by diagonals or by lines radiating from one or more internal points.

Avoid triangles with very acute or obtuse angles the intersection of the sides of which it is difficult to determine accurately.

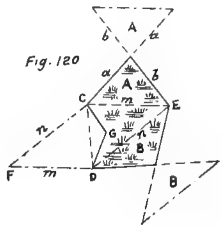


Fig. 120

Survey of a Field the Interior of which is Inaccessible (Fig. 120). Chain the sides and measure the angles as explained above. Sometimes all the triangles into which it might be divided by diagonals can be reproduced by symmetricals. Produce two adjacent sides by their own length **a** and **b** and chain **m** equal to diagonal **m**: Triangles **A** are equivalent; so are triangles **B**. For the central triangle **CED** run **CF=n** and parallel to it, and **DF=m** and parallel to it; **CDF** is equivalent to **CDE**; subtract triangle **CDG**.

Field Notes. For Chain Surveying, when the polygon is not very extensive or complicated, it is best to draw a diagram of the field and its division into triangles, lettering it for reference (Fig. 121). This

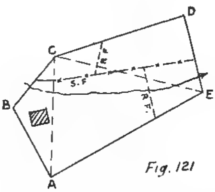


Fig. 121

diagram is drawn on one page and shows that the most economical chaining will be **AB, BC, CA, AE, EC, CD** and **DE**. On the following pages, (Fig. 122) (**A**), a center line running lengthwise will represent any side. Beginning near the bottom of the page, mark the starting point with a circle. **A=o** corner. Chain **AB** and noting that the side **a** of the house if produced would intersect **AB** at the reading 96, enter such a reading; then measure the distance to nearest corner 14; measure the house 31×24, then enter in book, to the right of the figure **96**: 14 **R** cor. House; 45 **R** cor. House, House 31×24. Continue the chaining to **B**, say 132; make a circle on line and mark it **B**132 cor. and a short distance away further on line an other circle which mark **B=o** cor. and so continue as indicated in the annexed diagram.

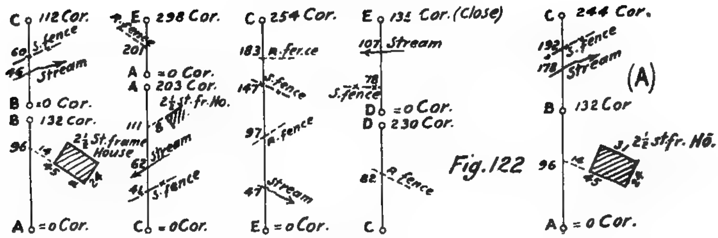


Fig. 122

Breaking Chain.—This method of setting the chain again at zero at every corner is called **breaking chain.**

Cumulative Distances.—Many operators do not break chain, but simply swing it around and continue the chaining to the end. The distances recorded being the total distances from the starting point or zero. They follow the method of **cumulative distances.**

In the above notes, the first two courses would read as shown in (A).

Offsets.—When the corners of a field are very numerous, some of them may be omitted in the chaining of outside boundaries; but they should be tied to the line of survey so that they can be plotted and the true area determined. This may be done: 1°. By tying those corners to two points in the line of survey; 2°. By letting a perpendicular fall from any one or all of them to the line of survey and chaining their distance R or L to that line.

The same methods are followed to determine any particular point, as a monument, a well, a rock, a tree, etc.

Irregular Boundary.—The method by perpendicular offsets is in fact the only one that may be applied to certain cases, as in the determination of an irregular boundary, such as the windings of a stream, the borders of a swamp, etc.

Office Work.—The chainman cleans the chains, tapes and pins, and assists in checking calculations. As a rodman he calls off field notes to the leveler who plots the profiles and cross-sections.

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THE ASSISTANT ENGINEER

BY

PROF. JEAN P. GENTHON

Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK III

THE RODMAN

WRITTEN FOR

THE CHIEF

Journal of the Civil Service

PUBLISHED BY

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THE RODMAN

Book III

PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with that most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3°. The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examination in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON,

July 1, 1907 New York.

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1°. Aptitude for mathematics.

2°. Habit of observation.

3°. Good memory.

4°. System.

5°. Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION.—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials, or new appliances and processes are used for old or new purposes. Notice the several kinds of labor, and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results: principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figure and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations, in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around you will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public

works have a Chief Engineer who prepares the work and directs its construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

Chief Engineer.			Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist.
	Division Engineers.		Inspectors.	
		Assistant Engineers	Transitmen.	Chainmen. Axemen.
			Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen.

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.	Memphis, Tenn.	St. Louis, Mo.
Boston, Mass.	Milwaukee, Wis.	St. Paul, Minn.
Buffalo, N. Y.	Mobile, Ala.	San Fancisco, Cal.
Charleston, S. C.	Nashville, Tenn.	Savannah, Ga.
Chattanooga, Tenn.	New London, Conn.	Seattle, Wash.
Chicago, Ill.	New Orleans, La.	Sioux City, Iowa.
Cincinnati, Ohio.	Newport, R. I.	Tampa, Fla.
Cleveland, Ohio.	New York, N. Y.	Vicksburg, Miss.
Detroit, Mich.	Norfolk, Va.	Washington, D. C.
Galveston, Tex.	Philadelphia, Pa.	Wheeling, W. Va.
Grand Rapids, Mich.	Pittsburg, Pa.	Wilmington, Del.
Jacksonville, Fla.	Portland, Me.	Wilmington, N. C.
Little Rock, Ark.	Portland, Oreg.	YellowstonePark,Wyo.
Louisville, Ky.	Rock Island, Ill.	

where examinations may be taken except that of Assistant Engineer which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken in

Albany.	Ithaca.	Ogdensburg.
Amsterdam.	Jamestown.	Olean.
Auburn.	Kingston.	Plattsburg.
Binghampton.	Lockport.	Poughkeepsie.
Buffalo.	Malone.	Rochester.
Elmira.	Newburg.	Utica.
Hornelsville.	New York.	Syracuse.
		Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewers or construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman; topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps.

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineeer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service appear regularly in "THE CHIEF."

THE ASSISTANT ENGINEER.

BOOK III

THE RODMAN

Rodman.—He who carries the rod, sets it up, takes the readings (when not taken directly by the instrument man) and records them.

Who His Superiors Are.—The rodman is directly under the orders of the leveler (or the topographer).

What He Carries.—He carries leveling rod, pocket rule and tape, hatchet, stakes, spikes, red chalk and note book.

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service—Panama Canal.

Title: Rodman.
Age limits: 21 to 45 years.
Salary: $75 and $83.33 per month.

Written Examination.

	Subjects.	weights.
1.	Pure and applied mathematics (elementary problems in mensuration, solution of plane triangles, and theoretical and applied mechanics, involving a fair knowledge of pure mathematics to and including calculus)	25
2.	Construction and care of instruments (comprising transit, including stadia work, level, plane table, rods, chain, tape, current meters, etc.)	25
3.	Theory and practice of surveying (comprising surveying, leveling, and other field work required in civil engineering and not covered in subjects 1 and 2)	20
4.	Design and construction (involving elementary knowledge of designing and constructing highways, railroads, dams, retaining walls, foundation work, trusses, etc.)	10
5.	Training and experience	20
	Total	100

Applicants must have had not less than two years' practical experience in connection with surveying work. Three years' study in a

school of civil engineering will be accepted as equivalent to the two years' experience required.

Two days will be required for this examination.

N. Y. State and County Civil Service.

Age limit: Not less than 21 years old.

Salary: $3.50 to $4.00 per day when employed.

Examination.

Subjects.	Relative weights.
1. Theoretical and practical questions, including mensuration and use of logarithms, plane trigonometry, elementary surveying and leveling, theory and use of rod, level and transit, highway construction	6
2. Experience	1
3. Education	3
Total	10

Specimen Questions.

The work of computation must be shown in full.
Slide-rules may be used.
Logarithm tables will be furnished by the examiners.

1. Solve completely the simultaneous equations

$$\text{and}\qquad \begin{aligned} \sqrt{xy} - \sqrt{x-y} &= 11 \\ x^2y - xy^2 &= 60 \end{aligned}$$

2. Compute to two places of decimals the value of **v** from the formula

$$v = \frac{\sqrt{2gh}}{\sqrt{1 + \frac{81}{256}\left(A + B + 2C + 4\frac{l}{d}\right) + D}} \quad \text{when} \begin{cases} g = 32.2;\ h = 25;\ A = .505; \\ B = 2.06;\ C = .294;\ f = .006; \\ l = 80\ ;\ d = \frac{1}{6} \text{ and } D = .183 \end{cases}$$

3. Find by logarithms the value of

$$\frac{\sqrt{.00281}}{.075^{4.2}}$$

4. **A** and **B** are two stations on opposite sides of an impassable gorge; they can both be seen from station **C** and their distances from **C** are 1,198 feet and 1,284 feet respectively. The angle at C between the transit lines AC and BC is 65° 10½'. Find the distance from A to B.

5. A road 12 feet wide is to be extended across between two parallel banks by filling in with earth and stone. The bank on one side has a batter of 1¼ to 1, and on the other of 1½ to 1; each slope down to the bed of a stream 5 feet wide and 14 feet below the level of the road. The filling is to have a batter of 1½ to 1 on each side; how many cubic yards will it contain?

6. If the safe tensile stress allowed per square inch for iron bars is 12,000 pounds, what should be the size of round bar selected to withstand a pull of 12½ tons, if the commercial sizes vary by 1-16 inch diameter up to 1½ inch and by ⅛ inch above that size?

7. Four wheels, each loaded with a weight of 5 tons and spaced 5 feet apart, roll over a beam of 18 feet span. What position of the wheels gives the maximum bending-moment, and what is the amount of that bending-moment?

8. Give in their proper order the adjustments of the engineer's transit.

9. If in running a transit line it should meet an obstruction, as a house, how would you continue the line beyond the obstruction? Describe three methods.

10. Given the following data, supply the missing distances, using the method of latitudes and departures:

Stations.	Bearings.	Distances.
1.	N. 35° E.	270
2.	N. 83½° E.	129
3.	S. 57° E.	?
4.	S. 34¼° W.	?
5.	N. 56½° W.	323

11. Write a set of transit notes for the center line of a highway a quarter-mile in length, to include at least two turns. Explain how you would measure the angles and the method you would use to eliminate errors in their measurements.

12. Write a set of level notes for the above road, giving turning-points and bench-marks, assuming that the road passes over a knoll and across a hollow.

13. Explain how you would take and record the elevations to the right and left of your transit line in the above.

14. Supposing that the grades on this highway are to be reduced by cuts and fills, explain how you would locate and set the slope stakes for that purpose.

15. Explain how you would select bench-marks and turning-points, in taking levels, and name and describe the rod you would prefer to use. What precautions should be taken in the use of the rod in order to make accurate measurements?

16. Sketch in pencil 5-foot contour lines from the elevation points given below; mark the elevations for each contour.

New York Municipal City Civil Service.

Assimilated to Chainman and Leveler q. v.

All city examinations are for Chainman or Rodman.

See questions proposed for the examinations of Chainman. (The Assistant Engineer. Book II. The Chainman.)

Scientific Requirements.

What precedes. Algebra. Geometry. Trigonometry.

ALGEBRA.

Properties of the Roots of a Quadratics.—Calling **x'** and **x"** the roots of the general equation.

$$ax^2+bx+c=0$$

which are $$x' = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

and $$x'' = \frac{-b-\sqrt{b^2-4ac}}{2a},$$ if we add them together we have (1) $x'+x'' = -\frac{b}{a}$, and if we multiply them together we have (2) $x'x'' = \frac{b^2-(b^2-4ac)}{4a^2} = \frac{b^2-b^2+4ac}{4a^2} = \frac{c}{a}$,

Which show that

1°. The sum of the roots of a quadratics equals the quotient of the coefficient of the term in **x** divided by the coefficient of the term in **x2**, taken with a sign contrary to that of the coefficient of the term in **x.**

2°. The product of the roots of a quadratics equals the quotient of the independent term divided by the coefficient of the term in **x2.**

Now we can write the general equation as follows by dividing both its members by **a** which is never zero:

Other form of a Quadratics.—

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

In this equation we may substitute $-(x'+x'')$ for $\frac{b}{a}$ and $x'x''$ for $\frac{c}{a}$; it then becomes $x^2-(x'+x'')x+x'x''=0.$

and shows how we can determine two quantities **x'** and **x"** when their sum and their product are given.

Find 2 Quantities, given their Sum and their Product.—Form a quadratics in which the sum of the given quantities taken with a contrary sign shall be the coefficient of the term in **x** and their product shall be the independent term; the term in **x2** shall have **1** for coefficient.

Example.—Find 2 numbers the sum of which is 10 and the product 21.

The unknown numbers will be the roots of the quadratics

$$x^2 - 10x + 21 = 0$$

Using formula of solution (**B**) we get

$$x = 5 \pm \sqrt{25-21} \text{ and separating the roots: } \begin{cases} x' = 5+\sqrt{4} = 5+2 = 7 \\ x'' = 5-\sqrt{4} = 5-2 = 3 \end{cases}$$

System of Two Quadratic Equations.—We can always take the value of one unknown in function of the other, thus reducing the system to one equation with one unknown. But that resulting equation is generally of the 4th degree. However, there are many systems that can be solved by means of artifices or particular considerations.

When one equation only is of the first degree in x and y, the system is simple. Take the value of **y** in the equation of the first degree and substitute it in the quadratics; the resulting equation will be a quadratics in **x.**

Homogeneous Equations.—They are equations where the sum of the exponents of **x** and **y** is the same in all the terms.

$$\text{Ex. } \begin{aligned} 5x^2 - 3xy + 4y^2 &= 100 \quad (1) \\ 2xy + 3x^2 &= 57 \quad (2) \end{aligned}$$

Assume $\frac{y}{x} = z$ which gives $y = xz$ (3); the system becomes

$$\begin{cases} 5x^2 - 3x^2z + 4x^2z^2 = 100 \\ 2x^2z + 3x^2 = 57 \end{cases} \text{ or } \begin{cases} x^2(5+3z+4z^2) = 100 & (4) \\ x^2(2z+3) = 57 & (5) \end{cases}$$

and by division and reduction $228z^2 - 371z - 15 = 0$

Get the values of **z**, substitute in (**5**) and thus get **x**; substitute the values of **z** and **x** in (**3**) and get **y.**

$$\text{Ex. Solve } \begin{cases} x + y + z = a & (1) \\ x^2 + y^2 + z^2 = b^2 & (2) \\ xy = cz & (3) \end{cases}$$

In (**1**), transfer **z** to the second member and square both members.

$x^2 + 2xy + y^2 = a^2 - 2az + z^2$. In this, substitute the value of x^2+y^2 taken from (2) and of xy taken from (3); you get $2z^2 - 2(a+c)z + a^2 - b^2 = 0$

From which get **z.**

Substitute the values of **z** in (**1**) thus obtaining **x + y** and in (**3**) thus obtaining **xy.** We revert to a case previously considered.

Bi-Quadratic Equation.—A trimonial equation which contains

terms in x^4, x^2 and independent. The general form is $ax^4+bx^2+c=0$. Assuming $x^2=y$ it then takes the form $ay^2+by+c=0$ from which

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \text{or} \quad x^2=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \text{and finally} \quad x=\pm\sqrt{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Some Binomial Equations.

Ex. I. $x^2-1=0$; $x^2=1$; $x=\pm\sqrt{1}$; $x'=1$ and $x''=-1$.

Ex. II. $x^2+1=0$; $x^2=-1$; $x=\pm\sqrt{-1}$; $x'=\sqrt{-1}$ and $x''=-\sqrt{-1}$

Ex. III. $x^3-1=0$; $(x-1)(x^2+x+1)=0$; from which $\begin{cases} x=1 \\ x^2+x+1=0 ; x=\frac{-1\pm\sqrt{-3}}{2} \end{cases}$

Ex. IV. $x^3+1=0$; $(x+1)(x^2-x+1)=0$; from which $x=-1$ and $x=\frac{1\pm\sqrt{-3}}{2}$

Ex. V. $x^4+1=0$; $x^4+2x^2+1=2x^2$; $(x^2+1)^2-(x\sqrt{2})^2=0$; $(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)=0$.
Each of the two factors now is a quadratics.

The factors may be separately equated to 0 and the resulting equations solved.

Progressions.

Arithmetical Progression.—A series of quantities **a, b, c,** ...... **m, n, p** such that the difference **r** between any term and the preceding one is a constant.

An arithmetical progression is indicated by the sign ÷ placed before it; a point is placed between the terms.

Arithmetical Ratio.—This constant difference **r** is called arithmetical ratio.

Increasing ÷ Progression.—A progression the terms of which increase from the first or in which the ratio is positive.

Ex. I. $\div\, 0 \cdot 1 \cdot 2 \cdot 3 \cdots\cdots$ in which $r=1$
Ex. II. $\div\, 6 \cdot 9 \cdot 12 \cdot 15 \cdot 18 \cdots\cdots$ in which $r=3$.

Decreasing ÷ Progression.—A progression the terms of which decrease from the first or in which the ratio is negative.

Ex. $\div\, 19 \cdot 17 \cdot 15 \cdot 13 \cdot 11 \cdots\cdots$ in which $r=-2$. $(15-17=-2)$

How to obtain any term.—Add to the first as many times the ratio as there are terms preceding it. If **a** is the 1st term and **r** the ratio, the **nth** term l will be **l=a+(n—1)r (1).**

Arithmetical Means.—Numbers (or quantities) inserted between two given numbers such as to form with them an arithmetical progression. Let **a** and **b** be 2 given numbers, **m** the number of means to be inserted. The total number of terms in the final series will be **m+2.** We have to find the ratio **r.** From **(1)** applied in this case, we have **b=a(m+1)r** from which

$$r=\frac{b-a}{m+1}$$

Sum of 2 terms equidistant from the extremes.—That sum equals the sum of the extremes $a+l$. For example n being the number of terms, the value of the mth term will be $a+(m-1)r$, and the value of the $n-(m-1)=(n-m+1)$th term will be $a+(n-m)r$ and by addition $2a+(m-1)r+(n-m)r=2a+mr-r+nr-mr=2a+nr-r=2a+(n-1)r=a+(a+(n-1)r)=a+l$.

Sum of the terms of an Arithmetical Progression.—Let s be the sum.

$$\left.\begin{array}{ll} & \div\ a+b+c+\ldots\ldots+h+k+l = S \\ \text{also} & \div\ l+k+h+\ldots\ldots+c+b+a = S \end{array}\right\}\text{adding these series term to term}$$

$(a+l)+(b+k)+(c+h)+\ldots\ldots+(h+c)+(k+b)+(l+a)=2S$. All these partial sums are each equal to $(a+l)$ and their number is n. We then have

$$n(a+l)=2S \quad \text{and} \quad S=\frac{(a+l)n}{2} \quad (2)$$

Geometrical Progression.—A series of quantities $a, b, c, \ldots\ldots h, k, l$ such that the quotient of any term by the preceding one is constant. The sign of a geom. progr. is $\div\!\!\div$ prefixed to the series and : is placed between the terms.

Geometrical Ratio.—The constant quotient of any term by the preceding one is called geometrical ratio q.

Increasing Geometrical Progression.

A $\div\!\!\div$ progression the terms of which increase from the first, or whose ratio is >1 as $\div\!\!\div 1:2:4:8:16:\ldots\ldots$ in which $q=\frac{8}{4}=2$.

Decreasing Geometrical Progression.

A $\div\!\!\div$ progression the terms of which decrease from the first, or whose ratio is <1 as $\div\!\!\div 81:27:9:3:1$ in which $q=\frac{9}{27}=\frac{1}{3}$.

How to obtain any term.—Multiply the first term by the ratio raised to a power indicated by the number of terms preceding the one sought. Therefore the last term is obtained by multiplying the first term by the ratio raised to a power marked by the number of terms less one in the series.

$$l=aq^{n-1} \ (1) \text{ from which } q=\sqrt[n-1]{\frac{l}{a}} \ (2) \text{ and } a=\frac{l}{q^{n-1}} \ (3)$$

Geometrical Means.—Quantities inserted between two other given ones, such as to form with them a geometrical progression. Let a and b be 2 given quantities, m the number of means to be inserted. We have to find the ratio q.

From (2) applied to this case we have

$$q=\sqrt[m+1]{\frac{b}{a}}$$

Product of 2 terms equidistant from the extremes.—That product is constant and equals the product of the extremes.

The m^{th} term will be aq^{m-1} and the $n-(m-1)=(n-m+1)^{th}$ term will be aq^{n-m}, and by multiplication $a \times aq^{m-1+n-m} = a \times aq^{n-1} = al$.

Product of all the terms of a geometrical progression.

We may write: $P = a \times b \times c \times \cdots \times h \times k \times l$ or again
$P = l \times k \times h \times \cdots \times c \times b \times a$ and by multiplication

$$P^2 = al \times bk \times ch \times \cdots \times hc \times kb \times la$$

Now each partial product is $= al$ and their number is n;
$P^2 = (al)^n$ and finally $P = \sqrt{(al)^n}$ (4)

Sum of all the terms of a geometrical progression.

Let S be that sum.

$S = a + b + c + \cdots + h + k + l$ Multiplying by q,

$Sq = aq + bq + cq + \cdots + hq + kq + lq$ which may be written

$Sq = b + c + d + \cdots + k + l + lq$; subtracting this from the first,

$Sq - S = lq - a$ or $S(q-1) = lq - a$ and $S = \frac{lq-a}{q-1}$ (5)

If the progression were decreasing we would have $S = \frac{a-lq}{1-q}$.

This last value of S shows that, although the number of the terms of a decreasing ÷ progression may increase indefinitely, their sum does not increase beyond $S = \frac{a}{1-q}$ when lq becomes zero.

This value of S is called a *limit*.

LOGARITHMS.

What Logarithms Are.—Given 2 progressions, one geometrical beginning with 1 and one arithmetical beginning with 0, the terms of the latter are called the logarithms of their corresponding terms in the former.

Ex. *Numbers* ÷÷ 1 : 3 : 9 : 27 : 81 : 243 : 729 : 2187 : 6541 : 19683...
Logarithms ÷ 0 . 2 . 4 . 6 . 8 . 10 . 12 . 14 . 16 . 18...

In this system, 6 is the **log.** of 27, or log. 27=6:

Also **log.** 19683=18.

A number may have more than one log.—Because a great many systems of two progressions may be formed agreeing with the definition given.

In a given system a number has only one log.—But when a system is adopted, one number has only one logarithm in that system.

Common System—or Brigg's Logs.—The Common or Brigg's System of logs. is a geometrical progression the first term of which is 1 and the ratio **q**=10; and an arithmetical progression beginning with **0** and with a ratio of 1: **r**=1; thus

Common or Brigg's System	Nos. $\div\!\div$ 1 : 10 : 100 : 1000 : 10 000 : 100 000 : 1 000 000 : .....
	Logs. $\div$ 0 . 1 . 2 . 3 . 4 . 5 . 6 . ....
which may be written :	Nos. $\div\!\div$ $10^0 : 10^1 : 10^2 : 10^3 : 10^4 : 10^5 : 10^6 : \ldots$
	Logs. $\div$ 0 . 1 . 2 . 3 . 4 . 5 . 6 . ....

All numbers have a log in a given system.—All the numbers not expressed in the geometrical progression have a log., because we can insert between any 2 consecutive terms of that progression a number of means so great that any term of the new progression shall be as little different from its neighbors as we wish; and we can insert in the $\div$ the same number of means between the logs. of the 2 terms selected, each of which will be the log. of its correspondent in the new geometrical progression.

Numbers less than 1 also have logs.—A system of logs. may be written as below and continued both ways from the terms 1 and 0 thus

$$(A)\begin{cases}\text{Nos.} \div\!\div \cdots \frac{1}{q^4} : \frac{1}{q^3} : \frac{1}{q^2} : \frac{1}{q} : 1 : q : q^2 : q^3 : q^4 : q^5 : \ldots q^m \ldots q^n \ldots q^{m+n} \ldots \\ \text{Logs.} \div \cdots -4r . -3r . -2r . -r . 0 . r . 2r . 3r . 4r . 5r \ldots mr \ldots nr \ldots (m+n)r \ldots\end{cases}$$

In this we see that numbers less than 1 have negative logs. But negative numbers have no logs.

Object of Logs.—To simplify calculations, transforming multiplications into additions, divisions into subtractions, powers into multiplications, and roots into divisions.

Properties of Logs.—1°. The log. of a product is the sum of the logs. of the factors.

$$\begin{cases}\text{Nos. } q^m \times q^n = q^{m+n} \\ \text{Logs. } mr + nr = (m+n)r\end{cases}$$ In System (A), q^{m+n} has the same rank in the $\div\!\div$ as $(m+n)\,r$ in the $\div$

Therefore $\log (q^m \times q^n) = \log q^m + \log q^n$

2°. The log. of a quotient is the difference of the logs. of the Divid. and divis.

$$\frac{a}{b} = Q \;;\quad a = bQ \;:\quad \log a = \log b + \log Q \;,\quad \log Q = \log a - \log b \,.$$

3°. The log. of a power equals the log. of the No. multiplied by the exponent.

$$\log a^m = \log (\overset{m \text{ factors}}{a \times a \times \cdots \times a}) = \log a + \log a + \log a + \cdots + \log a = m \log a$$

4°. The log of a root equals the log. of the No. divided by the index.

$$\sqrt[m]{a} = u \;;\quad a = u^m :\quad \log a = m \log u \;;\quad \log u = \log \sqrt[m]{a} = \frac{\log a}{m} \,.$$

Index or Characteristic of a log. is the integral portion of the log.

Mantissa of a log. is the decimal portion of the log.

Index.—In the Brigg's System, the index of the log. of a number is composed of as many units as there are figures less one in the integral part of the number.

Ex. 27 891. This number is such that $\begin{cases} \text{Nos.} & 10^4 < 27\,891 < 10^5 \\ \text{Logs} & 4 < \log 27891 < 5 \end{cases}$

Being less than 5 and greater than 4, log. 27891 will have 4 as an index.

Mantissa.—The logs. of two numbers, different but composed of the same figures placed in the same order, as for instance 7329.6 and 73.296, have the same mantissa but a different index. Two such numbers may be written.

n and $n \times 10^k$ in which n is the lesser.

$$\begin{array}{ll} \text{Numbers} & \text{Logs.} \\ n & \log n \\ n \times 10^k & \log(n \times 10^k) = \log n + k \log 10 = \log n + k. \end{array}$$

Now k is a whole number and only the integral part of $\log n$ will receive that increase to make the log of $n \times 10^k$.

Formation of a Table of Logarithms.—A Table of Logs. may be formed by inserting between the terms of the two progressions

$$\begin{cases} \div\div\ 1 : 10 \quad 100 . 1\,000 . 10\,000 \cdot 100\,000 : 1\,000\,000 \quad \ldots\ldots \\ \div\ 0 \cdot 1 \quad 2 . \quad 3 . \quad 4 \quad\quad 5 \quad\quad 6 \quad \ldots\ldots \end{cases}$$

a great many but the same number of means, so that two consecutive terms of the new geometrical progression may have 6, 7, 10 or more common figures, as 10.310765, 10.310766, 10.310767, etc., their correspondents in the new $\div$ will be their logs.

Logarithmic Tables were not so formed however. More rapid processes were made use of.

Arrangement of Logarithmic Tables.—Log. Tables are generally disposed on the same plan, so that the description of one, say Vega's, will illustrate them all.

1° The integral portion of logs. or their characteristic is not entered in the tables but only the decimal portion or mantissa. The computer knows, for instance, that log. 356.782=2.+a decimal portion, and log. 3.56782=0.+ the same decimal portion; so that only the mantissa of log. 3.56782 is entered.

2°. On a first vertical column headed **N** are entered numbers from 1000 to 9999 and their logs. are in the next vertical column headed **0**, given with 7 figures the first three of which are not repeated before each log. in order to facilitate the reading of the table, but they must be understood.

The logs. of the numbers of 6 figures are found in the following 9 vert. columns with their last 4 decimals only, the first three being the same as in column 0.

Find the Log. of a Number.—For example log. 35 having the same mantissa as that of 350, 3500, 35000, etc., it will be found opposite the number 3500 in the column marked **0** and prefixing to it the characteristic 1 we shall have log. 35=1.5440680. Likewise log. 1643 is found opposite that number and as the characteristic is 3, log. 1643=3.2156376. The log. 284.63 will be found opposite 2846 and in the column headed **3** (6th fig. of the No.); log. 284.63=2.4542807.

In a case like log. 405.56 for instance, notice that the first of the last 4 figs. in column 6 is surmounted with a dash —: $\bar{0}551$, this means that the first 3 figs. are to be taken below. they are 608, not 607; this notation dispenses with blank spaces used in other tables and it makes the table more compact.

In the case of a number of 6 figures, as 673.843; write down the log. of 673.84: log. 673.84=2.8285568; notice the difference between it and the next log. 8285632 (make that difference mentally), it is 64, which would be the increase of the log. for an increase of 1 unit of the fifth order in the number, or for an increase of 10 units of the sixth order. That difference would be 1-10 of 64=6.4 for 1 unit of the 6th order, and it will be 6.4×3 for 3 such units (6th fig. of the No.) or 19.2 which add to

$$\begin{array}{r} 2.8285568 \\ 19.2 \\ \hline \end{array}$$

so that log. 673.843=2.8285587.2.

Now the calculation for the 6th fig. is given in a small table headed 64 in the margin where, opposite 3, is found 19.2.

If there was a 7th fig. in the No., say another 3, the additive quantity would be 10 times less or 1.92 instead of 19.2, so that

Arrangement of the Calculation.

	log.	673.84	=	2.828 5568	
increment	for	.003	=	19.2	
"	"	.0003		1.92	
	log	673.8433	=	2.828 5589.12	
of which only				2.828 5589	is correct

Find a No. given its Log.—Let log. x=2.8401396. First we see that the integral portion of the number will be composed of 3 figs. (one more than the index).

Find in the tables the first three figs. 840, then in one of the columns opposite either 1396 or the next smaller No. We find 1375 and by referring to the columns of numbers we read 692.05. The difference between 1375 and the given log. 1396 is 21 on a difference of 62 between 1375 and the next 1437; looking in the margin under 62, we find 18.6 instead of 21 and it corresponds to the fig. 3 which will be the 6th of the number. The difrerence between 21 and 18.6 or 210 and 186 (for the 7th fig.) is 24 which very nearly corresponds to 4 and this may be taken as the 7th fig. of the required No. which will then be 692.0534.

Arrangement of the Calculation.

$$
\begin{array}{llll}
 & 2.840\ 1396 & = \log & 692.05 \\
 & 75 & & \\
\text{increment of} & \overline{21} & \text{corresponds to} & 0.003 \\
 & 18.6 & & \\
\text{"} \quad \text{"} & \overline{24} & \text{"} \quad \text{"} & 0.0004 \\
\hline
 & 2.840\ 1396 & = \log & 692.0534
\end{array}
$$

Logarithmic Calculations.—A few examples.

Ex. I. $x = 374.92 \times (1.06)^7$

Formula. $\log x = \log 374.92 + 7 \log 1.06$.

We shall begin with the last log which we have to multiply.

Operation.

$$
\begin{array}{lcll}
\log.\ 1.06 & \rightarrow & 0.025\ 3059 & \\
 & \times & \underline{7} & \\
7 \log\ 1.06 & = & 0.177\ 1413 & \\
\log\ 374.92 & = & 2.573\ 9386 & \\
\hline
\log\ x & = & 2.751\ 0799 & \text{for } 563.74 \\
 & & 89 & \\
 & & \overline{10} & \text{"} \qquad 1 \\
 & & \underline{7.7} & \\
 & & 23 & \text{"} \qquad \underline{3} \\
x & = & & 563.7413
\end{array}
$$

Ex. II. $x = \dfrac{0.927}{3.7728}$

We may write $x = \dfrac{9270}{37728}$; $10x = \dfrac{92\ 700}{37\ 728}$ which fraction is > 1

Formula. $\log 10x = \log 92\ 700 - \log 37\ 728$.

Operation.

$$
\begin{array}{lcl}
\log.\ 92\ 700 & = & 4.967\ 0797 \\
\log\ 37\ 728 & = & 4.576\ 6638 \\
\hline
\log\ 10x & = & 0.390\ 4159 \\
 & & \underline{052} \\
 & & 107 \\
 & & \\
10x & = & 2.4576 \\
\text{hence} \quad x & & 0.24576.
\end{array}
$$

The calculation is never done in that way in practice; we make use of the following **remarks:**

Logs. of Decimal Numbers less than 1.—It is agreed to call log. of a decimal number less than 1 the same positive mantissa as the log. of the given decimal number considered as a whole No., prefixed with a negative index formed with a number of units equal to the rank of the first decimal fig. after the decimal point. To show that the index alone is to be negative we surmount it with the sign —.

Ex. III $\quad x = 351 \times 0.03467 = \dfrac{351 \times 3.467}{100}$

$\log x \quad \log 351 + \log 3.467 - \log 100 = \log 351 + 0.539\,9538 - 2$

which we can write $\quad \log x = \log 351 + \bar{2}\,.\,539\,9538$.

Arithmetical Complement of a Decimal No.—What it is lacking to be equal to 1.

Thus 1—0.06391=0.93609.

0.93609 is the arithmetical complement of 0.06391 and vice versa.

0.06391 is the arithmetical complement of 0.93609.

Find the Arith. Comp. of a Decimal.—Rule: Subtract from 10 the last fig. to the right and from 9 all the others.

Subtraction of a Log. by means of an addition.—Let **A** be a number from which the logarithm **(i+m)** is to be subtracted, **i** being the index and **m** the mantissa or decimal portion (as for instance 2.7493012 may be written 2+0.7493012); we have

$$A - (i+m) = A - i - m = A - i - m + 1 - 1 = A - i - 1 + 1 - m$$
$$= A - (i+1) + (1-m)$$

Rule: To subtract a log. from a number, add the complement of the mantissa preceded by the index increased by + 1 and taken with a contrary sign.

Ex. IV. $\quad x = \dfrac{36.28}{742.68}\ ; \quad \log x = \log 36.28 + c^t \log 742.68$

Operation.

$\log 36.28 \qquad 1.559\,6673$

$\log 742.68 = 2.870\,8017;\ c^t \log 742.68 \quad = \bar{3}.129\,1983$ {this is obtained mentally from the Tables.

$\log x \qquad \bar{2}.688\,8656$

$\begin{array}{r} 46 \\ \hline 10 \\ 8.8 \\ \hline 12 \end{array}$

$x = 0.048\,85011$

Ex. V. $\;l = \dfrac{\pi R n}{180}$ is a formula giving the length of an arc of n degrees and radius R.

Let us suppose $\quad n = 25°\,18'\,26''$

$R = 948.65$ ft.

$\pi = 3.1416$

Reducing n and 180 into seconds we have the equation of the problem :

$$l = \frac{3.1416 \times 948.65 \times 91\,106}{648\,000}$$

Formula of Solution : $\log l = \log 3.1416 + \log 948.65 + \log 91106 - \log 648\,000$.

Operation.

$$\begin{aligned}
\log 3.1416 &= 0.497\,1499 \\
\log 948.65 &= 2.977\,1060 \\
\log 91\,106 &= 4.959\,5470 \\
-\log 648\,000 &= \bar{6}.188\,4250 \\
\hline
\log Z &= 2.6222279
\end{aligned}$$

$$\begin{array}{rr}
44 & \\
\hline
35 & 3 \\
312 & \\
\hline
38 & 4
\end{array}$$

$$Z = 419.0134$$

Combinations.—We call combinations **n** to **n** of **m** objects the several groups that can be formed with **n** of these objects.

Combinations are divided into **arrangements** and **distinct products.**

Arrangements.—Combinations are called arrangements when groups containing the same objects, but placed in a different order, are considered as distinct.

Ex.: **a, b, c, d e,** f taken 2 and 2 may be so arranged **ab, ac, ad, ae, af, ba, bc, bd, be, bf, ca, cb, cd, ce, cf, da, db, dc, de, df, ea, eb, ec, ed, ef, fa, fb, fc, fd, fe.**

Arrangements differ from each other by the order of the objects.

Permutations.—Arrangements take the name of **permutations** when the **m** objects are taken **m** and **m.**

Ex.: **a, b, c, d** taken 4 and 4 is a number of permutations:

abcd, abdc, acbd, acdb, adcb, adbc, bacd, bcad, bcda, badc, bdca, etc.

Distinct Products.—of **m** objects **n** and **n** are combinations differing by one object at least; they are arrangements in which the groups containing the same objects are counted as one only.

Ex.: **a, b, c, d, e, f** taken 2 and 2 give the distinct products, **ab, ac, ad, ae, af, bc, bd, de, bf, cd, ce, cf, de, df, ef.**

The name of **Combinations** is generally reserved for that of **distinct products.**

We shall represent arrangements of m objects n and n by A^m_n.

We shall represent permutations of m objects by P_m.

By definition $A^m_m = P_m$.

We shall represent distinct products or combinations by C^m_n

Number of Arrangements.—Given **m** objects, we know only **m** ways of taking them, 1 and 1.

so we write $$A_1^m = m$$
If, to the right of each of these m arrangements 1 and 1, we place one of the $m-1$ other objects we shall have $A_2^m = m(m-1)$
We would see in like manner that $A_3^m = m(m-1)(m-2)$
Generally $A_n^m = m(m-1)(m-2)\ldots\ldots[m-(n-1)]$ or

$$\text{(a)} \quad A_n^m = m(m-1)(m-2)\ldots\ldots(m-n+1)$$

Rule: The number of arrangements of **m** objects **n** and **n** is the product of **n** consecutive integers decreasing from **m.**

Number of Permutations.—

$$P_m = A_m^m = m(m-1)(m-2)\ldots\ldots(m-m+1)$$
$P_m = m(m-1)(m-2)\ldots\ldots 3.\ 2.\ 1$ which is generally written

$$\text{(b)} \quad P_m = 1\cdot 2\cdot 3\ldots\ldots(m-2)(m-1)\,m\ .$$

Rule: The number of permutations of **m** objects is the product of the first **m** numbers (integers).

Number of Combinations or distinct products.

Let C_n^m be the number of combinations n and n of m objects.

These groups of **n** objects each, differ by at least one object.

Let us now consider one of these groups and suppose that we make in it all the permutations we can, then pass to the second group and do the same and so continue to the last group, we shall have made all the arrangements **n** and **n** of the **m** objects.

Each group will furnish **1.2.3....n** arrangements because

it is a P_n, and the C_n^m groups will furnish $1\cdot 2\cdot 3\ldots\ldots n\times C_n^m$ and this is the number of arrangements A_n^m or
$$m(m-1)(m-2)\ldots\ldots(m-n+1)$$
Therefore we have
$$1\cdot 2\cdot 3\ldots\ldots n\times C_n^m = m(m-1)(m-2)\ldots\ldots(m-n+1)$$
from which
$$\text{(c)} \quad C_n^m = \frac{m(m-1)(m-2)\ldots\ldots(m-n+1)}{1.\ 2\cdot 3\ldots\ldots n}$$

Rule: The number of combinations **n** and **n** of **m** objects is the product of **n** consecutive integers decreasing from **m** divided by the product of the first **n** integers.

The Number of Combinations n and n of m objects is the same as the number of combinations of these objects m—n and m—n.

If we consider one combination **n** and **n**, there will be left **m—n** objects, and so it will be for each and every combination; hence

$$C_n^m = C_{m-n}^m$$

Binomial Formula (Newton's).

It consists in establishing a rule to raise (evolve) rapidly and without actually performing the multiplication, a binomial to any power.

as for instance $(x+a)^m$

Let it be proposed to find the product P of m binomials, as

$$P=(x+a)(x+b)(x+c)\cdots\cdots(x+l)$$

Any term of P is the product of m factors different taken one from each binomial.

1° The first term is x^m product of the first m terms.

2° To obtain a term in x^{m-1} we take x in $m-1$ binomials with the second term in the remaining binomial. The coefficient of x^{m-1} will be the sum S_1 of all the second terms: S_1x^{m-1}.

3° To obtain a term in x^{m-2} we take x in $m-2$ binomials with the second term in the remaining 2 binomials. The coefficient of x^{m-2} will be the sum S_2 of the products 2 and 2 of the second terms: . . . . S_2x^{m-2}

4° Generally the coefficient of x^{m-p} is the sum S_p of the products p and p of the second terms: S_px^{m-p}.

5° Finally, the last term of P is the product $(abc\cdots\cdot l)$ of the second terms which we may call S_m.

So we have:

$$P=(x-a)(x-b)(x-c)\cdots\cdot(x-l)=x^m+S_1x^{m-1}+S_2x^{m-2}+\cdots+S_px^{m-p}+\cdots\cdot+S_m.$$

or, developing the calculations:

$$P = x^m \begin{array}{l|} +a\\ +b\\ +c\\ +\vdots\\ \\ +p\\ +\vdots\\ +l \end{array} x^{m-1} \begin{array}{l|} +ab\\ +ac\\ +\vdots\\ +bc\\ +bd\\ \vdots\\ +kl\\ \\ \end{array} x^{m-2} \begin{array}{l|} +abc\\ +abd\\ +\vdots\\ +bcd\\ \vdots\\ +hkl\\ \\ \\ \end{array} x^{m-3} + \cdots\cdot + abcd\cdots hkl.$$

EVOLUTION OF $(x+a)^m$. In the above formula, if we make $a=b=c=d=\cdots\cdot=k=l$, we have $P=(x+a)^m$, and at the same time

1° $S_1=a+b+c+\cdots\cdot+l$ is the product of a by the number of terms m, so that $S_1=\frac{m}{1}a$,

2° $S_2=ab+bc+\cdots\cdot+kl$ is the product of a^2 by the number of combinations 2 and 2 of the m second terms, so that $S_2=\frac{m(m-1)}{1\cdot 2}a^2$; etc.

3° Generally S_p is the product of a^p by the number of combinations p and p of the m second terms, so that $S_p=\frac{m(m-1)(m-2)\cdots\cdot(m-p+1)}{1\cdot 2\cdot 3\cdots\cdot p}$;

4° Finally $S_m=a^m$; therefore:

$$P=(x+a)^m=x^m+\frac{m}{1}ax^{m-1}+\frac{m(m-1)}{1\cdot 2}a^2x^{m-2}+\frac{m(m-1)(m-2)}{1\cdot 2\cdot 3}a^3x^{m-3}+\cdots\cdot+a^m. \quad (1)$$

The general term in the above development is $\frac{m(m-1)\cdots\cdot(m-p+1)}{1\cdot 2\cdot 3\cdots\cdot p}a^px^{m-p}$.

How to obtain easily the successive terms:

Rule: Multiply the coefficient of any term by the exponent of **x,** divide the product by the rank, this will be the coefficient of the following term to complete which increase by **1** the exponent of **a** and diminish by **1** the exponent of **x.**

The series is homogeneous.—The sum of the exponents of **a** and **x** in any term is **m.** The series contains **m+1** terms.

The coefficients of terms equally distant from the extremes are equal. The number of terms being **m+1,** any term that we may consider having **p** terms before it will have **m—p** terms after it; its coefficient then is

C_p^m and we have seen that $C_p^m = C_{m-p}^m$.

EVOLUTION OF $(x-a)^m$. In formula (1) we make $a=-a$; it becomes.:

$$(x-a)^m = x^m - \frac{m}{1}ax^{m-1} + \frac{m(m-1)}{1\cdot 2}a^2x^{m-2} - \frac{m(m-1)(m-2)}{1\cdot 2\cdot 3}a^3x^{m-3} + \ldots\ldots \pm a^m. \quad (2)$$

Sum of all the Coefficients of $(x+a)^m$. Make $a=1$ and $x=1$ in (1)

$$(x+a)^m = 2^m = 1 + \frac{m}{1} + \frac{m(m-1)}{1\cdot 2} + \frac{m(m-1)(m-2)}{1\cdot 2\cdot 3} + \ldots\ldots + 1 \,.\, . \quad (3)$$

The Sum of the even rank Coefficients equals the Sum of the odd rank Coefficients.—Make **a=1** and **x=1** in formula **(2).**

$$(x-a)^m = 0 = 1 - \frac{m}{1} + \frac{m(m-1)}{1\cdot 2} - \frac{m(m-1)(m-2)}{1\cdot 2\cdot 3} + \ldots\ldots \pm 1$$, which proves it.

Total Number of Combinations of m objects.—Formula **(3),** in which the first **1** is not a number of combinations, may be written:

$2^m = 1 + C_1^m + C_2^m + C_3^m + \ldots\ldots + C_m^m$; from which
$C_1^m + C_2^m + C_3^m + \ldots + C_m^m = 2^m - 1$ = *Total number of combinations which can be made with m objects.*

SOLID GEOMETRY.

Definitions.—A line **AB** (Fig. 123) is perpendicular to a plane **P** when it is perpendicular to all the lines **BC—BD,** etc., drawn through its foot **B** in the plane **P.** It is the shortest dist. from a pt. **A** to a plane **P.**

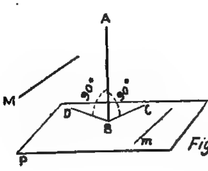


Fig. 123

A line **M** is parallel to a plane **P** when it is parallel to a line **m** in the plane. That parallel will never meet the plane.

Two planes **P—Q** (Fig. 124) are parallel when they are perpendicular to the same line **AB.**

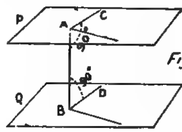


Fig. 124

Any line **AC** drawn in one of the planes **P** is parallel to the other plane **Q.**

A **dihedral angle APB** (Fig. 125) is that formed by two interesting planes **APQ—BPQ** and is measured by the plane angle **m** formed by two perpendiculars to the intersection **PQ** drawn from a pt. in it **C** and in each plane.

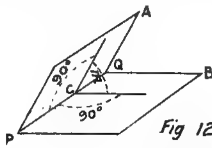


Fig 125

The planes **APQ—BPQ** forming a dihedral angle **APB** are the faces of the angle and their intersection **PQ** is the **arriss** of the angle.

A solid angle **A** (Fig. 126) or **polyhedral angle** is the figure formed by more than two planes passing through a common point called **vertex..** The faces are the plane angles **BAC—BAD—CAD** and the **arrisses AB—AC—AD** the intersections of the planes 2 and 2.

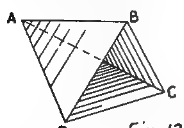


Fig. 126

A **trihedral** angle is a solid angle formed with 3 faces.

SOLIDS—Polyhedron.—A portion of space entirely limited by plane faces.

Tetrahedron (Fig. 127).—Polyhedron of 4 faces.

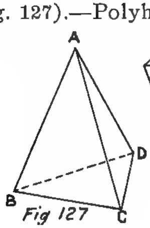


Fig 127

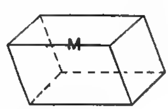


Fig. 128

Hexahedron (Fig. 128).—Polyhedron of 6 faces.

Octahedron.—Polyhedron of 8 faces.

Dodecahedron.—Polyhedron of 12 faces.

Icosahedron.—Polyhedron of 20 faces.

Regular Polyhedron.—One the faces of which are equal regular polygons. Their dihedral and polyhedral angles, and their arrisses are equal to each other.

Prism (Fig. 129).—Polyhedron two opposite faces of which called **bases** are equal parallel polygons. The **lateral faces** are parallelograms.

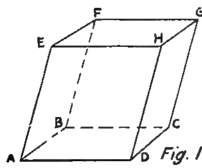


Fig. 129

Right Prism (Fig. 130).—When the lateral arrisses are perpend. to the bases. When not so perpend., the prism is **oblique.**

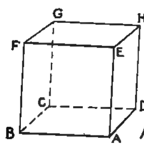


Fig. 130

Lateral arrisses of a prism are parallel and equal.

The lateral faces of a right prism are rectangles.

A prism is **triangular, quadrangular,** etc., when its bases are triangles, quadrilaterals, etc.

Regular Prism.—The bases of a right prism are regular polygons.

Height of a Prism (Fig. 131).—Perpend. dist. **h** betw. the bases.

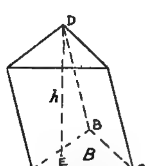


Fig. 131

Truncated Prism or Frustrun of a Prism (Fig. 132).—Portion of a prism bet. one base and a sectional plane not parallel to the base.

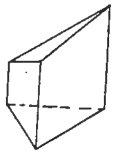


Fig. 132

Parallelopipedon (Fig. 133).—Prim the bases of which are parallelograms.

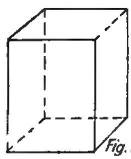


Fig. 134

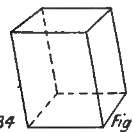


Fig 133

Rectangular Parallelopipedon (Fig. 134).—Right prism the bases of which are rectangles.

Cube (Fig. 135).—Rectangular parallelopipedon all the faces of which are squares. The cube is the regular **hexahedron.**

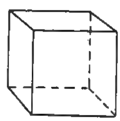


Fig. 135

Right Section of a Prism (Fig. 136).—The section made by a plane perpendicular to the lateral arrisses.

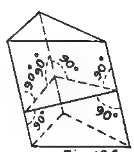


Fig. 136

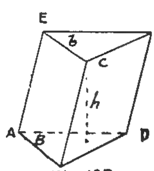


Fig 137

Lateral Surface of a Prism (Fig. 137).—The total surface of all its lateral faces: **AC+CD+DE.**

Total Surface of a Prism.—The lateral surface plus that of the two bases: **AC+CD+DE+B+b.**

Volume of a Solid.—The amount of space it occupies; or the ratio between that volume and the volume unit.

Volume Unit.—The unit of volume is the cube constructed on the length unit. Its base is the area unit and its height the linear unit.

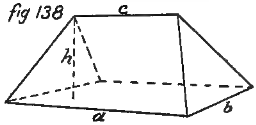
Equal Solids.—Solids of same shape which can coincide by superposition.

Volume of a Prism.—Product of the base **B** by the height **h**: **V=Bh.**

Volume of a Wedge (Fig. 138).

$V = \frac{bh}{6}(2a + c)$; in which a = long side of base ; b = short side of base ; c = length of edge ; h = perp. distance from edge to base .

fig 138



Volume of a Cube (Fig. 139).—The cube of the side **a.**

$$V = a^2 \times a = a^3 .$$

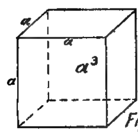


Fig. 139

Total Surface of a Cube.—

$$A = 6a^2$$

Equivalent Solids.—They cannot coincide, but they have the same volume.

Pyramid (Fig. 140).—Polyhedron one face of which **B**, called **base** is any polygon, and the others or lateral faces are triangles **F** having a common vertex **V** called the vertex of the pyramid.

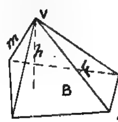


Fig 140

A pyramid is triangular, quadrandular, pentagonal, etc., according as its base is a triangle, a quadrilateral, a pentagon, etc.

Tetrahedron.—A triangular pyramid.

Height of a Pyramid.—Perpendicular distance **h** from the vertex **V** to the base **B** (also called altitude).

Regular Pyramid (Fig. 141).—Its base **AD** is a regular polygon, and its height **h** falls at the center **O** of the base.

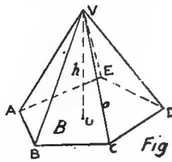


Fig 141

The **apothem** of a regular pyramid is the apothem of one of the faces (which is different from the apothem of the base).

Volume of a Pyramid.—One-third of the product of the base **B** by the height **h**, or the product of the base by one-third the height.

$$V = \frac{Bh}{3} = B\frac{h}{3}.$$

Truncated Pyramid or Frustrum of a Pyramid (Fig. 142).—Portion of a pyramid between its base **B** and a section **b** meeting all the lateral arrisses.

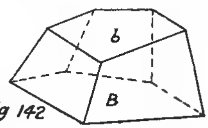


Fig 142

Regular Frustrum of a Pyramid.—In a regular pyramid, when the section is parallel to the base. In a regular frustrum the lateral faces are equal trapezoids.

Volume of a Regular Frustrum.—

$V = \frac{h}{3}(B + b + \sqrt{Bb})$, in which h = height of the frustrum; B = greater base; b = smaller base.

Similar Polyhedrons (Fig. 143).—Their solid angles are equal **A=M; B=N; C=P;** etc., and their faces are similar and similarly placed **ABE** similar to **MNR; AED** to **MRQ;** etc. The homologous dihedrals are equal **AB=MN; BE=NR;** etc. All the homologous lines are in the same ratio of similitude.

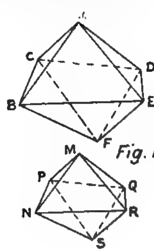


Fig. 143

Surface of Revolution (Figs. 144, 145, 146, 147).—Surface generated by a plane line **AB** revolving about a line **xy** called **axis** situated in the same plane.

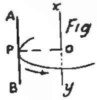


Fig 144

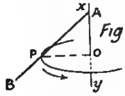


Fig 145

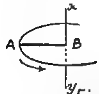


Fig. 146

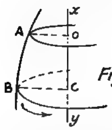


Fig. 147

Solid of Revolution (Fig 148).—Solid generated by a plane figure **ABCD** revolving about a line **xy** called **axis** situated in the same plane.

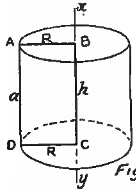


Fig 148

In a solid of revolution the perimeter of the generating surface (or a portion of that perimeter) generates the surface of the solid.

Each point of the generating surface describes a circle.

The points of the perimeter describe **parallels,** and planes drawn through the axis determine **meridians** in the surface of revolution.

Principal Solids of Revolution or Three Round Bodies.—They are the **cylinder,** the **cone** and the **sphere.**

Cylinder of Revolution—or Circular Right Cylinder (Fig. 148).—The axis is one side **BC** of a rectangle **ABCD;** the generating surface is that rectangle. The axis is the height **h** of the cylinder. The side **AD** opposite the axis generates the lateral surface and the other two sides generate the bases of which they are the radii **R.**

Directrices or Elements.—The successive positions of the side opposite the axis.

Truncated Cylinder or Frustrum of a Cylinder (Fig. 149).—Portion of a cylinder between one base **B** and any sectional plane **M** cutting all the directrices.

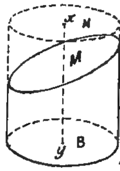


Fig 149

A right section **N** is perpendicular to the axis **xy** and is therefore parallel to the base **B.**

Lateral Area of a Right Cylinder.

Product of the circumference of base $2\pi R$ by the height h:

$$Lat.\ A = 2\pi Rh$$

Total Area.—

$$A = 2\pi Rh + 2\pi R^2 = 2\pi R(R+h).$$

Volume of a Cylinder of Revolution.—Product of the area of base πR^2 by height h: $V = \pi R^2 h$.

Volume of an Oblique Cylinder (Fig. 150).—Product of area of right section **S** by a directrix or element **a: V=Sec.Sxa.**

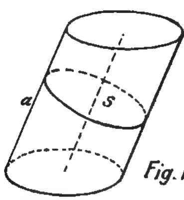


Fig. 150

Volume of a Frustrum of a Cylinder of Revolution.—Product of right section **B** by axis **d. V=Sec.Bxd.**

Cone of Revolution (Fig. 151).—Solid generated by a revolution of a right angled triangle **ABS** turning around one side **BS** called **axis**; the other side **AB** generates the **base** of which it is the radius **R**

and the hypothuse **SA** generates the lateral surface. The hypothenuse in any of its positions is called an element, generatrix or side of the cone.

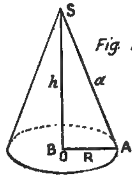


Fig. 151

Lateral Area of a Cone of Revolution.—½ product of circumference of base

$2\pi R$ by side a : $\quad Lat.\ A = \frac{1}{2}\,2\pi R a = \pi R a$.

Total Area of Cone.—Lateral area

$\pi R a$ plus base πR^2 : $\quad A = \pi R a + \pi R^2 = \pi R(a+R)$.

Volume of a Cone of Revolution (Fig. 152).—⅓ product of base

πR^2 by height h : $\quad V = \dfrac{\pi R^2 h}{3}$.

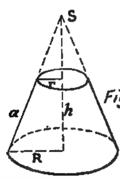


Fig. 152

Lateral Area of a Frustrum of a Cone (Fig. 152).—(Formed by a secant plane parallel to base.) ½ product of an element **a** by the sum of the circumferences of the bases

which is $2\pi(R+r)$. $\quad Lat.\ A = \pi a (R+r)$.

Total Area.—Lateral area plus area of bases

$\pi R^2 + \pi r^2 = \pi(R^2+r^2)$: $\quad A = \pi[a(R+r) + (R^2+r^2)]$

Volume.—⅓ the product of the height **h** by the sum of the bases

πR^2 and πr^2, and a mean proportional $\sqrt{\pi R^2 \pi r^2} = \pi R r$ between them:

$$V = \frac{\pi h}{3}(R^2 + r^2 + Rr)$$

Sphere (Fig. 153).—Solid generated by a semi-circle **ABC** revolving about its diam. **AB** as an axis. The center **O** of the semi-circle is the center **O** of the sphere; its radius **OA** is the radius **R** of the sphere; the semi-circumference **ACB** generates the surface of the sphere. All the points of the surface are equally distant from the center.

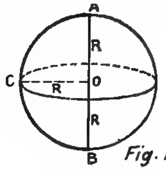


Fig. 153

Area.—Four great circles:

$$A = 4 \times \pi R^2 = 4\pi R^2 \quad \text{or} \quad A = \pi D^2.$$

Volume.—⅓ the product of area by radius:

$$V = \frac{4\pi R^3}{3} = \frac{\pi D^3}{6}.$$

Prismoidal Formula (Thomas Simpson).—The volume of a solid comprised between 2 parallel bases **B** and **b** (Fig. 154) (one of which may be zero) equals ⅙ the product of the height **h**, by the sum of the bases **B** and **b** plus 4 times a section **S** equidistant (half way) from the bases:

$$V = \frac{h\,(B + b + 4S)}{6}.$$

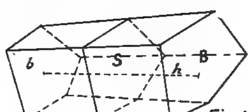


Fig. 154

Guldin's Formulas.—To obtain volumes of solids of revolution.—A solid of revolution may be considered as generated by the revolution of a polygon **S** (Fig. 155). Considering that polygon and connecting the middles of its sides, we form a new and smaller polygon. Connecting the middles of the new sides, then continuing indefinitely, these successive smaller and smaller polygons tend towards a certain fixed point **O** called in geometry **point of mean distance** and in mechanics **center of gravity** of the polygon.

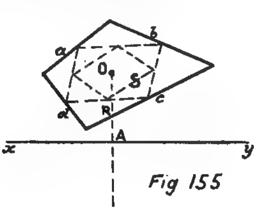


Fig 155

1°. The area **A** of a surface revolution equals the product of the length of the generating line **l=(a+b+c+d)** by the circumference **2πR** described by the center of gravity **O** (**R** being the dist. from the center of gravity to the axis **R=AO**):

$$A = 2\pi R l .$$

2° The volume **V** of a solid of revolution equals the product of the area **S** of the generating surface by the circumference **2πR** described by the center of gravity.

$$V = 2\pi R S$$

TRIGONOMETRY.

Trigonometry.—Science of trigons or triangles.

Object.—1° Calculating lines when lines and angles are given

2°. Calculating angles when lines or lines and angles are given.

Hypotheses.—A point **o** (Fig. 156) given in a plane is supposed to be fixed and a line **oA**, equal to the unit of length **oA=1**, and lying in that plane is supposed to revolve around **o**, say from right to left from an original, say horizontal, position.

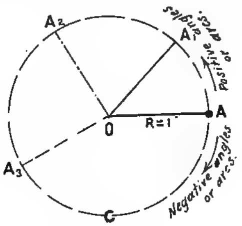


Fig. 156

Generation of Angles.—In that motion, **oA** will form, with its original position, angles gradually increasing to 90°, to 180°, to 270° and to 360° when a full revolution has been accomplished. The line. even then may continue its motion and generate angles greater than 360°.

Generation of Arcs.—In that motion, the point **A** describes, from the point of origin, arcs gradually increasing in the same ratio as the central angles corresponding to them.

Length of the Circumference.—

$C = 2\pi R$; but as $R = OA = 1$, that formula becomes $C = 2\pi$, whence $\frac{C}{2} = \pi$ and $\frac{C}{4} = \frac{\pi}{2}$. This is always so in trigonometry :
A circumference is always equal to 2π, a semi-circle to π and a quadrant to $\frac{\pi}{2}$, because the radius is always taken equal to 1.

Negative Angles and Arcs.—Those described by a contrary motion (left to right) of line **oA,** as indicated in figure by a descending arrow.

Complementary Angles and Arcs (Fig. 157).—Two angles or arcs the algebraic sum of which is 90°

or $\frac{\pi}{2}$: $A+B=90°$; $a+b=\frac{\pi}{2}$; one or both may be negative.

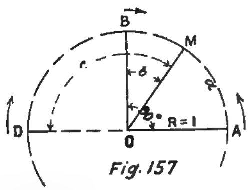


Fig. 157

Supplementary Angles and Arcs.—Two angles or arcs the algebraic sum of which is 180° or

π : $A+B=180°$; $a+b=\pi$; one or both may be negative.

Trigonometrical Circle (Fig. 158).—In a circle with **R=1,** draw 2 diam. perp. to each other, one horizontal, the other vertical. Let **o** be the center, **A** the right hand extremity of the horizontal diam., **C** its left hand extremity, **B** the top extremity of the vertical diam. and **D** its bottom extremity. Now consider an arc **AM=a** less than 90° laid off from **A** towards **B;** the arc **AM=a** being positive and less than 90°, its complement is $\frac{1}{2}\pi$—**a** will also be positive. That difference $\frac{1}{2}\pi$—**a** is the arc **BM** which completes the quadrant. We are then led to consider the point **B** as the origin of complementary arcs and these will be positive when reckoned from **B** to the right and negative in the opposite direction.

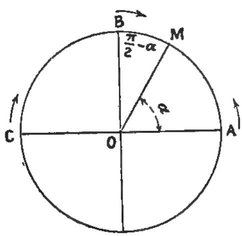


Fig. 158

Supplementary arcs might be considered as having their origin in **C,** but the same result is attained by admitting their origin to be at **A.**

Trigonometrical lines or circular functions of an arc (or angle) (Fig. 159).—Draw the trigonometrical circle and consider arc **AM=a.**

Draw a perpend. **MF** from pt. **M** to diam. of origin **CA.** Draw radius **oM** to the extremity of **a** and produce it. Draw a tangent at the origin **A** till it intersects at **E** the radius **oM** produced.

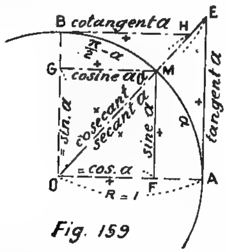


Fig. 159

Line **MF** is called the **sine** of arc **a** or of angle **MoA,** which is written **MF=sin, a.**

Line **AE** is called the **tangent** of arc **a** or of angle **MoA,** which is written **AE=tg. a.**

Line **oE** is called the **secant** of arc **a** or of angle **MoA,** which is written **oE=sec. a.**

Here we mean that the measures of **MF, AE, oE,** that is the ratios

$$\frac{MF}{OA},\ \frac{AE}{oA},\ \frac{oE}{oA}\quad \text{or}\quad \frac{MF}{R},\ \frac{AE}{R},\ \frac{oE}{R}\quad \text{or}\quad \frac{MF}{1},\ \frac{AE}{1},\ \frac{oE}{1}$$

are respectively **sin. a, tg. a, sec. a** which three lines are called the **direct circular functions** of **a.**

Consider now the complement

$$\frac{\pi}{2}-a = BM \quad \text{of} \quad AM$$

its origin is **B** and **M** is its extremity. The **sine** of **BM** will be **MG,** its **tangent BH** and its **secant oH** and these three **direct circular functions** of **BM** or **½π—a** are called **inverse circular functions** of the first angle or arc **a** of which **BM** is the complement, and they are respectively called **MG cosine** of **AM, BH cotangent** of **AM** and **oH cosecant** of **AM** and vice versa, and by definition:

$$\left.\begin{aligned} \sin a &= \cos\left(\tfrac{\pi}{2}-a\right) \\ \text{tang}\, a &= \cot\left(\tfrac{\pi}{2}-a\right) \\ \sec a &= \text{cosec}\left(\tfrac{\pi}{2}-a\right) \end{aligned}\right\} \text{and conversely} \left\{\begin{aligned} \cos a &= \sin\left(\tfrac{\pi}{2}-a\right) \\ \cot a &= tg\left(\tfrac{\pi}{2}-a\right) \\ \text{cosec}\, a &= \sec\left(\tfrac{\pi}{2}-a\right) \end{aligned}\right.$$

Note.—As **MG=Fo,** we may consider the cosine as the abscissa corresponding to the sine or ordinate of the extremity of the arc.

Sine.—The ordinate at the extremity of the arc.

Tangent.—The tangent at the origin limited by the radius of the extremity.

Secant.—Distance between the center and the extremity of the tangent.

Signs of the functions (Fig. 160).—They are supposed to be all

positive for an arc less than $\frac{1}{2}\pi$ and to be negative when they have a contrary direction. Hence:

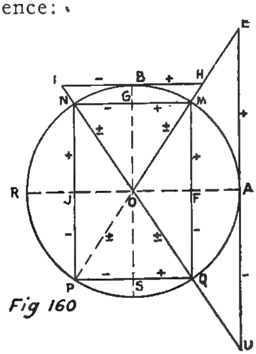


Fig 160

A sine is + when perpend. above and — when perp. below, the diam. of origin.

A tangent is + when perpend. above and — when perp. below, the diam. of origin.

A secant is + when it does and — when it does not, pass through the extremity of the arc.

A cosine is + when to the right and — when to the left, of the vertical diam.

A cotg. is + when to the right and — when to the left, of the vertical diam.

A cosec. is + when it does and — when it does not, pass through the extremity of the arc.

Table of Signs.

Arc ending in	sine	cosine	tangent	cotangent	secant	cosecant
1 st. Quadrant	+	+	+	+	+	+
2nd. "	+	—	—	—	—	+
3rd. "	—	—	+	+	—	—
4th. "	—	+	—	—	+	—

Table of the Variations of the Functions.

Functions	when passing gradually from	0 to	$\frac{\pi}{2}$ to	π to	$\frac{3\pi}{2}$ to	$2\pi=0$
sine	pass gradually from first to last value with signs shown.	$0 + <$	$1 + >$	$0 - >$	$-1 - <$	0
cosine		$1 + >$	$0 - >$	$-1 - <$	$0 + <$	1
tangent		$0 + <$	$\pm\infty - <$	$0 + <$	$\pm\infty - <$	0
cotangent		$\mp\infty + >$	$0 - >$	$\mp\infty + >$	$0 - >$	$\mp\infty$
secant		$1 + <$	$\pm\infty - <$	$-1 - <$	$\mp\infty + <$	1
cosecant						

REMARK : *The change from $+\infty$ to $-\infty$, or from $-\infty$ to $+\infty$ is instantaneous.*

Let the reader study these variations carefully.

Functions of Supplementary Angles.—By referring to the trigometrical circle we see that supplementary angles have all their lines equal and of contrary sign except sine and cosec. which have same sign. We then have these sets of formulas:

$$\sin a=\sin(\pi-a);\ \sin(\tfrac{\pi}{2}-a)=\sin(\tfrac{\pi}{2}+a);\ \sin a=\sin(180^\circ-a);\ \sin(90^\circ-a)=\sin(90^\circ+a).$$
$$tg\, a=-tg(\pi-a);\ tg(\tfrac{\pi}{2}-a)=-tg(\tfrac{\pi}{2}+a);\ tg\, a=-tg(180^\circ-a);\ tg(90^\circ-a)=-tg(90^\circ+a).$$
$$\sec a=-\sec(\pi-a);\ \sec(\tfrac{\pi}{2}-a)=-\sec(\tfrac{\pi}{2}+a);\ \sec a=-\sec(180^\circ-a);\ \sec(90^\circ-a)=-\sec(90^\circ+a).$$
$$\cos a=-\cos(\pi-a);\ \cos(\tfrac{\pi}{2}-a)=-\cos(\tfrac{\pi}{2}+a);\ \cos a=-\cos(180^\circ-a);\ \cos(90^\circ-a)=-\cos(90^\circ+a).$$
$$\cot a=-\cot(\pi-a);\ \cot(\tfrac{\pi}{2}-a)=-\cot(\tfrac{\pi}{2}+a);\ \cot a=-\cot(180^\circ-a);\ \cot(90^\circ-a)=-\cot(90^\circ+a).$$
$$\mathrm{cosec}\, a=\mathrm{cosec}(\pi-a);\ \mathrm{cosec}(\tfrac{\pi}{2}-a)=\mathrm{cosec}(\tfrac{\pi}{2}+a);\ \mathrm{cosec}\, a=\mathrm{cosec}(180^\circ-a);\ \mathrm{cosec}(90^\circ-a)=\mathrm{cosec}(90^\circ+a).$$

Five Fundamental Formulas.

Draw the trigonometrical circle and all the lines of an arc **AM** and of its complement **BM.** Remember that **R=oA=oM=oB=1.**

1°. Triangle **oMF** gives:

$$\overline{MF}^2+\overline{OF}^2=\overline{OM}^2 \quad \text{or} \quad \sin^2 a+\cos^2 a=1 \qquad (1)$$

2°. Similar triangles **oMF** and **oEA** give:

$$\frac{AE}{OA}=\frac{MF}{OF} \quad \text{or} \quad tg\, a=\frac{\sin a}{\cos a} \qquad (2)$$
$$\frac{oE}{OM}=\frac{OA}{OF} \quad \text{or} \quad \sec a=\frac{1}{\cos a} \qquad (3)$$

3°. Similar triangles **oBH** and **oGM** give:

$$\frac{BH}{oB}=\frac{MG}{oG} \quad \text{or} \quad \cot g\, a=\frac{\cos a}{\sin a} \qquad (4)$$
$$\frac{oH}{oM}=\frac{oB}{oG} \quad \text{or} \quad \mathrm{cosec}\, a=\frac{1}{\sin a} \qquad (5)$$

There can be only five equations between the six functions.—These relations are independent of the arc and are only expressed between the functions in terms of each other, so if there were 6 different relations they would form a system of equations admitting only of definite values for each and these functions would not be variable as the arc.

Other formulas may be deducted.—By combining the 5 fundamental formulas, we may obtain new (but not different) formulas.

1° Squaring **(3)** and **(2)** and subtracting we get:

$$\sec^2 a - tg^2 a=\frac{1}{\cos^2 a}-\frac{\sin^2 a}{\cos^2 a}=\frac{1-\sin^2 a}{\cos^2 a}=\frac{\cos^2 a}{\cos^2 a};$$
$$\sec^2 a - tg^2 a=1 \qquad (6)$$

2° Squaring **(2):**

$$tg^2 a=\frac{\sin^2 a}{\cos^2 a};\quad 1+tg^2 a=1+\frac{\sin^2 a}{\cos^2 a}=\frac{\sin^2 a+\cos^2 a}{\cos^2 a}=\frac{1}{\cos^2 a}=\sec^2 a;$$
$$1+tg^2 a=\sec^2 a \qquad (7)\ \text{same as (6).}$$

3°. The product of **(2)** and **(4)** gives: **tg. a cotg. a = 1 (8)**

4°. Equation **(3)** may be written: **sec. a cos. a = 1 (9)**

5°. Equation (5) may be written: **cosec. a sin. a = 1 (10)**

Note: Tangent and cotangent, secant and cosine, sosecant and sine are inverse or reciprocal of each other and have always the same sign.

Sine and Cosine of the sum of two arcs (Fig. 161).—Take two arcs **AM=a** and **MN=b** set one after the other so that **A** being the origin of **a, M** will be the origin of **b.** Draw **MF=sin. a** perp. from **M** to diam. **oA** of origin of **a; oF** will be **cos. a.** Draw **NI=sin. b** perp. from **N** to diam. **oM** of origin of **b; oI** will be **cos. b.** Now arc **a+b** is **AMN:** draw **NJ=sin (a+b)** perp. from **N** to diam. **oA** of origin of **(a+b); oJ** will be **cos. (a+b).** In order to calculate **sin. (a+b)** and **cos. (a+b),** draw **IK** perp. to **oA** and **IL** parallel to **oA.** By comparing the similar triangles **NIL** and **MoF,** we have:

$$\frac{LI}{FM} = \frac{NI}{oM} = \frac{NL}{oF} \quad \text{or} \quad \frac{LI}{\sin a} = \frac{\sin b}{1} = \frac{NL}{\cos a}$$

from which $LI = JK = \sin a \sin b$ and $NL = \cos a \sin b$

Similar Δ^s IOK and MoF give: $\frac{IK}{MF} = \frac{oI}{oM} = \frac{oK}{oF}$ or $\frac{IK}{\sin a} = \frac{\cos b}{1} = \frac{oK}{\cos a}$

from which $IK = LJ = \sin a \cos b$ and $oK = \cos a \cos b$.

We have to calculate $\left\{\begin{array}{l} \sin(a+b) = NJ = NL + LJ \\ \cos(a+b) = oJ = oK - JK \end{array}\right\}$ substituting the values of NL, LJ, oK and JK just found, we get:

$$\left\{\begin{array}{ll} \sin(a+b) = \sin a \cos b + \cos a \sin b & (11) \\ \cos(a+b) = \cos a \cos b - \sin a \sin b & (12) \end{array}\right.$$

which are proved to be general.

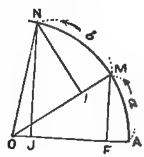


Fig. 161

Sine and Cosine of the difference of two arcs (Fig. 162).—By placing the second arc **b** from **M** towards **A** and studying the case in a similar manner, we would obtain the formulas:

$$\left\{\begin{array}{ll} \sin(a-b) = \sin a \cos b - \cos a \sin b & (13) \\ \cos(a-b) = \cos a \cos b + \sin a \sin b & (14) \end{array}\right.$$

which are also proved to be general.

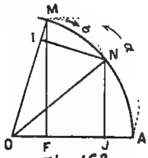


Fig. 162

Tangent of the sum of two arcs.—

$$tg\,(a+b)=\frac{\sin(a+b)}{\cos(a+b)}=\frac{\sin a\cos b+\cos a\sin b}{\cos a\cos b-\sin a\sin b}=\frac{\frac{\sin a\cos b}{\cos a\cos b}+\frac{\cos a\sin b}{\cos a\cos b}}{\frac{\cos a\cos b}{\cos a\cos b}-\frac{\sin a\sin b}{\cos a\cos b}}$$

$$=\frac{\frac{\sin a}{\cos a}+\frac{\sin b}{\cos b}}{1-\frac{\sin a}{\cos a}\times\frac{\sin b}{\cos b}}\quad\therefore\quad tg\,(a+b)=\frac{tg\,a+tg\,b}{1-tg\,a\,tg\,b}\qquad(15)$$

Tangent of the difference of two arcs.—We would in like manner obtain

$$tg\,(a-b)=\frac{tg\,a-tg\,b}{1+tg\,a\,tg\,b}\qquad(16)$$

Cotangent of the sum and the difference of two arcs.—

$$cotg\,(a+b)=\frac{\cot a\cot b-1}{\cot a+\cot b}\qquad(17)$$

$$\cot\,(a-b)=\frac{\cot a\cot b+1}{\cot b-\cot a}\qquad(18)$$

In like manner we might obtain **sec (a+b)** and **sec (a—b).**

Multiple Arcs, Sine 2 a, cos. 2 a, etc.—Making **b=a** in formulas **(11) (12),** they become:

$$\left.\begin{cases}\sin 2a=2\sin a\cos a & (19)\\ \cos 2a=\cos^2 a-\sin^2 a & (20)\end{cases}\right\}\text{and in function of one of them:}$$

$$\left\{\begin{array}{l}\sin 2a=\pm\,2\sin a\sqrt{1-\sin^2 a}\\ \cos 2a=1-2\sin^2 a\end{array}\right\}\text{and}\left\{\begin{array}{ll}\sin 2a=\pm\,2\cos a\sqrt{1-\cos^2 a} & (21)\\ \cos 2a=2\cos^2 a-1 & (22)\end{array}\right.$$

Likewise by making **b=2 a** in formulas **(11) (12),** they become:

$$\begin{cases}\sin 3a=\sin a\cos^2 a-\sin^3 a+2\sin a\cos^2 a=3\sin a\cos^2 a-\sin^3 a & (23)\\ \cos 3a=\cos^3 a-\cos a\sin^2 a-2\cos a\sin^2 a=\cos^3 a-3\cos a\sin^2 a & (24)\end{cases}$$

Tangent 2 a, cot. 2a, etc.—Making **b=a** in formula **(15),** it becomes:

$$tg\,2a=\frac{2\,tg\,a}{1-tg^2 a}\qquad(25)\qquad\text{Likewise by making } b=2a$$

$$tg\,3a=\frac{tg\,a+tg\,2a}{1-tg\,a\,tg\,2a}=\frac{3\,tg\,a-tg^3 a}{1-3\,tg^2 a}\qquad(26)$$

Making **b=a** in formula **(17):**

$$\cot.2a=\frac{\cot^2 a-1}{2\cot a}\qquad(27)\qquad\text{Likewise by making } b=2a\ :$$

$$\cot 3a=\frac{\cot a\cot 2a-1}{\cot a+\cot 2a}=\frac{\cot^3 a-3\cot a}{3\cot^2 a-1}\qquad(28)$$

Sub-multiple Arcs.—

$\sin\frac{a}{2}$ and $\cos\frac{a}{2}$. In formulas (1) and (20), replace a by $\frac{a}{2}$; they become:

$$\begin{cases}\sin^2\frac{a}{2}+\cos^2\frac{a}{2}=1\\ \cos^2\frac{a}{2}-\sin^2\frac{a}{2}=\cos a\end{cases}$$

By subtracting them : $2\sin^2\frac{a}{2} = 1-\cos a \quad ; \quad \sin\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{2}}$ (29)

By adding them : $2\cos^2\frac{a}{2} = 1+\cos a \quad ; \quad \cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}}$ (30)

By replacing a by $\frac{a}{2}$ in formulas (1) and (19), they become :

$$\left\{\begin{array}{l} \sin^2\frac{a}{2}+\cos^2\frac{a}{2} = 1 \\ 2\sin\frac{a}{2}\cos\frac{a}{2} = \sin a \end{array}\right\} \text{by adding \& subtracting these:} \left\{\begin{array}{l} \sin^2\frac{a}{2}+2\sin\frac{a}{2}\cos\frac{a}{2}+\cos^2\frac{a}{2} = 1+\sin a \\ \sin^2\frac{a}{2}-2\sin\frac{a}{2}\cos\frac{a}{2}+\cos^2\frac{a}{2} = 1-\sin a \end{array}\right.$$

$$\text{or} \left\{\begin{array}{l} (\sin\frac{a}{2}+\cos\frac{a}{2})^2 = 1+\sin a \\ (\sin\frac{a}{2}-\cos\frac{a}{2})^2 = 1-\sin a \end{array}\right\} \text{or} \left\{\begin{array}{l} \sin\frac{a}{2}+\cos\frac{a}{2} = \pm\sqrt{1+\sin a} \\ \sin\frac{a}{2}-\cos\frac{a}{2} = \pm\sqrt{1-\sin a} \end{array}\right\} \text{again by adding and subtracting :}$$

$$\left\{\begin{array}{l} 2\sin\frac{a}{2} = \pm\sqrt{1+\sin a}\pm\sqrt{1-\sin a} \\ 2\cos\frac{a}{2} = \pm\sqrt{1+\sin a}\mp\sqrt{1-\sin a} \end{array}\right\} \text{or} \left\{\begin{array}{ll} \sin\frac{a}{2} = \pm\dfrac{\sqrt{1+\sin a}\pm\sqrt{1-\sin a}}{2} & (31) \\ \cos\frac{a}{2} = \pm\dfrac{\sqrt{1+\sin a}\mp\sqrt{1-\sin a}}{2} & (32) \end{array}\right.$$

Tangent $\frac{a}{2}$.

Change a to $\frac{a}{2}$ in formula (25), it becomes: $tg\,a = \dfrac{2\,tg\frac{a}{2}}{1-tg^2\frac{a}{2}}$

or $tg\,a\,tg^2\frac{a}{2}+2\,tg\frac{a}{2}-tg\,a = 0$, from which: $tg\frac{a}{2} = \dfrac{-1\pm\sqrt{1+tg^2 a}}{tg\,a}$ (33)

In terms of $\sin a$ and $\cos a$:

$$tg\frac{a}{2} = \frac{\sin\frac{a}{2}}{\cos\frac{a}{2}} = \frac{\pm\sqrt{\frac{1-\cos a}{2}}}{\pm\sqrt{\frac{1+\cos a}{2}}} \text{ (See (29) and (30)) } = \pm\sqrt{\frac{1-\cos a}{1+\cos a}}$$

$$tg\frac{a}{2} = \left\{\begin{array}{lll} = \pm\sqrt{\dfrac{(1-\cos a)^2}{1-\cos^2 a}} & = \pm\dfrac{1-\cos a}{\sin a} & (34) \\ = \pm\sqrt{\dfrac{1-\cos^2 a}{(1+\cos a)^2}} & = \pm\dfrac{\sin a}{1+\cos a} & (35) \end{array}\right.$$

Other formulas deducted from (11), (12), (13) and (14).—Writing these formulas in this order:

$$\left.\begin{array}{ll} (11) & \sin(a+b) = \sin a\cos b+\cos a\sin b \\ (12) & \sin(a-b) = \sin a\cos b-\cos a\sin b \\ (13) & \cos(a+b) = \cos a\cos b-\sin a\sin b \\ (14) & \cos(a-b) = \cos a\cos b+\sin a\sin b \end{array}\right\} \text{By addition and subtraction we obtain the following :}$$

$$(36)\left\{\begin{array}{l} \sin(a+b)+\sin(a-b) = 2\sin a\cos b \\ \sin(a+b)-\sin(a-b) = 2\cos a\sin b \\ \cos(a+b)+\cos(a-b) = 2\cos a\cos b \\ \cos(a-b)-\cos(a+b) = 2\sin a\sin b \end{array}\right\}$$

Making $a+b = p$ and $a-b = q$, which gives $a = \frac{p+q}{2}$; $b = \frac{p-q}{2}$, the last formulas become

$$(37)\left\{\begin{array}{l} \sin p+\sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2} \\ \sin p-\sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2} \\ \cos p+\cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2} \\ \cos q-\cos p = 2\sin\frac{p+q}{2}\sin\frac{p-q}{2} \end{array}\right\}$$

Dividing these two and two, we get ·

$$
(38)\left\{\begin{aligned}
\frac{\sin p+\sin q}{\sin p-\sin q} &= \frac{2\sin\frac{p+q}{2}\cos\frac{p-q}{2}}{2\cos\frac{p+q}{2}\sin\frac{p-q}{2}} = \frac{tg\,\frac{p+q}{2}}{tg\,\frac{p-q}{2}}\\
\frac{\sin p+\sin q}{\cos p+\cos q} &= \frac{2\sin\frac{p+q}{2}\cos\frac{p-q}{2}}{2\cos\frac{p+q}{2}\cos\frac{p-q}{2}} = tg\,\frac{p+q}{2}\\
\frac{\sin p+\sin q}{\cos q-\cos p} &= \frac{2\sin\frac{p+q}{2}\cos\frac{p-q}{2}}{2\sin\frac{p+q}{2}\sin\frac{p-q}{2}} = \cot\frac{p-q}{2}\\
\frac{\sin p-\sin q}{\cos p+\cos q} &= \frac{2\cos\frac{p+q}{2}\sin\frac{p-q}{2}}{2\cos\frac{p+q}{2}\cos\frac{p-q}{2}} = tg\,\frac{p-q}{2}\\
\frac{\sin p-\sin q}{\cos q-\cos p} &= \frac{2\cos\frac{p+q}{2}\sin\frac{p-q}{2}}{2\sin\frac{p+q}{2}\sin\frac{p-q}{2}} = \cot\frac{p+q}{2}\\
\frac{\cos p+\cos q}{\cos q-\cos p} &= \frac{2\cos\frac{p+q}{2}\cos\frac{p-q}{2}}{2\sin\frac{p+q}{2}\sin\frac{p-q}{2}} = \cot\frac{p+q}{2}\cot\frac{p-q}{2}
\end{aligned}\right.
$$

Trigonometrical Tables.

From the following: $\frac{a^3}{4} > a - \sin a$ and $1-\frac{a^2}{2} < \cos a < 1-\frac{a^2}{2}+\frac{a^4}{16}$, the value of an arc of 10", which, calculated from the geometrical formula $l=\frac{\pi R n}{180}$ or $l=\frac{\pi n}{180}$ (because $R=1$), or $l=\frac{\pi\times 10}{180\times 60\times 60}=\frac{\pi}{64800}$;

1=0.000048481368110..... may be assumed as the value of **sin. 10"** to the 12th decimal place; **sin. 10"=0.0000484813681** with an error less than one-half of a unit of the last decimal.

Cos. 10" may be obtained likewise with 13 decimals: **Cos. 10"= 0.9999999988248**..... Now adding **(13)** and **(14)** to **(11)** and **(12)** respectively, we have:

$$
(36)\left\{\begin{aligned}
\sin(a+b)+\sin(a-b) &= 2\sin a\cos b\\
\cos(a+b)+\cos(a-b) &= 2\cos a\cos b
\end{aligned}\right\}\text{ in which make } a=mb\text{; they become:}
$$

$$
(39)\left\{\begin{aligned}
\sin(m+1)b &- 2\sin mb\cos b-\sin(m-1)b\\
\cos(m+1)b &= 2\cos mb\cos b-\cos(m-1)b
\end{aligned}\right\}\text{then giving } m \text{ successive values } 1,2,3, \text{etc., and making } b=10''
$$

$$
(40)\left\{\begin{aligned}
m=1&\left\{\begin{aligned}\sin 20'' &= 2\sin 10''\cos 10''-\sin 0 = 2\sin 10''\cos 10''\\ \cos 20'' &= 2\cos 10''\cos 10''-1 = 2\cos^2 10''-1\end{aligned}\right.\\
m=2&\left\{\begin{aligned}\sin 30'' &= 2\sin 20''\cos 10''-\sin 10''\\ \cos 30'' &= 2\cos 20''\cos 10''-\cos 10''\end{aligned}\right.\\
m=3&\left\{\begin{aligned}\sin 40'' &= 2\sin 30''\cos 10''-\sin 20''\\ \cos 40'' &= 2\cos 30''\cos 10''-\cos 20''\end{aligned}\right.
\end{aligned}\right.\quad\text{and so on.}
$$

Calculations may be simplified from the remark that the constant factor **2 cos. 10"** differs very little from **2**, and by representing the difference by **K**; **K=2—2 cos. 10"**, we have **K=0.0000000023504**.... By making **2 cos. 10"=2—K**; formula **(39)** becomes:

$$
\left\{\begin{aligned}
\sin(m+1)b &= 2\sin mb-\sin(m-1)b-k\sin mb\\
\cos(m+1)b &= 2\cos mb-\cos(m-1)b-k\cos mb
\end{aligned}\right.
$$

and

$$
\left\{\begin{aligned}
\sin(m+1)b-\sin mb &= \sin mb-\sin(m-1)b-k\sin mb\\
\cos(m+1)b-\cos mb &= \cos mb-\cos(m-1)b-k\cos mb
\end{aligned}\right\}\text{ and formulas (40) become}
$$

$$
(41)\left\{\begin{aligned}
m=1&\left\{\begin{aligned}\sin 20''-\sin 10'' &= \sin 10''-k\sin 10''\\ \cos 20''-\cos 10'' &= \cos 10''-k\cos 10''-1\end{aligned}\right.\\
m=2&\left\{\begin{aligned}\sin 30''-\sin 20'' &= \sin 20''-k\sin 20''-\sin 10''\\ \cos 30''-\cos 20'' &= \cos 20''-\cos 10''-k\cos 20''\end{aligned}\right.\\
m=3&\left\{\begin{aligned}\sin 40''-\sin 30'' &= \sin 30''-\sin 20''-k\sin 30''\\ \cos 40''-\cos 30'' &= \cos 30''-\cos 20''-k\cos 30''\end{aligned}\right.
\end{aligned}\right.\quad\text{and so on.}
$$

These formulas are due to Thomas Simpson.

It is necessary to verify results frequently. To that end we calculate directly **sin.** and **cos.** of 9° 18° 27°; 30° 36° 45° which it is easy to obtain with any number of decimals. After calculating the sines and cosines of arcs up to 30°, those of arcs less than 45° can easily be calculated by subtraction. If we notice that **sin. 30°=½,** we have

$$\left\{\begin{array}{l} \sin(30°+x) + \sin(30°-x) = 2\cos x \sin 30° = \cos x \\ \cos(30°-x) - \cos(30°+x) = 2\sin x \sin 30° = \sin x \end{array}\right\} \quad \text{and}$$

$$(42) \quad \left\{\begin{array}{l} \sin(30°+x) = \cos x - \sin(30°-x) \\ \cos(30°+x) = \cos(30°-x) - \sin x \end{array}\right.$$

From 45°to 90° the sin. and cos. are given by formulas

$$\left\{\begin{array}{l} \sin.(45°+a) = \cos(45°-a) \\ \cos(45°+a) = \sin(45°-a) \end{array}\right.$$

Tangents and cotangents could be obtained from **(2)** and **(4)** which give:

$$\left.\begin{array}{l} \log tg\, a = \log \sin a - \log \cos a \\ \log cot\, a = \log \cos a - \log \sin a \end{array}\right\} \text{from which } tg\, a \text{ and } cot\, a.$$

Tables of Natural Lines.—These considerations might lead to the formation of tables of natural sines, cosines, tangents and cotangents.

Logarithms of Trigonometrical Functions.—Having a Table of Natural Lines we can find their **logs.** in a Table of Logarithms and thus form a **Table of Logs.** of Trigometrical Functions. However, this work, although possible, would be very tedious; also these lines and their logs. were obtained differently by means of series converging very rapidly.

Value of the Radius in the Formation of Logarithmic Trigonometrical Tables.—As **sine** and **cosine** vary only between 0 and 1, their logarithms would all be negative; to obviate this, the radius is assumed to be 10 000 000 000=10^{10}; then the logs. will all be greater than 0. In working the tables, the characteristic should always be diminished by 10 which may bring a negative index with positive mantissa.

Remark.—Logarithms of cotangents between 0° and 45° and of tangents between 45° and 90° were not increased and their logs. must be used as found; it is because of the fact that within these limits **tg. a** and **cotg. a** are greater than 1 and their logs. are therefore positive.

Arrangement of Tables of Logarithms.—Vega's arrangement is as follows:

1°. The tables contain logarithms of sin., cos., tang. and cotg. of angles between 0° and 90°, increasing by 10".

2°. Degrees from 0° to 45° are entered at top of page; for these angles the page must be read from top down.

Degrees from 45° to 90° are entered at bottom of page; for these angles the page must be read from bottom up.

3°. Minutes are entered in first left hand vertical column reading downwards, for angles less than 45° and in first right hand vertical column reading upwards for angles of 45° and more. A page covers ten minutes.

4° Seconds are entered in second left hand vertical column reading downwards for angles less than 45° and in second right hand vertical column reading upwards for angles of 45° and more. Seconds are entered only as 0, 10, 20, 30, 40, 50.

Remark.—Degrees and minutes, although written but once, are understood to be written at the beginning of every line whether to left or right.

5°. In the wide columns following each angle will be found. in succession, and from left to right, the logarithms of the **sine,** the **tangent,** the **cotangent** and the **cosine,** as indicated at the top of each column, for angles less than 45° These same columns become successively the logarithms of the cos., the cotangent, the tangent and the sine, as indicated at the bottom of each column, for angles of 45° and more. Note that the sum of two angles, one with its degrees taken at the top of the page, its minutes and seconds on the left hand, and the other with its degrees taken at the bottom of the page, its minutes and seconds on the right hand, and on the same horizontal line as seconds of the first angle, is 90°. These angles are complementary and the sine of one becomes the cosine of the other, and the tangent of one becomes the cotangent of the other.'

6°. Smaller columns headed **d.** (differences) follow those containing the logs of sines and cosines, and one headed **d. c.** (common difference) is between the columns containing the logs. of tangents and cotangents. The numbers which they contain are written between the lines of the logs. and they indicate, as may easily be ascertained by subtraction, the difference between consecutive logs., 86 for instance, that is the diff. between logs. of lines of angles differing by 10" The difference for one second is 1-10 which is obtained by simply placing a decimal point before the last figure to the right 8.6. and the difference for say 3" will be obtained by multiplying by 3 the difference 8.6 found for 1". The product 25.8 is an **increment** for logs. of sines and tangents and a **reduction** for logs. of cosines and cotangents because these last two lines decrease when the angle increases and conversely.

Ex. I. Let lg. sin. 23° 18' 27.6" and lg. cos. 23° 18' 27.6" be sought, Look for 23° at top of page; it is written on 6 pages; look for the page the minutes of which begin with 10 on the first left hand column; follow that column until you reach 18'; between 18 and 19 you look on the next column for the number of seconds—find 20.

in the column of Sines find log sin 23°18'20"	= 9.597 2942
In the small column d following find 48.9 for 1"	
and for 7.6" it will be 48.9 × 7.6	371.6
which add. You get log sin 23°18'27.6"	9.597 3314
or, reducing the characteristic (index) by 10,	$\bar{1}$.597 3314

Now for log. cos. 23° 18' 27.6" you proceed a little differently; as 27.6=20+7.6, it also equal 30—2.4. In order to avoid a subtraction:

look for $\log\cos 23°18'30'' = \bar{1}.963\,0266$
add to it the *diff.* for 2.4" which is $9.1 \times 2.4 = 21.84$
You get $\log\cos 23°18'27.6'' = \bar{1}.963\,0288$

You would do the same for $\log\cot 8°02'32.7''$
$\log\cot 8°02'40'' \quad 0.849\,7597$
diff. for 7.3" (because 32.7 = 40 − 7.3) is $151.9 \times 7.3 = 1018.87$
Therefore $\log\cot 8°02'32.7'' = 0.849\,8616$

Ex. 11.

Find x and y, given $\begin{cases}\log\sin x = \bar{1}.864\,9237 \\ \log\cot y \quad 1.124\,9216\end{cases}$

1° $\bar{1}$ corresponding to 9, look in the *Tables*, in the column of *sines*, the number nearest to but *smaller* than $9.864\,9237$
You find it in page 547 and in the last column marked *Sin* at bottom; it is $9.864\,9113$ for $47°06'40''$
Their *difference* is $= 124$ for $6.3''$
Therefore the angle $x = 47°06'46.3''$

6.3" is obtained by dividing 124 by the *diff.* for 1" which is 19.6, thus: $\frac{124}{19.6} = \frac{1240}{196} = 6.3$

2° Look in the column of *cotangents* for the number nearest to but *greater* than $1.124\,9216$
You find it in page 310; it is $1.124\,9656$ for $4°17'20''$
Their difference is $= 440$ for $1.5''$

Therefore the angle $y = 4°17'21.5''$

Triangles.

Representation of Sides and Angles.—In any triangle we represent the sides by **a, b** and **c**, and the angles opposite by **A, B** and **C.**

In a right angled triangle **a** is the hypothenuse and **A** the right angle.

Right Angled Triangles.—Theorems.—I. In any right angled triangle (Fig. 163), each side equals the product of the hypothenuse by the sine of the angle opposite or by the cosine of the adjacent angle.

II. In any right angled triangle, each side equals the product of the other side by the tangent of the angle opposite or by the cotg. of the adjacent angle.

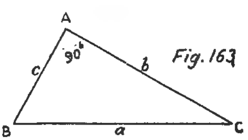


Fig. 163

These theorems may easily be proved by drawing from a vertex of the triangle an arc of a circle with **R=1,** drawing also its **sine** and **tangent,** and comparing each triangle thus formed with the given one to which they are similar.

From these 2 theorems we can write

$$(43)\ \begin{Bmatrix} b = a \sin B \\ c = a \sin C \end{Bmatrix}. \text{ But } \begin{Bmatrix} \sin B = \cos C \\ \sin C = \cos B \end{Bmatrix}; \text{ therefore, } \begin{Bmatrix} b = a \cos C \\ c = a \cos B \end{Bmatrix}\ (44)$$

$$(45)\ \begin{Bmatrix} b = c \operatorname{tg} B \\ c = b \operatorname{tg} C \end{Bmatrix}. \text{ But } \begin{Bmatrix} \operatorname{tg} B = \cot C \\ \operatorname{tg} C = \cot B \end{Bmatrix}; \text{ therefore } \begin{Bmatrix} b = c \cot C \\ c = b \cot B \end{Bmatrix}\ (46)$$

Solution of Right Angled Triangles.—

1°. Given the hypothenuse a and one angle B, find C, b, c and Area S.

$C = 90° - B$; $b = a \sin B$; $c = a \cos B$; and as area $= \frac{bc}{2}$,

$$S = \frac{a \sin B \times a \cos B}{2} = \frac{a^2 \sin B \cos B}{2} = \frac{a^2 \times 2 \sin B \cos B}{4} \text{ and as}$$

$$2 \sin B \cos B = \sin 2B,$$

$$S = \frac{a^2 \sin 2B}{4}; \qquad \log S = 2 \log a + \log \sin 2B - \log 4$$

2°. Given one side b and angle B, find C, a, c and Area S.

$C = 90° - B$; $c = \frac{b}{\sin B}$; $c = b \cot B$; and as area $= \frac{bc}{2}$,

$$S = \frac{b \times b \cot B}{2} = \frac{b^2 \cot B}{2}; \qquad \log S = 2 \log b + \log \cot B - \log 2.$$

3°. Given the hypothenuse a and one side b, find c, B, C and Area S.

$$\sin B = \frac{b}{a}; \quad c^2 = a^2 - b^2 = (a+b)(a-b); \quad c = \sqrt{(a+b)(a-b)};$$

$$\log c = \frac{\log(a+b) + \log(a-b)}{2}.$$

$$S = \frac{bc}{2} = \frac{b\sqrt{(a+b)(a-b)}}{2}; \quad \log S = \log b + \frac{\log(a+b) + (\log(a-b)}{2} - \log 2;$$

$$\log S = \frac{2 \log b + \log(a+b) + \log(a-b) - 2 \log 2}{2}$$

4°. Given b and c, find a, B, C and Area S.

We can get a from the formula $a = \sqrt{b^2 + c^2}$, but it is not calculable by logs; it is preferable to first find the angles.

$\operatorname{tg} B = \frac{b}{c} = \cot C$; $\log \operatorname{tg} B = \log \cot C = \log b - \log c$; then

$a = \frac{b}{\sin B}$; $\log a = \log b - \log \sin B$; $S = \frac{bc}{2}$; $\log S = \log b + \log c - \log 2$

Oblique Angled Triangles.

Theorems.—I—In any triangle (Fig. 164) the sines of the angles are proportional to the opposite sides. Draw a perpend. **AD** from **A**

to **a** for instance; that perpend. is common to two right angled triangles and its values are **AD=b sin. C** and **AD=c sin. B**; therefore **b sin. C=c sin. B** from which

$\frac{b}{\sin B} \quad \frac{c}{\sin C}$ If an other perp. than AD were drawn we would have .

$\frac{b}{\sin B} = \frac{a}{\sin A}$; therefore $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (47)

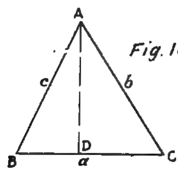


Fig. 164

Theorem II.—In any triangle, the square of a side equals the sum of the squares of the other two sides plus twice their product by the cosine of their angle.

Referring to the same triangle as above, with the perpend. **AD,** we have by geometry

$a^2 = b^2 + c^2 - 2b \times AD$, if the angle A is acute ; but $AD = c \cos A$; therefore $a^2 = b^2 + c^2 - 2bc \cos A$.

If A is obtuse, we have : $a^2 = b^2 + c^2 + 2b \times AD$; but $AD = c \cos BAD$ and $\cos BAD = -\cos A$;

therefore,
$$\left\{\begin{array}{l} a^2 = b^2 + c^2 - 2bc \cos A \\ b^2 = a^2 + c^2 - 2ac \cos B \\ c^2 = a^2 + b^2 - 2ab \cos C \end{array}\right\} \qquad (48)$$

From system **(48)** we deduce

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$; but from (29), $\sin^2 \frac{A}{2} = \frac{1 - \cos A}{2}$, and from (30), $\cos^2 \frac{A}{2} = \frac{1 + \cos A}{2}$

In these last two formulas replace **cos. A** by its value, they become

$$\left.\begin{array}{l} \sin^2 \frac{A}{2} = \dfrac{1 - \frac{b^2 + c^2 - a^2}{2bc}}{2} = \dfrac{a^2 - (b^2 + c^2 - 2bc)}{4bc} \text{ and } \sin \frac{A}{2} = \sqrt{\dfrac{(a+b-c)(a+c-b)}{4bc}} \\ \cos^2 \frac{A}{2} = \dfrac{1 + \frac{b^2 + c^2 - a^2}{2bc}}{2} = \dfrac{b^2 + c^2 + 2bc - a^2}{4bc} \text{ and } \cos \frac{A}{2} = \sqrt{\dfrac{(a+b+c)(b+c-a)}{4bc}} \end{array}\right\} (49)$$

We would also get from **(48)**:

$$\left\{\begin{array}{l} \sin \frac{B}{2} = \sqrt{\dfrac{(b+c-a)(b+a-c)}{4ac}} \\ \cos \frac{B}{2} \quad \sqrt{\dfrac{(a+b+c)(a+c-b)}{4ab}} \end{array}\right\} \text{and} \left\{\begin{array}{l} \sin \frac{C}{2} = \sqrt{\dfrac{(b+c-a)(a+c-b)}{4ab}} \\ \cos \frac{C}{2} = \sqrt{\dfrac{(a+b+c)(a+b-c)}{4bc}} \end{array}\right\} (49)$$

Now if we assume:

$$a+b+c = 2p,$$

we shall have: $\left\{\begin{aligned} b+c-a &= 2(p-a)\\ a+c-b &= 2(p-b)\\ a+b-c &= 2(p-c)\end{aligned}\right\}$, and formulas (49) become .

$$\left\{\begin{aligned} \sin\frac{A}{2} &= \sqrt{\frac{(p-b)(p-c)}{bc}}\\ \sin\frac{B}{2} &= \sqrt{\frac{(p-a)(p-c)}{ac}}\\ \sin\frac{C}{2} &= \sqrt{\frac{(p-a)(p-b)}{ab}}\end{aligned}\right\} \quad (50) \qquad \text{and} \qquad \left\{\begin{aligned} \cos\frac{A}{2} &= \sqrt{\frac{p(p-a)}{bc}}\\ \cos\frac{B}{2} &= \sqrt{\frac{p(p-b)}{ac}}\\ \cos\frac{C}{2} &= \sqrt{\frac{p(p-c)}{ab}}\end{aligned}\right\} \quad (51)$$

Dividing the equations in **(50)** by the corresponding in **(51)**, we get:

$$\left.\begin{aligned} tg\frac{A}{2} &= \sqrt{\frac{(p-b)(p-c)}{p(p-a)}}\\ tg\frac{B}{2} &= \sqrt{\frac{(p-a)(p-c)}{p(p-b)}}\\ tg\frac{C}{2} &= \sqrt{\frac{(p-a)(p-b)}{p(p-c)}}\end{aligned}\right\} \quad (52)$$

By applying formula **(19)**

to A and $\frac{A}{2}$, it becomes $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$, and by formulas (50) and (51),

$$\sin A \quad 2\sqrt{\frac{(p-b)(p-c)}{bc}}\sqrt{\frac{p(p-a)}{bc}}$$

$$\left.\begin{aligned} \sin A &= \frac{2}{bc}\sqrt{p(p-a)(p-b)(p-c)}\\ \sin B &= \frac{2}{ac}\sqrt{p(p-a)(p-b)(p-c)}\\ \sin C &= \frac{2}{ab}\sqrt{p(p-a)(p-b)(p-c)}\end{aligned}\right\} \quad (53)$$

Theorem III.—In any triangle, a side is equal to the sum of the products of the **cosines** of its **adjacent angles** each multiplied by the corresponding **adjacent side.**

Referring to the same triangle as above and taking the values of each portion of **a** into which it is divided by the perpendicular and adding them, we get:

$$\left.\begin{aligned} a &= b\cos C + c\cos B\\ b &= a\cos C + c\cos A\\ c &= a\cos B + b\cos A\end{aligned}\right\} \quad (54)$$

Solution of Oblique Angled Triangles.

1°. Given two angles B, C and side a, find A, b, c and Area S.

$$A = 180° - (B+C)\ ; \quad b = \frac{a\sin B}{\sin A}\ ; \quad c = \frac{a\sin C}{\sin A}\ ;$$

$$S = \frac{a \times AD}{2} = \frac{a\,b\sin C}{2} = \frac{a\sin C \times a\sin B}{2\sin A} = \frac{a^2\sin B\sin C}{2\sin A} = \frac{a^2\sin B\sin C}{2\sin(B+C)}\ .$$

2°. Given two sides a and b and the angle C which they form; find c, A, B and Area S.

From $\frac{a}{\sin A} = \frac{b}{\sin B}$: $\frac{a}{b} = \frac{\sin A}{\sin B}$; $\frac{a-b}{a+b} = \frac{\sin A - \sin B}{\sin A + \sin B} = \frac{tg\frac{A-B}{2}}{tg\frac{A+B}{2}} = \frac{tg\frac{A-B}{2}}{\cot\frac{C}{2}}$ by (38);

$$tg\,\frac{A-B}{2} = \frac{(a-b)\cot\frac{C}{2}}{a+b} \quad (55)$$

from which $\frac{A-B}{2}$ or $\frac{A}{2} - \frac{B}{2}$ is calculated:

$$A+B = 180° - C \quad \text{and} \quad \frac{A}{2} + \frac{B}{2} = \frac{180°-C}{2}$$

Suppose $\frac{A}{2} - \frac{B}{2} = m$

by addition $A = \frac{180°-C}{2} + m$;

by subtraction $B = \frac{180°-C}{2} - m$.

c may be computed by the formula $c = \frac{a \sin C}{\sin A}$

$$S = \frac{ab \sin C}{2} \quad (56)$$

3°. Given two sides a and b and the Angle A opposite one of them; find c, B, C and Area S.

$$\sin B = \frac{b \sin A}{a} \; ; \; C = 180° - (A+B) \; ; \; c = \frac{a \sin C}{\sin A} \; ; \; S = \frac{ab \sin C}{2} .$$

4°. Given the three sides a, b, c; find the three angles A, B, C, and Area S.

Using formula **(50)** we shall get

$\frac{A}{2}$, $\frac{B}{2}$, $\frac{C}{2}$ and therefore A, B and C.

We have $S = \frac{ab \sin C}{2}$ and by formula (53): $S = \frac{ab}{2} \times \frac{2}{ab}\sqrt{p(p-a)(p-b)(p-c)}$

Therefore : $S = \sqrt{p(p-a)(p-b)(p-c)}$ (57)

Example I.—Solve a right angled triangle—given the hypothenuse a=264.73 ft. and one angle B=34° 18' 50"

Formulas to use:

$$C = 90° - B \; ; \; b = a \sin B \; ; \; c = a \cos B \; , \; S = \frac{a^2 \sin B \cos B}{2}$$

Arrangement of Calculations.

$$\begin{array}{r} 90° \\ 34°\ 18'\ 50'' \\ \hline \end{array}$$

$C = 90° - 34° 18' 50'' = 55° 41' 10''$

$c = 264.73 \cos 34° 18' 50''$;

log 264.73	=	2.422 8032
log cos 34° 18' 50"	=	$\bar{1}$.916 9599
log c	=	2.339 7631
c	=	218.66 ft.

$$b = 264.73 \sin 34°18'50'';$$

$$\begin{aligned}
\log 264.73 &= 2.422\ 8032 \\
\log \sin 34°18'50'' &= \bar{1}.751\ 0683 \\
\hline
\log b &= 2.173\ 8715 \\
b &= 149.24 \text{ ft.}
\end{aligned}$$

$$S = \frac{\overline{264.73}^2 \sin 34°18'50'' \cos 34°18'50''}{2}$$

$$\begin{aligned}
\log 264.73 &= 2.422\ 8032 \\
\hline
2 \log 264.73 &= 4.845\ 6064 \\
\log \sin 34°18'50'' &= \bar{1}.751\ 0683 \\
\log \cos 34°18'50'' &= \bar{1}.916\ 9599 \\
-\log 2 &= \bar{1}.698\ 9700 \\
\hline
\log S &= 4.212\ 6046 \\
& \quad\ \underline{5871} \\
& \quad\ 175 \\
S &= 16\,315.7 \text{ Sq ft.}
\end{aligned}$$

Example II.—Solve a triangle given **a**=189.17; **b**=164.39 and **C**=43° 06' 30"

Formulas to use:

$$\text{Formulas to use: } \begin{cases} tg\,\dfrac{A-B}{2} = \dfrac{(a-b)\cot\frac{C}{2}}{a+b} ; \quad A+B = 180° - C \\ c = \dfrac{a \sin C}{\sin A} \qquad \dfrac{A+B}{2} = 90° - \dfrac{C}{2} \\ S = \dfrac{ab \sin C}{2} \end{cases}$$

Arrangement of Calculations:

$$\text{Data: } \begin{cases} a = 189.17 \\ b = 164.39 \end{cases} \quad C = 43°06'30''$$

$$\begin{aligned}
a-b &= 24.78 & \frac{C}{2} &= 21°33'15'' \\
a+b &= 353.56 & \frac{A+B}{2} &= 68°26'45''
\end{aligned}$$

$$\begin{aligned}
\log (a-b) = \log 24.78 &= 1.394\ 1013 \\
-\log (a+b) = -\log 353.56 &= \bar{3}.451\ 5369 \\
\log \cot \tfrac{C}{2} = \log \cot 21°33'15'' &= 0.403\ 3688 \\
& \quad + \underline{\quad 308} \\
\log tg\,\tfrac{A-B}{2} &= \bar{1}.249\ 0378 \\
\tfrac{A-B}{2} &= 10°03'40''
\end{aligned}$$

$$\begin{aligned}
2A &= 78°30'25'' & A &= 39°15'12.5'' \\
2B &= 58°23'05'' & B &= 29°11'32.5''
\end{aligned}$$

$$\begin{aligned}
\log a = \log 189.17 &= 2.276\ 8523 \\
\log \sin C = \log \sin 43°06'30'' &= \bar{1}.834\ 6622 \\
-\log \sin A = -\log \sin 39°15'12.5'' &= 0.198\ 7663 \\
\hline
\log c &= 2.310\ 2808 \\
c &= 204.31 \text{ ft.}
\end{aligned}$$

$$\begin{aligned}
\log a = \log 189.17 &= 2.276\ 8523 \\
\log b = \log 164.39 &= 2.215\ 8754 \\
\log \sin C = \log \sin 43°06'30'' &= \bar{1}.834\ 6622 \\
-\log 2 &= \bar{1}.698\ 9700 \\
\hline
\log S &= 4.026\ 3599 \\
& \quad\ \underline{289} \\
S &= 10\,625.8 \text{ Sq.ft.} \quad 310
\end{aligned}$$

TECHNICAL REQUIREMENTS.
SURVEYING.

Checking Stations.—As the leveling party follows the transit party, the stations are supposed to be properly staked. However, errors may creep in, and it is one of the duties of the Rodman to see that the stakes are properly numbered.

The Rodman must pace the distance when going from a station to another and check the stations to ascertain that there is neither omission nor duplication.

This pacing is particularly necessary in thick underbrush.

Correcting Wrong Stationing.—If he finds a mistake, he calls the attention of the leveler to it, goes back several stations to make sure that he is right, then corrects the wrong station.

How to Correct Stations.—If the transit party is not too far ahead in the work, the rodman should catch up to them and notify the transitman who will order the station numbers changed.

Station Equation.—If too much work has been done already, a station equation is written on the stakes where the error occurs.

Omission of a Station.—The station stakes are found marked thus (for instance): 120, 121, 123, 124.... As station 123 should have been marked 122, the stake may be raised and reset with the figures 123 facing the **F. S.** On the rear face the rodman then writes this station equation:

122
= 123

Duplication of a Station.—The stakes are found to read (for instance) 16, 17, 18, 18, 19, 20, 21.... In this case the station equation should be **19**
= 18 and the original marking 18 should be made to face the **F. S.**

Level Stakes.—Stakes on which levels are taken. They should be driven flush with the ground.

Bench Mark.—A permanent point such as the door-sill of a house, the coping of a culvert, a large boulder, the root of a tree, etc., the elevation of which is determined and recorded for reference.

How to Make a Bench Mark.—1°. On the root of a tree—Cut off a root so as to form a knob; if possible drive a spike into it on which to rest the rod. Blaze the trunk (which consists in cutting off a portion of the bark) for a surface on which to write the initials **B. M.** (Bench mark) **N° 18** (the number of it, if required) and **204.723** (the elevation of the Bench).

2°. On rock or stone—A cross may be made with a hatchet or a cold chisel.

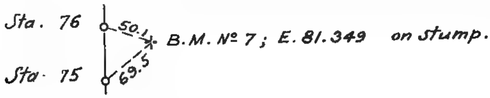
3°. On a vertical face—Draw a horizontal line of a permanent character.

The markings on rock or stone may be made with red chalk if they cannot be carved.

Locate Bench Marks.—They should be carefully located and the location entered in the field book, as for instance:

1°. Sta. 6+13.4 B.M. No. 3, E. 81.349 Rock on line.

2°. Sta. 31+27.4 18.0R. B.M. No. 5, E.139.086 Lower gate hinge.



Where Should Bench Marks Be Established.—Outside of possible disturbance and: 1°. Near the starting point of a line of levels. 2°. Near the end of the same. 3°. Generally about every half-mile. 4°. On each side of a large stream crossing the line. 5°. On hills. 6°. In depressions. 7°. At or near road crossings. 8°. Where a good opportunity for a very permanent mark presents itself.

Leveling Rod.—A long graduated ruler made of hard wood, to be held vertically when set at a point of the ground the elevation of which is desired.

How Constructed.—The foot has a ferrule to prevent wear.

The rod is generally built in two sections of equal lengths (for convenience in transportation). The rear section slides up in a groove or between guides, along the front section, and it can be set to any height by means of a bridle encircling both parts and a clamping screw.

Target.—The top of the ruler often carries a disc showing, by two lines drawn across its face, the exact end of the rod. That disc, called the Target, is movable and sometimes carries a vernier. The upper clamp also carries a vernier.

Short Rod.—When it is not necessary to extend the rod, the rod is called a short rod. The target is slid down to the right point.

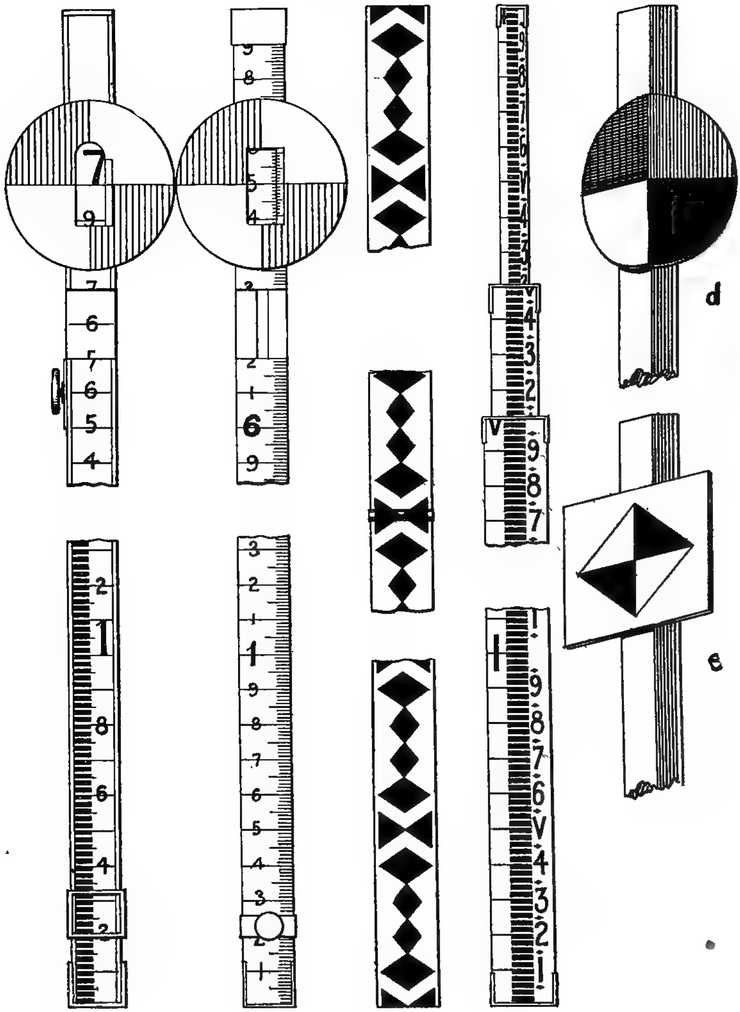
Long Rod.—When it is necessary to extend the rod, the rod is called a long rod. The target is clamped at the end of the rod before it is extended to the right point.

Self-reading Rod.—One which is read directly by the leveler; it does not carry a target. The markings on such a rod are heavy enough to be easily seen through the telescope.

Principal Kinds of Rods.—The Philadelphia rod, the New York rod and the Boston rod.

Philadelphia Rod.—(Fig. A—a)—Seven feet long, sliding out to 12 ft. Two strips. Divided into feet, tenths and hundredths. Oval target divided by a horizontal and a vertical line into four compartments painted alternately white and black; its center is cut open and carries a vernier used for readings of short rods. Such a target is

FIG. A



a b c f

called a quadrant target. The fig. shows two kinds of graduations. On long rods the upper clamp vernier is used. The verniers read to 1-1000 foot. The rod is heavy, the divisions plain, and it can be used as a self-reading rod.

New York Rod.—**(Fig. A—b)**—Lighter than the Philadelphia rod. Two strips. Six and a half ft. long, sliding out to 12 ft.; divided on the face to hundredths of a foot and reading to 1-1000 foot by the target vernier. Figures not as plain as in the Philadelphia. The two portions of the rod slide by means of groove and tongue, which is a draw back because sometimes affected by dampness. Target similar to the Philadelphia's. The rear half of the rod reads downward. It may be used as a self-reading rod only on short rods.

Boston Rod.—Six feet and a half long extending to 11 ft. 4 in. Two strips, one sliding in a groove. Diamond target rigidly fixed **(Fig. A—e).** For short rods it is held upside down and the target strip is moved up. For long rods in the usual way. Reads on one or the other side. Vernier at each end about the height of the eye.

Objection: An objection to this rod is that, by reversal from short to long rod, any error in the height of the target is doubled.

Diamond Targets and Their Advantage.—Diamond targets are those which show, on their face, lines forming a diamond instead of rectangular axial lines. Their advantage lies in the fact that the leveler can better determine the bisection of an angle by the cross-hairs than the coincidence of the cross-hairs with horizontal and vertical lines which they may overlap by their full thickness without detection.

Angle Target.—**(Fig. A—d)**—An oval target bent at right angles along the vertical axial line so that the half-faces fit against two adjacent faces of the rod. The leveler can detect if the rod is held vertically, because only then the horizontal axial line will appear as a single line in the field of the telescope. He may therefore control and direct a careless rodman.

Other Forms of Rod Markings.—**(Fig. A—f)** shows a telescopic self reading rod sometimes called **telemeter rod** when used on stadia survey. Divided into one-hundredths of one foot. Such a rod is often made in two sections unioned with strong hinges. Instead of horizontal lines some rods are divided by triangles the vertices of which are the divisions. **(Fig. A—c)** the leveler can more easily estimate the sub-divisions by the distance from a vertex where the cross-hairs intersect the side of a triangle. The diamonds shown in the fig. have a height of two inches.

How to Hold the Rod.—The ferrule is rested on the stake, bench-mark or point where a level is to be taken, the rodman standing squarely on his feet behind the rod. This is held plumb by supporting it with the ends of the fingers of both hands.

Or a round level may be attached to the rod for very accurate work. When holding the rod, balance it gently front and back within a small arc of a circle.

How to Move the Target.—Unclamp it; move it fast at first, then slowly when nearing the right point. When signaled all right clamp it and show again, then call out the height and record it.

Frequent Errors.—Be careful 1°, to clamp the target exactly on the end division for long rods; 2°, not to make a reading error of 1 foot or .1 ft.

What Affects Rods.—Moisture affects rods. The wood of the rods should have been treated with paraffin. Temperature affects them but little.

Be Quick.—A rodman should be active and try to catch up with the transit party ahead.

Notes to be Kept.—Elevation at the starting point; station number, readings of rod in the column marked **B. S.** or + (back sight), or in the column marked **F. S.** or '(fore sight), as the case may be, at each station or point where a level is taken. Bench marks, their location and elevation.

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The Assistant Engineer

BY

PROF. JEAN P. GENTHON

Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK IV

THE LEVELER

WRITTEN FOR

THE CHIEF

Journal of the Civil Service

PUBLISHED BY

THE CHIEF PUBLISHING CO.

45 CENTRE ST., NEW YORK

1908

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PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with the most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3°. The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examination in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON.

July 1, 1907. New York.

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1°. Aptitude for mathematics.

2°. Habit of observation.

3° Good memory.

4°. System.

5°. Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION.—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials or new appliances and processes are used for old or new purposes. Notice the several kinds of labor, and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory, which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results; principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books, always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figures and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations, in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around you will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public works have a Chief Engineer who prepares the work and directs its

construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

Chief Engineer.	Division Engineers.		Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist
			Inspectors.	
		Assistant Engineers.	Transitmen.	Chainmen. Axemen.
			Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen.

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.	Memphis, Tenn.	St. Louis, Mo.
Boston, Mass.	Milwaukee, Wis.	St. Paul, Minn.
Buffalo, N. Y.	Mobile, Ala.	San Francisco, Cal.
Charleston, S. C.	Nashville, Tenn.	Savannah, Ga.
Chattanooga, Tenn.	New London, Conn.	Seattle, Wash.
Chicago, Ill.	New Orleans, La.	Sioux City, Iowa.
Cincinnati, Ohio.	Newport, R. I.	Tampa, Fla.
Cleveland, Ohio.	New York, N. Y.	Vicksburg, Miss.
Detroit, Mich.	Norfolk, Va.	Washington, D. C.
Galveston, Tex.	Philadelphia, Pa.	Wheeling, W. Va.
Grand Rapids, Mich.	Pittsburg, Pa.	Wilmington, Del.
Jacksonville, Fla.	Portland, Me.	Wilmington, N. C.
Little Rock, Ark.	Portland, Oreg.	Yellowstone Park, Wyo.
Louisville, Ky.	Rock Island, Ill.	

where examinations may be taken except that of Assistant Engineer, which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above-named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken at

Albany.	Ithaca.	Ogdensburg.
Amsterdam.	Jamestown.	Olean.
Auburn.	Kingston.	Plattsburg.
Binghampton.	Lockport.	Poughkeepsie.
Buffalo.	Malone.	Rochester.
Elmira.	Newburg.	Utica.
Hornelsville.	New York.	Syracuse.
		Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewer construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman; topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps.

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service, appear regularly in "THE CHIEF."

THE ASSISTANT ENGINEER

BOOK IV

THE LEVELER

Leveler.—He who carries and uses the level, and records all the data necessary to determine the elevations of points of the ground.

Who His Superiors Are.—The leveler is under orders of the **Chief of party** or of the **Assistant Engineer.**

What He Carries.—He carries the **level,** an **adjusting pin** and a **level book.**

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service—Panama Canal.

Title: Level man.

Age limits: 21 to 45 years at examination.

Salary: $100 to $125 per month.

Written Examination.

	Subjects.	Weights.
1.	Pure and applied mathematics (elementary problems in measuration, solution of plane triangles, and theoretical and applied mechanics, involving a fair knowledge of pure mathematics to and including calculus)	20
2.	Construction and care of instruments (comprising transit, including stadia work, level, plane table, rods, chain, tape, current meters, etc.)	20
3.	Theory and practice of surveying (comprising surveying, leveling, and other field work required in engineering and not covered in subjects 1 and 2)	20
4.	Design and construction (involving elementary knowledge of designing and constructing highways, railroads, dams, retaining walls, foundation work, trusses, etc.)	10
5.	Training and experience	30
	Total	100

Three years' practical experience in use and care of instruments in surveying.

After one year's satisfactory service, appointee will be eligible for promotion as a transit man without further examination.

New York State and County Civil Service.

Title: Leveler.

Age Limit: Not less than 21 years old.

Salary: $4.50 to $5 per day when employed.

Examination.

	Subjects.	Relative weights.
1.	Theoretical and practical questions, including mensuration and use of logarithms, plane trigonometry, topographical surveying and leveling, elementary mechanics, theory and use of rod, level and transit, highway construction, etc.	5
2.	Experience	3
3.	Education	2
	Total	10

SPECIMEN QUESTIONS.

1. Calculate the number of gallons of water per day that would be delivered by a new 10-inch cast-iron pipe, 4,000 feet long, on a grade of 5 feet in a thousand, flowing full, if the pipe has one right angle turn with a 3 foot radius and one 45 degree turn with a 9 foot radius.

2. A hole 6 inches square is made in a ship's bottom 20 feet below the water line. What force must be exerted to keep the water out by holding a piece of board against the hole, if sea-water weighs 64 pounds per cubic foot?

3. What amount of water could we reasonably expect would be discharged by a smooth, open wooden flume of rectangular cross-section, if the flume is uniformly 4 feet wide, the water 2 feet deep, and the slope of the water surface be taken as 1 foot per thousand?

4. Find the slope to be given to the bottom of a waterway cut in gravelly soil so that it may carry 180 cubic feet of water per second. The dimensions of the waterway are 35 feet top width, 7 feet depth, and 9 feet bottom width. Use Kutter's Formula.

5. Calculate the opening necessary in a road culvert, having a fall of 3 inches in 30 feet, to carry the flood-flow of a brook, draining 1.25 square miles of farming country, having a general slope of 5 feet in a thousand .

6. In order to find the quantity of water conveyed by a ditch 3 feet wide, a weir with a rectangular notch 2 feet wide and 1 foot deep, with sharp edges, is placed across the ditch, causing the water to have a depth of 2¼ feet above the bottom of the ditch and 8½ inches above the crest of the notch. Find the discharge in cubic feet per second.

7. What depth and bottom width should be given the transverse profile of a canal feeder, whose banks are of coarse gravel with a few weeds and slope at 40 degrees to the horizontal, in order that it may conduct 75 cubic feet of water per second with a mean velocity of 3 feet per second? The feeder has a fall of 2 in 1,000.

8. A canal in clean coarse gravel is 20 feet wide at the bottom and its side slopes are 1½ to 1, it has a longitudinal slope or fall of 1 in 360 and a depth of 8 feet. If a surmerged weir 2 feet high be built across the canal, what will be the increase in the depth of the water?

9. The horizontal section of a canal lock has an area of 12,150 square feet and the difference of level between the surface of the water in the lock and in the upper reach is 9 feet. Each of the two gates is to have one sluice or valve whose centre is to be 20 feet below the surface of the upper reach, and the water is to be leveled up in 2 minutes 48 seconds. Determine the proper area of each valve.

10. A cast-iron beam of rectangular section, 12 inches deep, 6 inches wide and 20 feet long, carries, in addition to its own weight, a simple load **P**; the safe allowable tensile stress is 2,000 pounds per square inch. Find the maximum allowable value of **P** when it is placed (a) at the middle point, (b) at 2½ feet from one end.

11. A hollow cast-iron pillar 12 feet in height has to support a dead load of 35,000 pounds, and a live load of 20,000 pounds; its internal diameter is 6½ inches. Find the required thickness of the metal, taking 6 as the factor of safety.

New York City Municipal Civil Service.

Note: Apparently suppressed in new classification. Dec. 17, 1907.

Questions Given at Previous Examinations.

Mathematics.

Give all the figures on ruled sheets.

1. A lot of ground is 97 feet or 4 and 5-8 inches long and 24 feet 7 and 3-8 inches wide. How many square feet are there in the lot? Do **all** the work by decimals.

2. The rise of a street in a distance of 5,763 feet is 12.967 feet. What is the percentage of the grade?

3. The section across the embankment made in building a road is as follows: Width at top of road-bed 40 feet; slopes of embankment at each side one and a half horizontal to one vertical. Left hand slope stake 10 feet below road-bed; left hand angle point or beginning of slope, 8½ feet below; centre point 8 feet below; right hand point 9¼ feet below; right slope stake, 8¾ feet below. What is the area of the section? Make a sketch and give all the figures.

4. The original surface of a road is a rapid rise, but is uniform, and the surface of the cross sections are level and slope 1½ to 1. The cut at one end is 3 feet; width at bottom 40 feet, and at top 49 feet. At the other end the cut is 7 feet with width at bottom of 40 feet and top 61 feet. The length between the sections is 100 feet. Compute by the prismoidal formula the contents in cubic yards.

5. Draw an isometric cube.

6. Multiply by logs. 61.2347×0.0007581.

7. Determine without tables the nat. sine and cosine of angle "a" in a right angled triangle "a" "b" "c" of which the Base "a b" is 119 and the vertical side "b c" 71.5.

8. Divide 27 and 2 tenths by 10 and 625 thousandths.

9. Extract the square root of .00390625.

10. The two ends of a floor are 19 feet and 16 feet long. One side perpendicular to the end is 28 feet long. Find the area of the floor and the length of the fourth side.

11. Find the solid contents of the frustrum of a cone whose altitude is 18 feet; diameters of bases being 8 feet and 2 feet.

Technical.

1. Adjustment of "Y" Level.

2. What is a Traverse Table, and what functions of an angle do the tabulated figures represent?

3. What is the greatest allowable error leveling a mile through a crowded street?

4. What error of angle and dist. would you allow in running transit line same distance—same conditions.

5. Give sketch of Double Page of Transit Book.

6. What are the sources of error in running transit lines and what means to avoid it?

7. Give sketch of method of calculating contents of survey.

8. What is the fundamental formula for flow of water?

9. How would you calculate the forces tending to overturn a retaining wall?

10. Given 2 cross sections 100 feet apart slopes 1½ to 1, calculate by prismoidal formula and mean areas.

11. Under what conditions is formula used?

12. What are the requisites for good granite paving construction?

13. Describe brick masonry construction: Bond, mortar, workmanship and method of construction.

14. Give same for heavy masonry construction.

15. What is concrete—selection of materials—proportions; method of mixing and how placed in work.

16. Of what parts does a log consist and what rule governs the integral parts.

17. How many bricks in Sewer 4 feet external diameter, 12 inches thick, 230 feet long. Describe brick sewer construction, bond, placing material, etc.

18. How would you locate soundings for pier?

19. What steps necessary for laying out a new street, and state in detail how slope stakes 1½ to 1 are set.

20. Which holds best in trestle a ragged bolt or straight bar of iron —same length and cheek?

21. How much shrinkage in embankment?

22. How would you know when a transit is accurate?

23. How in relation to a Level?

24. Give transit notes for a 2° curve with angle 22° 56'.

25. From elev. of grade of 103' and 0.2 fall, show cross-section notes for cut and fill represented by + and— and show where grade crosses natural surface.

26. Give square feet of surface: Cylinder 2' diameter and 3' high.

27. Solid contents of same.

28. How would you use a Plane Table?

29. What is the weight of good wrought iron?

30. What is the weight of good cast-iron?

31. What is the weight of good steel?

32. Parallelogram of Forces.

33. What is a Truss?

34 What is lime?

35. What is hydraulic cement?

36. Difference between Portland and Rosendale cement?

37. How do you test cement?

38. In what part of a building would you use lime?

39. In what part of a building would you use cement?

40. How do you preserve iron from rust?

41. How do you preserve wood in contact with earth?

42. How do you preserve wood in salt water?

43. 50' beam, load 250 lbs. per foot; what weight would be supported at 5' and 19' ft. from end?

44. Show sketch for timbering a tunnel.

45. Show sketch for timbering a shaft.

46. Explain survey line in a tunnel.

47. How would you calculate base of a retaining wall?

48. What is English method of timbering a tunnel?

49. How would you bond masonry?

50. Under what conditions would you use a tunnel heading?

51. How would you know the size of timber to use in a tunnel?

52. Would you use hard or soft wood in tunnel?

53. H. P. Locomotive: Cyl. 18" diam., 22" stroke; press. 80 lbs.; speed 10 miles per h.; wheels 4½' diam.

54. Calculate.

55. What is H. P. of stream; flow 500 cu. ft. per. sec., 10' fall.

56. What is a vernier?

57. Why should degrees run from 0° to 180°?

58. Where do you use cast iron?

59. Where do you use wrought iron in structures?

60. At what temperature has water its greatest density?

61. (a) What is the parallax in a level? (b) What are the two causes of the parallax and how are they overcome?

62. Describe the adjustment of the line of collimation of an engineer's level.

63. (a) Describe the method of adjusting the bubble on an engineer's level. by reversion. (b) What instrumental imperfection may make this method inaccurate?

64. Describe a method of adjusting an engineer's level by means of the pegs.

65. Can a bubble be correctly adjusted by the peg method before the line of collimation is adjusted? Give your reasons.

66. (a) In very accurate leveling what would be the longest sight you would think it best to take? (b) Give two or more reasons for this.

67. Aside from the apparent "dancing" of the target caused by radiation of the heat from the earth, state in what other ways the sun affects the work of the leveler.

68. State clearly what errors in leveling are eliminated by keeping the lengths of backsights and foresights equal.

69. Describe the operation of carrying a line of levels across a deep stream, say 1,500 feet wide, with reasonable accuracy.

70. (a) Where a roadway is to have slopes on each side of one and a half horizontal to one vertical, the cuts on the center line stakes being given, describe the operation of setting slope stakes for the contractor. (b) Show the form of the notes you would keep as you lay out the work.

71. In what distance does the error in elevation of the target due to curvature of the earth amount to .001 of a foot? Give figures of the computation.

72. Suppose the leveler to have taken an accurate sight of the target, what may happen to cause a wrong elevation of the target to be recorded?

73. Give all the reasons for sighting the target again after giving the signal all right to the rodman.

74. (a) How do you fix the grades for a sewer? (b) What is the greatest distance that should exist between grade points where the grade is very flat? (c) Why is this?

75. In an extensive rock cutting how would the work be laid out, and how would the quantities for a monthly estimate of the work done be obtained?

76. Give an example of notes taken in the case of a deep rock cut 700 feet by 80 feet for a street.

77. State in their proper order, without describing them, the several adjustments of an engineer's level.

78. State generally what are the two methods of adjusting the bubble to parallelism with the line of sight.

79. (a) Describe the "peg" method of making this adjustment. (b) Which is the most exact method of making this adjustment, and why?

80. (a) Is it possible to do correct work with an instrument out of adjustment? (b) If so, explain how it is that errors are eliminated.

81. (a) In very exact or "precise" leveling, what would be the limit of length of sight you would take? (b) Explain why you would adopt this limit.

82. Describe the proper method of adjusting for "parallax" when using a level.

83. (a) What are the causes of parallax? (b) How would they affect the adjustments.

84. (a) An instrument being in perfect adjustment, does it sight a truly level line to a distant point, or does it require correction? If so, state how. (b) Describe the errors, if any.

85. (a) What is the law of increase or effect due to curvature? (b) Assuming eight inches in a mile, how much would it be in 500 feet?

86. How do frost and cold tend to cause errors in leveling?

87. How does heat cause errors in leveling?

88. How may sunlight cause trouble?

89. Under what conditions or times of day are those troubles the least?

90. What other difficulties or causes of error are there in leveling in a crowded city?

91. Give a form of record of levels on a long street and explain the terms used.

The following questions were given in Buffalo:

92. A sewer is to be placed in a certain street in Buffalo not now paved. After construction of sewer, asphalt pavement is to be laid. The sewer is to be brick oval and will empty into larger sewer also of brick. The top of the trunk sewer is about 17 feet below pavement. The new sewer to be built will be about a mile in length and will receive the discharge of lateral sewers of 10 intersecting streets. Describe all the work of preparing plans for this new sewer from the beginning of field work up and through construction, leaving the street in proper condition to begin the preparation for the pavement. It is desired that this question be answered very fully, describing as far as possible the character of the material and labor which may be put into it.

93. Given to excavate a trench 50 feet wide on the bottom with sloping banks of 2 to 1 depth of center cut at one end of cross section 23 feet—transverse slope level. Center cut at other end 17 feet transverse slope level. Length on C. L. 757 feet with parallel end sections. Assuming ground along C. L. to be uniform grade and bottom of trench level, find quantity of earth to be excavated by two methods.

94. Sketch a topographical map of a tract of land bounded by 7 straight lines containing about 5 square miles. Through the area a river falling at the rate of 1 inch in 3,000 feet. The highest point of land is 200 feet above the lowest level of river. Sketch 20 contours for entire area and show three highways crossing it, one north and south, two east and west; the grade of highway not to exceed 8 per cent. at any place. Show by conventional signs the topography of the tract.

SCIENTIFIC REQUIREMENTS.

All that which precedes (see Book I, The Axeman; Book II, The Chainman; Book III. The Rodman) Mechanics, Hydrostatics, Pneumatics.

MECHANICS.

Mechanics.—Science of equilibrium and motion.

Mechanics:
- **Statics.**—Part of Mechanics treating of the laws of equilibrium of forces.
- **Kinematics**—Part of Mechanics treating of motion considered in itself.
- **Dynamics.**—Part of Mechanics treating of motion as an effect of forces acting upon a body.

Matter.—All that may be apprehended by the senses. It may assume three aspects.

Solid.—Matter the components of which strongly adhere to each other.

Liquid.—Matter the components of which easily change position while remaining together.

Gas.—Matter the components of which tend to separate and occupy larger spaces when nothing confines them.

Boay.—A definite portion of matter all the elements of which are supposed tied by rigid lines.

Rest.—State of a body without motion. There is no absolute rest, but only

Relative Rest.—State of a body all the material points of which remain at a constant distance from a point supposed at rest.

Motion—Action of a body occupying in soace several successive positions.

Free Body.—One which can assume any position in space when solicited by a force.

Hampered Body.—One prevented from following the direction of a force acting upon it, by the interposition of an obstacle.

Postulatum.—A body cannot pass from rest to motion or vice versa without being solicited to it by an external cause.

Force.—External cause capable of imparting or modifying motion in a body.

Weight or Gravity.—A force soliciting unsupported bodies to follow a vertical direction.
Molecular Attraction or Repulsion.—Force manifested in springs.

Muscular Action.—Force in men and animals.

Force:
- **Point of Application.**—Infinitely small material point upon which a force exerts its action.
- **Direction.**—Element of straight line which the point of application tends to follow at the moment the force acts.
- **Intensity.**—Greater or less vigorous action of a force.

Equal Forces.—Two forces are equal if no motion results in a point on which both act in opposite directions.

Equilibrium.—Forces are in equilibrium when, being applied to the same point, they produce no motion.

Multiple Forces.—A force is twice, three times, etc., another when it equilibrates 2, 3, etc., forces equal to the other when applied to the same point in opposite directions.

Measurement of Forces.—To measure a force we compare its effect to the effect of another force supposed to be known, as the force of gravity.

Weight.—The weight of a body is the pressure, resulting from gravity, which the body exerts upon any obstacle which prevents its fall or motion.

Dynamometer.—Instrument used to measure the intensity of forces.

Representation of Forces.—We may represent the intensity of forces by straight proportional lines drawn in the direction of the forces.

Resultant Force.—Single force the effect of which is the same as that of several others.

Component Forces.—Several forces the effect of which is replaced by that of a single other force called **Resultant.** Therefore

Theorem.—When several forces, acting upon a free body, are in equilibrium, any of these forces is equal to and opposite the resultant of all the others, and conversely.

Composition of Forces.—Process of finding the resultant of given forces.

Resolution of Forces.—Process of replacing a single force by several others capable of producing the same action.

Theorems.—1°. The point of application of a force may be displaced to any point of its direction. (The new point is supposed to be rigidly tied to the first.)

2°. Two equal forces acting in opposite directions upon a body are in equilibrium.

3°. A force may be added to (or taken from) those acting upon a body, provided an equal force be also introduced (or suppressed) acting in opposite direction.

4°. When an acting force may be replaced by another force acting upon a fixed point of the same system, the two forces are equal and the direction of the first was through the fixed point.

5°. If a system of forces admits of a resultant, it has only one such.

6°. If a body is free to move about a fixed point and is solicited by a force whose direction does not pass through that point, the body will move.

7°. Two forces applied to the same point (a) have a resultant; (b) it is in the plane of the two forces; (c) when the two forces are equal the direction of the resultant is the bisectrix of the angle.

Parallelogram of Forces (Fig. 165).—Any two forces applied to the same point have a resultant, represented in intensity and direction by the diagonal of the parallelogram drawn on the lines representing the intensity and direction of the two forces.

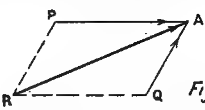


Fig. 165

Representing the two forces by **P** and **Q**, the resultant by **R**, and by **(P Q)** **(Q R)** and **(P R)** the angles formed by their directions, we have:

$$\frac{P}{\sin(QR)} = \frac{Q}{\sin(PR)} = \frac{R}{\sin(PQ)} \qquad (1)$$

and

$$R^2 = P^2 + Q^2 + 2PQ\cos(PQ) \qquad (2)$$

When $P = Q$, then $R = 2P\cos\frac{(PQ)}{2}$; and when $(PQ) = 90°$, then $R = 0$

Resolution of a Force Into Two Others With Given Directions (Fig. 166).—These directions may be drawn from one end of the given force which may be considered as the resultant of the two

forces sought. We may then complete the parallelogram of forces in which the known quantities are **R, (P R), (Q R), (P R).**

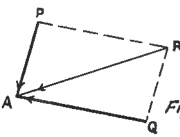


Fig. 166

From (1) we get
$$P = R\,\frac{\sin(QR)}{\sin(PQ)}$$
and
$$Q = R\,\frac{\sin(PR)}{\sin(PQ)} \qquad (3)$$

and if $PQ = 90°$, then
$$P = R\cos(PR)$$
$$Q = R\cos(QR) \qquad (4)$$

Moment of a Force With Regard to a Point (Fig. 167)—Product of the force **P** by the perpendicular **p** drawn from the point to the direction of the force: **M=Pp.**

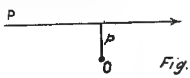


Fig. 167

Theorems of Moments.—1°. The moments of two forces with regard to any point of their resultant are equal.

Draw the parallelogram of forces (Fig. 168) and the perpendiculars **p** to **AP, q** to **AQ** from the extremity of force **R.** We have **triangle APR=triangle AQR** and therefore **(5) Pp=Qq** which is twice the area of each triangle.

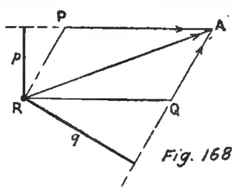


Fig. 168

2°. The moment of the resultant of two forces applied to the same point, with regard to any point in their plane (Fig. 169), equals the sum of the moments of the components. **(6) Rr=Pp+Qq.**

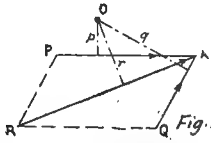


Fig. 169

Which is deduced from this Geometrical Theorem: The area of a triangle constructed upon the diagonal of a parallelogram equals

the sum of the areas of the triangles constructed upon two adjacent sides, the vertex being common.

Signs of Moments.—Considering an equilibrium system of two forces **P, Q,** and their resultant **R,** we have as above **(6)** with regards to a point **O, Rr=Pp+Qq.** Considering separately the action of each force, it tends to revolve **p, q** and **r** about the point **O** either in the same or in opposite direction. If we agree that the moments of forces tending to move **p, q** and **r** in a certain direction are positive, they will be negative when **p, q** or **r** shall move in the opposite direction.

If **o** is upon the resultant, **r=o** and **(6)** becomes **o=Pp+Qq** or **Pp=—Qq** and the moments of the forces are equal and of contrary signs conversely. If the moments of two component forces are equal and of contrary signs with regard to a point in their plane, that point is upon the direction of their resultant.

Polygon of Forces (Fig. 170).—Given any number of forces applied to the same point, we can replace the first two **P** and **Q** by their resultant **R**; we can replace **R** and the third force **S** by a new resultant **R'** and continue until we have operated on the last force. The last resultant will be that of the system and is the same in whatever order we compose the forces.

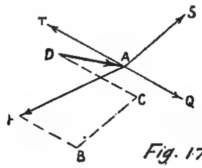


Fig. 170

If we perform graphically the operation just described but without actually drawing the successive resultants, we are led to the construction in space of a plane or wharped polygonal line whose successive sides are proportional and parallel to the given forces. The line that would close the polygon is parallel and proportional to the resultant of the system.

Condition of Equilibrium of a System of Forces.—The resultant must be zero. Therefore the polygonal contour must naturally close and form a polygon which is called **Polygon of Forces.**

Parallelopipedon of Forces (Fig. 171).—In the case of three forces not in the same plane, the line closing the polygonal contour is the resultant of the three forces is the diagonal of a parallelopipedon, the adjacent arrisses of which are three given forces.

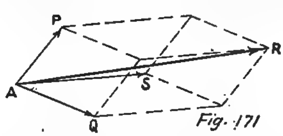


Fig. 171

Conditions of Equilibrium of Three Forces.—1°. They must be in one plane.

2° According to formula **(1)** they must each be proportional to the sine of the angle formed by the direction of the other two.

Resolution of a Force Into Three Others Having Given Directions (Fig. 172).—Through the point of application **O** of the force **OR** draw the three directions **OX, OY, OZ,** which will form a trihedral angle. Draw **RA** parallel to **OZ** till it intersects the plane **OXY** of the other two forces; **AB** parallel to **OY,** we form the polygonal contour **RABO** the sides of which **OB, BA** and **AR** represent the components wanted. To have them in direction we would complete the parallelopipedon of which these lines are the arrisses.

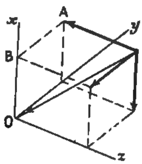


Fig. 172

If the directions are rectangular, we have, by representing the components by **X, Y, Z**:

$R = \sqrt{X^2 + Y^2 + Z^2}$ (7), and the angles which the resultant makes with the axes by a, b, c; $X = R\cos A$, $Y = R\cos b$, $Z = R\cos c$ (8)

Reduction of a System to Three Rectangular Forces.—Through the point of application **o** draw three rectangular axes **ox, oy, oz.** Any force **P** may be resolved into three others:

$P\cos a$, $P\cos b$, $P\cos c$ acting one along each axis. Any other force P' may likewise be resolved into $P'\cos a'$, $P'\cos b'$, $P'\cos c'$ and so with the others. All the

$$\left.\begin{array}{lll} \text{forces acting along} & ox \text{ will have a component} & X = P\cos a + P'\cos a' + P''\cos a'' + \cdots \\ \text{those acting along} & oy \text{ " " " "} & Y = P\cos b + P'\cos b' + P''\cos b'' + \cdots \\ \text{" " "} & oz \text{ " " " "} & Z = P\cos c + P'\cos c' + P''\cos c'' + \cdots \end{array}\right\} (9)$$

By composing the three forces **X, Y, Z** into a single force **R,** this will be the final resultant of the given system and its value will be calculated by formulas **(7) and (8).**

Projections of the Resultant Upon an Axis.—Formulas **(9)** prove the following theorem:

Theorem.—Several forces being applied to the same point, the projection of their resultant upon a straight line equals the sum of the projections of its components.

General Condition of Equilibrium.—In order that the resultant of a system be zero, the three componants **X, Y, Z** must also be zero.

It is not necessary that the axes be rectangular, but only that the sum of the projections of the forces upon three directions forming the arrisses of any trihedral be zero for each direction.

PARALLEL FORCES.

Resultant of Two Parallel Forces Acting in the Same Direction (Fig. 173).—Suppose two forces **P** and **Q** applied to a point **O**, draw from the extremity of **R** perpendiculars **p** and **q** on **P** and **Q**; we have seen that **Pp=—Qq.** Suppose that the forces be applied at the feet of **p** and **q**; also that the force **P** retains its position and that the force **Q** rotates about the foot of **q** describing a circle, the angle **a** of the forces decreases indefinitely and tends to zero when the forces are parallel, and their resultant is parallel to them. In all these positions, the intensity of the resultant is given by formula **(2)**:

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos POQ} \text{ ; therefore } \lim. R = \lim. \sqrt{P^2 + Q^2 + 2PQ \cos POQ}$$
$$= \sqrt{P^2 + Q^2 + 2PQ \lim. \cos POQ} \text{ ;}$$
now $\lim. \cos POQ = 1$ because $\lim. POQ = 0$; therefore $\lim. R = \sqrt{P^2 + Q^2 + 2PQ} = P + Q$.

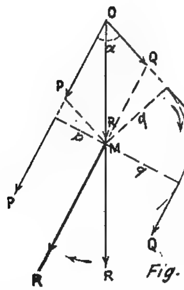


Fig. 173

The resultant equals the sum of the components.

The point **M** divides the common perpendicular in an inverse ratio to the forces,

because $Pp = Qq$ may be written $\frac{P}{Q} = \frac{q}{p}$

Any other secant which might be assumed as a line of application would similarly be divided.

The point **M** which so divides a secant of two parallel forces acting in the same direction in the inverse ratio of the forces is the point of application of the resultant, and the point of application of the components may be assumed to be at the extremities of that secant.

From $\frac{P}{Q} = \frac{q}{p}$ we have $\frac{P}{Q+P} = \frac{q}{p+q}$ or $\frac{P}{R} = \frac{q}{p+q}$ from which $\frac{P}{q} = \frac{Q}{p} = \frac{R}{p+q}$;

hence *each force may be represented by the distance of the points of application of the other two forces.*

Resolution of a Force R Into Two Forces P and Q Parallel to the First (Fig. 174).—These forces may pass through two given points **M** and **N** in a plane with **R**.

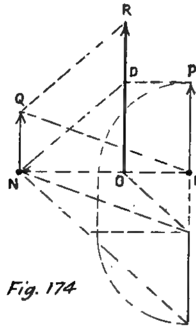


Fig. 174

Draw line **MN**; the point of application of **R** may be supposed to be at **O** intersection of **MN** with the direction of **R**; through **M** and **N** draw parallels to **R** on which lay off distances **PM** and **QN** such that **PM×OM=QN×NO,** which means the division of **RO** into two portions **OD** and **DR** proportional to the known distances **ON—OM.**

Resultant of Two Forces Parallel and of Opposite Directions (Fig. 175).

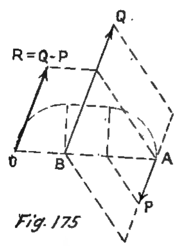


Fig. 175

Let $Q > P$ and B and A the points of application. Connect B-A and produce. Resolve Q into two forces P and Q-P applied P at A and Q-P at point O such that $BO = \frac{P \times AB}{Q-P}$. The forces at A being equal and of opposite direction neutralize eachother and there remains only force Q-P which is the required resultant. From the above equation we deduce

$$\frac{BO}{AB} = \frac{P}{Q-P}; \quad \frac{BO}{BO+AB} = \frac{P}{P+Q-P}; \quad \frac{BO}{AO} = \frac{P}{Q}; \text{ therefore}$$

Two forces parallel and of opposite directions have a resultant parallel to them, in the direction of the greater, equal to their difference and applied at a point of the line of their points of application such that the distances to that point are inversely proportional to the forces.

If we write Q-$P = R$, we still have $\frac{P}{BO} = \frac{Q}{AO} = \frac{R}{AB}$

so that each force may yet be represented by the distance of the points of application of the other two forces.

Couple.—When two parallel forces of opposite directions are equal they have no resultant.

The above equation $BO = \frac{P \times AB}{Q-P}$ becomes in this case $BO = \frac{P \times AB}{0}$ and $R = Q-P = 0$ which shows *impossibility*.

This particular case is called a **couple.**

Moments.—The theorem of moments is extended to the case of parallel forces. (Fig. 176). Let **D** be a point in the plane of two parallel forces. Draw a common perpendicular through **D** to the forces and call **p** the distance from **D** to **P, q** that from **D** to **Q** and **r** that from **D** to **R.** We have seen that

$$P \times AO = Q \times BO \text{ therefore also } P(p-r) = Q(r-q)\,;\; Pp - Pr = Qr - Qq\,,$$
$$Pp + Qq = r(P+Q) = Rr\,, \quad \text{finally} \quad Pp + Qq = Rr\,.$$

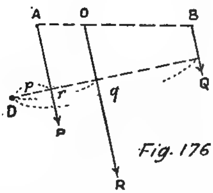


Fig. 176

Were the forces in opposite directions, the theorem would still be applicable if all the quantities be taken with their respective signs, that is, forces in opposite directions will be of contrary signs, and so will the perpendiculars drawn from the center of moments.

Resultant of Any Number of Forces Situated in a Plane.—Compose two forces **P and R** not forming a couple, the system of **m** forces is reduced to an equivalent system of **m-1** forces, and the sum of the moments did not change because the moments **Pp+Qq** are replaced in it by their equal **Rr.** The second system is then reduced to a new one of **m-2** forces and so on until a system of two forces is obtained, and if these don't form a couple, their resultant will be that of the original system and its moment is equal to the sum of the moments of the given forces. Therefore: The moment of the resultant of a system of forces acting in a plane, with regard to any point in the plane, equals the algebraical sum of the moments of the components.

Composition of Any Number of Parallel Forces.—Some forces may act in one direction, and the rest in the direction opposite. Compose into one force all those of the first group, their resultant **P** will be equal to their sum; then do the same for the second group, their resultant **Q** will also be their sum. Finally compose the forces **P, Q** into a final resultant **R** which will be that of the given system.

Should these forces vary in intensity but remain in the same ratio and should they rotate about their point of application while remaining parallel to one another, their resultant will still pass

through the same point, be parallel to them and have the same intensity in all cases. The point of application of the resultant is also called the **center of the parallel forces.**

Moments of Parallel Forces With Regard to a Plane (Fig. 177).— The moment of a force **P** with regard to a plane is the product of the force by the distance **p** from its point of application to the plane.

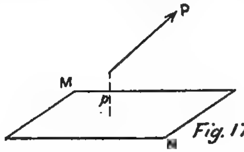


Fig. 177

Theorem.—The moment of the resultant of a system of parallel forces, with regard to a plane (Fig. 178), is the algebraical sum of the moments of the components:

$Rr = Pp + Qq + \cdots$ (10) from which we deduce $r = \dfrac{Pp + Qq + \cdots}{R}$

or $r = \dfrac{Pp + Qq + \cdots}{P + Q + \cdots}$ (11) which is the distance from the center of the parallel forces to the plane.

Center of Mean Distances. If the system is one of n equal forces acting in the same direction, (11) becomes $r = \dfrac{P(r + q + \cdots)}{nP}$ or $r = \dfrac{r + q + \cdots}{n}$ (12)

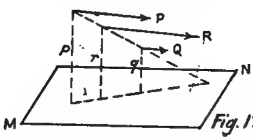


Fig. 178

Gravity.—Force exerted by all the points of the earth upon bodies on its surface. All bodies are heavy and so are their particles, and if left free they will fall, following a vertical direction.

A thread supporting a body is in a vertical direction and bears a tension which can be measured with a dynamometer.

The intensity of gravity varies with the latitude increasing from the equator to the pole; it also diminishes with the elevation.

Weight of a Body.—We may consider a body as the sum of an indefinite number of material equal particles, all solicited equally by the force of gravity and therefore parallel. These parallel forces act in the same direction and have a resultant parallel to them and equal to their sum. That resultant is the **weight** of the body.

Center of Gravity.—Center of the parallel forces due to gravity.

This center is unique whatever be the position of the body.

If the center of gravity of a body be fixed, the body will be in equilibrium in all the positions it may assume.

Experimental Determination of the Center of Gravity of a Body.—Suspend the body to a thread in two different positions; the intersection of the two verticals will be its center of gravity.

Definitions.

Center of Gravity of a Line.—Center of the parallel forces supposed to be applied to all the elements of the line.

Center of Gravity of a Surface.—Center of the parallel forces supposed to be applied to all the elements of the surface.

Center of Gravity of Regular Solids.—When a solid has a center, that center is also the center of gravity.

When a solid has an axis or a plane of symmetry, the center of gravity is situated upon that axis or that plane. The reason is that the solid is supposed to be homogeneous and therefore the determination of the center of gravity depends only on the geometrical figure of the solid. Therefore:

Center of Gravity of a Straight Line.—The middle point.

Center of Gravity of a Parallelogram.—Intersection of the diagonals.

Center of Gravity of a Circle, Ellipse, Sphere, etc.—The center of the figure.

Center of Gravity of a Cylinder.—Middle of the line of centers of the bases.

Center of Gravity of a Parallelopipedon.—Intersection of the diagonals.

Center of Gravity of the Perimeter of a Triangle (Fig. 179).—Center of the circle inscribed in the triangle formed by connecting the centers of the sides of the given triangle.

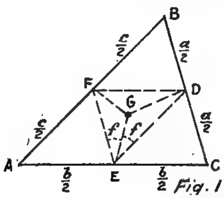


Fig. 179

Center of Gravity of the Area of a Triangle (Fig. 180).—Point of intersection of the medians. It is situated at one-third of each from the corresponding side, or at two-thirds of each from the corresponding vertex.

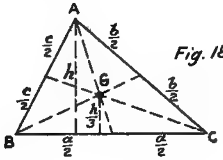


Fig. 180

Center of Gravity of a Trapezoid (Fig. 181).—Draw the median of the bases; produce the bases in opposite directions on which set distances equal to the opposite base; join the points thus obtained; the intersection of that line with the median is the center of gravity.

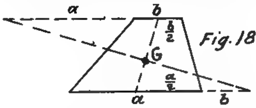


Fig. 181

Center of Gravity of Any Quadrilateral (Fig. 182).—It may be decomposed into two triangles by a diagonal. The center of gravity of each may be obtained, and on them parallel forces may be applied proportional to the respective areas of the triangles; then these forces may be composed into one resultant equal to their sum, parallel to them and the point of application of which is the center of gravity of the whole figure. Here is a short graphical process to obtain it: Draw the two diagonals and mark their middle points. Each diagonal divides the other into two unequal segments; set off the smaller segment of each on the diagonal to which it belongs from the other extremity, then connect crosswise these points to the middle of the other diagonal. The intersection will be the center of gravity of the quadrilateral.

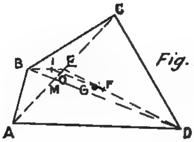


Fig. 182

Center of Gravity of Any Irregular Polygon.—Divide it into triangles by diagonals drawn from one vertex and suppose that parallel forces proportional to the areas of the triangles be applied to their centers of gravity. Compose these forces; their resultant will be equal to their sum, parallel to them, and its point of application will be the center of gravity of the polygon.

Center of Gravity of a Prism.—Middle of the line connecting the centers of gravity of the bases.

Center of Gravity of a Pyramid.—It is situated upon the line connecting the vertex with the center of gravity of the base and at one-fourth of that line from the base.

Center of Gravity of a Cone.—It is on the line connecting the vertex with the center of the base and at one-fourth of that line from the base.

Center of Gravity of a Frustrum of a Pyramid or Cone with Parallel Bases.—Let **B, b** be the bases and **h** the height of the frustrum. The center of gravity must be on the line connecting the center of gravity of the bases.

If x represents the dist. of the center of gravity from B and $h-x=y$ the dist. of the same from b,

we have $\frac{x}{y} \quad \frac{B+3b+2\sqrt{Bb}}{b+3B+2\sqrt{Bb}}$; but as B and b are proportional to the squares of two homologous sides A and a, we can write:

$$\frac{x}{y} = \frac{A^2+3a^2+2Aa}{a^2+3A^2+2Aa} \qquad (13).$$

Center of Gravity of a Polyhedron.—The polyhedron is decomposed into tetrahedrons and the center of gravity of each is obtained; to these centers of gravity parallel forces proportional to the volumes of the tetrahedrons are applied; they are then resolved into a single resultant equal to their sum, parallel to them and the point of application of which is at the center of gravity of the polyhedron.

Leibnitz Theorem.— A system of equal weights being given, if forces are applied to its center of gravity, represented in intensity and direction by the straight lines connecting it with the center of gravity of each weight, these forces will be in equilibrium.

There is equilibrium between three forces applied to the center of gravity of a triangle, represented in intensity and direction by the lines connecting it with each vertex, and conversely.

There is equilibrium between four forces applied to the center of gravity of a triangular pyramid, represented in intensity and direction by the lines connecting it with each vertex.

There is equilibrium between six forces applied to the center of gravity of a triangular prism, represented in intensity and direction by the lines connecting it with each vertex.

General Condition of Equilibrium of a Solid.—All the forces of the system being reduced to two, these must be equal, act at the same point and in opposite directions.

SIMPLE MACHINES.

Machines.—Instruments used to equilibrate given resistances by means of forces or potentials neither equal nor directly opposite those resistances. The parts composing the machines are not free, but hampered by fixed obstacles in their motions.

Simple Machines.—Machines formed with a single part.

When the obstacle to its motion is

$$\left.\begin{array}{l}\text{a fixed point}\\ \text{a fixed axis}\\ \text{a fixed plane}\end{array}\right\} \begin{array}{c}\text{the machine}\\ \text{is called}\end{array} \left\{\begin{array}{l}\text{lever}\\ \text{windlass}\\ \text{inclined plane}\end{array}\right.$$

Compound Machines.—Formed with several simple machines reacting upon each other.

Lever (Fig. 183).—Solid body of any shape, free to move in any direction about a fixed point. Or it is a rod moving in a plane about one of its points called **fulcrum.**

Fig. 183

Generally two forces only are applied to a lever; the one **P** at our disposal called **power,** the other **R** called **resistance,** which is opposed by the body to be displaced. To these two for e be added the weight of the lever itself; unless the vertical from its center of gravity passes through the fulcrum when in equilibrium.

General Condition of Equilibrium.—All the forces must have a single resultant passing through the fulcrum.

Conditions of Equilibrium of a Lever Solicited by Two Forces Only (Fig. 184).—Let **O** be the fulcrum of lever **A O B;** let a power **P** be applied to **A** to overcome a resistance **Q** applied to **B.** In order that the forces **P** and **Q** have a resultant passing through the fulcrum **O** they must be in the same plane with that point. Then the moments of the two forces **P** and **Q** with regard to **O** must be equal and of contrary signs. Let **p** and **q** be the perpendiculars drawn from **O** to the directions of the forces, we must have

$$Pp = Qq \quad \text{or} \quad \frac{P}{Q} = \frac{q}{p}.$$ Hence the two conditions of equilibrium

1°. The power and resistance must be in the same plane with the fulcrum.

2°. The power and resistance must be inversely proportional to their distances from the fulcrum and each tending to revolve the lever in opposite directions.

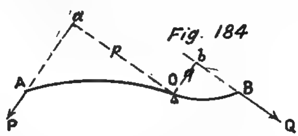


Fig. 184

We can at will change the magnitude and position of the power provided that its moment with regard to the fulcrum be not changed. The lever will still be in equilibrium.

Load on the Fulcrum.—In case of two forces **P** and **Q**

it will be $R = \sqrt{P^2 + Q^2 + 2PQ \cos POQ}$.

When the power **P** and resistance **Q** are parallel, and act in the same direction, **R=P+Q.** because **POQ=o.**

First Kind of Lever (Fig. 185).—The **fulcrum is situated between P and Q.**..This lever is used by laborers and in railroading when straightening the rails or displacing ties.

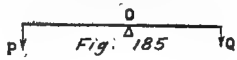


Fig. 185

Note that in a straight lever the perpendicular **p** and **q** are proportional to the section of lever on their side from the fulcrum and called arm (power arm or resistance arm). Therefore we say that in a state of equilibrium of a straight lever, the power and resistance are inversely proportional to their respective arms.

Second Kind of Lever (Fig. 186).

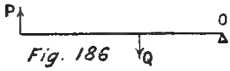


Fig. 186

The forces **P** and **Q** are parallel and act in contrary directions.

$$R = \pm (P - Q)$$

in that case **P** and **Q** are on the same side of the fulcrum. The pressure on the fulcrum is then a minimum. When the resistance **Q** is nearer the fulcrum, the lever is of the **second kind**

and always $P < Q$. This kind of lever is applied in the *weelbarrow*.

Third Kind of Lever (Fig. 187)—When the power **P** is nearer the fulcrum, the lever is of the **third kind**

and always $P > Q$.

This kind of lever is found in the **pedal** of a **reel organ.**

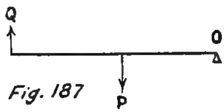


Fig. 187

The several kinds of levers have their application in the several forms of scales.

Scales.—Instruments used to weigh bodies.

Balance Scales.—In the Balance scales with two plates, one of which contains the body to be weighed **Q,** and the other the standard weights used **P** to establish equilibrium, the point of suspension is in the middle of the lever.

Conditions of Accuracy.—1° The two lever arms must be of equal length. 2° The vertical of the lever center of gravity must pass through the fulcrum.

Condition of Sensitiveness.—The Balance scales will be sensitive in proportion as the lever center of gravity, although situated below the fulcrum will be nearer to it.

Beam Scales.—Also a lever of the first kind; they have unequal arms.

Platform Scales.—Have two or more different levers, generally one of the first and one of the third kind.

Pulley.—A wheel free to move around its center in its plane. The circumference of the wheel is hollowed out to receive and maintain in position a chain or rope. This rope moving as the pulley revolves, the forces applied to it act along its center, so that in order to have the radius of the pulley, half the thickness of the rope is added to the radius of the gorge.

Pulley a Lever.—A pulley is a bent lever with equal arms which are the radii of the tangent points where the rope enters and leaves the pulley.

Condition of Equilibrium of a Fixed Pulley (Fig. 188). **Pr=Qr,** or **P=Q** in whatever position, or, the power must equal the resistance.

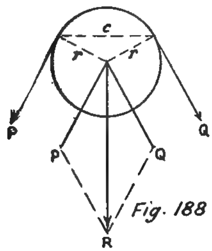


Fig. 188

Pressure at the Center.—The resultant **R** is the sum of two equal forces **P** and **Q.** By drawing from the center the parallelogram of forces and comparing the similar triangles formed by drawing also the radii **r** and the chord **c** of tangency.

we obtain $\frac{P}{R} = \frac{r}{c}$,

or the power is to the pressure at the center as the radius of the pulley is to the chord of tangency; when the forces **P** and **Q** become parallel, **c** becomes a diameter **2r** and we have

$\frac{P}{R} = \frac{r}{2r} = \frac{1}{2}$ or $R = 2P$ which value is its *maximum*.

Conditions of Equilibrium of a Movable Pulley.—1° The chord of tangency must be horizontal. 2° The power must be to the suspended weight as the radius of the pulley is to the chord of tangency.

Case of Least Power P for a Given Weight W.—The equation

$\frac{P}{R} \quad \frac{r}{c}$ may be written $\frac{P}{W} = \frac{r}{c}$ and $P = \frac{r}{c} W$

Now the chord **c** has its greatest value when **P** and **Q** are parallel,

then $c = 2r$ and we have a *minimum* for $P \;\therefore\; P = \frac{1}{2} W$

Train of Pulleys.—Several pulleys free to move independently two or more axes the upper one being fixed and the lower movable.

A chain or rope attached at one end to the fixed axis is wound around continuously from one pulley to the next and at its free end the power **P** is applied.

The weight **W** is suspended to the axis of the lower pulley or set.

Condition of Equilibrium of a Train of Pulleys.—The weight **W** is supported by a system of parallel forces equal to the number **n** of pulleys and each equal to **P**;

$$\text{therefore} \quad P = \frac{W}{n},$$

or **the power equals the quotient of the weight by the number of pulleys.**

Windlass.—A cylinder or drum subject to revolve about its axis supported at one or both ends, by means of one or two cranks, or radial arms, movable bars, etc.

Winch.—A horizontal windlass generally used for hoisting. The resistance to be overcome is a weight **W** suspended to a rope or chain secured to the drum at the other end and wound around the drum several times. The power **P** is applied at the end of the crank. The center of gravity of the winch, which is symmetrical in its parts, falls on a point of the axis so that the weight of the machine does not affect the conditions of equilibrium.

Capstan.—A vertical windlass secured at its lower end and moved with radial arms. Used aboard vessels to lower or raise anchor.

Condition of Equilibrium of the Winch.—**R** being the radius of the cylinder and **r** that of the crank,

$$\text{we have} \quad \frac{P}{W} = \frac{R}{r}; \quad \text{or}$$

The power is to the weight as the radius of the cylinder is to that of the crank.

Pressure Exerted on the Supports.—The total pressure is 1° that of the weight **W**; 2° that of the power P (equal to **W** in the state of equilibrium) and 3° the weight **w** of the machine. If the weight be suspended in the middle of the drum, each support will sustain a pressure of

$$\frac{W+P}{2} + \frac{w}{2} - \frac{2W}{2} + \frac{w}{2} = W + \frac{w}{2}$$

If the weight be at a distance **a** from support **S** and that **l** be the distance between supports **S** and **S'**, the pressures will be inversely proportional to **a** and **l-a.**

$$\text{Pressure on } S = 2W(l-a) + \frac{w}{2}$$
$$\text{" \quad " } S' = 2Wa + \frac{w}{2}$$

Differential Winch.—The weight **W** is suspended from a pulley; the ends of the rope supporting the pulley are wound in opposite ways one around each of two cylinders of different radii

$R > R'$ mounted on the same axis In this case we have :

$$Pr = \frac{W}{2}(R-R'), \text{ or } \frac{P}{W} = \frac{R-R'}{2r}, \text{ or}$$

The power is to the weight as the difference of the radii of the cylinders is to double the radius of the crank.

The last equation may be written

$$P = \frac{W}{2r}(R-R').$$

The value of **P** for a given weight will be the smaller as the difference of the radii is the smaller.

Crane.—A combination of winch and pulley generally built in the form of an **A,** two heavy pieces of timber forming the sides, the winch forming the cross piece and the pulley being placed at the apex.

The machine has secure footings and is maintained in a nearly vertical position by guy ropes. It allows of the weights being raised higher than with the windlass.

Derrick.—A crane mounted on a vertical axis around which it may revolve, thus allowing the weights raised to be deposited within a certain radius from the machine.

Contract Wheels (Fig. 189).—Two wheels **A** and **A'** mounted on parallel shafts whose distance is **d** and of radii **R** and **R'** such that **R+R'=d,** the wheels moving in the same plane. When one wheel revolves, friction communicates an opposite motion to the other wheel

$$\text{and the} \quad \frac{\text{number of revolutions of } A}{\text{number of revolutions of } B} = \frac{R'}{R}, \text{ or}$$

The number of revolutions of the wheels are inversely proportional to their radii (and also to their circumferences).

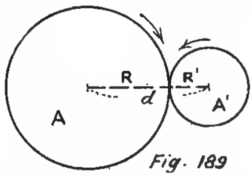


Fig. 189

Such a machine would be of very little use practically because a small resistance would overcome friction. In order to insure permanency of contact, teeth and hollows are practiced in the circumferences of both wheels.

Toothed Wheel (Fig. 190).—One the circumference of which is divided into equal teeth and spaces.

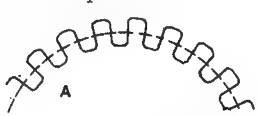


Fig. 190

Toothed Gears.—Two toothed wheels intended to act together in a piece of machinery.

Pitch Circles (Fig. 191).—Circles drawn through the middle of the teeth; they are tangent to each other at a point called the **pitch point.** The diameters of these circles are called the **pitch diameters** of the gears.

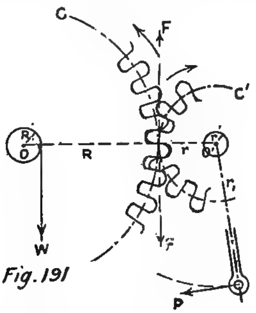


Fig. 191

Face of a Tooth.—That portion outside the pitch circle.

Root of a Tooth.—That portion within the pitch circle.

The root is made a little deeper than the length of the face and this for **clearance.** On rough gears the width of the tooth is made a fraction smaller than the space, that difference is called **back-lash.**

Pitch of a Gear.—Space occupied by a tooth and a space (or between the centers of two consecutive teeth).

Pinion.—One of the toothed wheels much smaller than the other and often actuated by means of a crank.

Condition of Equilibrium.—A **toothed gears** is a double winch.

Let **R** be the radius of the large wheel and **R'** that of the shaft; **r** the radius of the pinion and **r'** that of the crank. Let also **F** two forces equal and opposite in directions which, applied to the pitch point would equilibrate either wheel (action of one wheel equals the reaction of the other). The equations of equilibrium of the two winches are

$\frac{P}{F} = \frac{r}{r'}$ (1) for the pignon and $\frac{F}{W} = \frac{R'}{R}$ (2) for the wheel. Eliminating F we finally obtain $\frac{P}{W} = \frac{R'r}{Rr'}$ as the equation of equilibrium of the system, or

The power is to the weight (or resistance) as the product of the radii of the pinion and drum is to the product of the radii of the wheel and crank.

In practice the ratio

$\frac{r}{R}$ is replaced by $\frac{n}{N}$.

n and **N** being the number of teeth in the pinion and the wheel. If we suppose the shaft replaced by another pinion geared to the toothed wheel of a third winch, and the weight suspended from the drum of this last, we see that

The power is to the weight as the product of the radii of the pinions and drum is to the product of the radii of the wheels and crank.

Rack and Pinion.—A straight bar cogged on one side and geared to a pinion actuated by a crank. Often, in order to increase the efficiency of the machine, the pinion is geared to a wheel on the shaft of which a pinion actuating the rack is mounted. In all cases

The power is to the weight, as the product of the radii of the pinions is to the product of the radii of the wheel and crank.

Inclined Plane.—A plane oblique to the horizon or not perpendicular to the vertical of the place.

Conditions of Equilibrium of a Body Resting on a Rigid Plane by One Point Only.—All the forces soliciting the body must have a single resultant passing through that point and of a direction normal to the plane. That normal resultant will be the pressure exerted upon the plane.

Friction.—When the resultant is not normal to the plane it may be resolved into two forces—one normal and destroyed by the plane, the other in the direction of the plane and soliciting the body to slide, unless the surface of the plane opposes a sufficient resistance. Experience shows that this resistance exists; it is called **friction.**

Causes of Friction.—Molecular attraction, unevenness of surfaces which all have projections and cavities, thus tending to gear in each other, and deformation of the surface at the points of contact caused by pressure. The conditions of equilibrium given above suppose a perfect polish.

Equilibrium of a Body Resting by Several Points upon a Rigid Plane (Fig. 192).—The forces applied must have a single resultant normal to the plane and whose direction meets the plane within the convex polygon formed by the several points of rest.

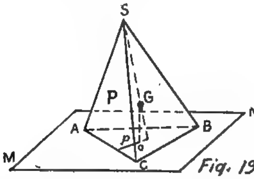


Fig. 192

Solid Body Resting on a Horizontal Plane.—When a body rests on a plane it may have an infinity of points of contact, that is to say it may have a face of contact with the plane. As long as the vertical of the center of gravity intersects the plane within the face of contact, the body will be in equilibrium.

Axis of Rotation (Fig. 193).—When the vertical of the center of gravity falls without the face of contact, the body tips over, rotating about the nearest edge which is an **axis of rotation.**

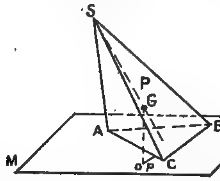


Fig. 193

In order to prevent the overturning of a body a force **Q** could be applied to it such that its moment **Qq** with regard to the axis of rotation would be equal and of contrary sign to the moment **Pp** of the weight of the body.

Moment of Stability.—The moment of the weight of a solid body **Pp** with regard to the arris of the surface of contact nearest the vertical of the center of gravity (**P** is the weight of the body and **p** the perpendicular distance from the point of intersection of the vertical of the center of gravity with the plane to the nearest arris.)

Equilibrium of a Heavy Body Resting on an Inclined Plane.—Left to the action of gravity the body will not be in equilibrium because the action of gravity which is vertical is not normal to the plane. This however can be resolved into two forces, one normal and destroyed by the plane, the other parallel to the plane and whose effect will be to cause the sliding of the body.

Normal Pressure (Fig. 194).—**a** being the angle which the inclined plane makes with the horizontal, the first component, normal to the plane, will be equal to **P cos a** which value is called the **normal pressure** of the body supported by the plane.

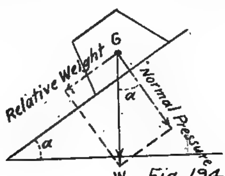


Fig. 194

Relative Weight.—The second component, parallel to the plane, will be equal to **P sin a** which value is called **relative weight** of the body.

Angle of friction.—When the angle **a** is very small no displacement takes place; this is due to friction. If we suppose **a** gradually increasing, there will be a moment when sliding will begin; let **A** be the value of that limit of **a**; the angle **A** is the **angle of friction.**

Friction is then exactly equal to the component **P sin A.**

If we represent by **C** the ratio of the friction **P sin A** to the pressure **P cos A** we have **C=tan A.**

The angle of friction and therefore the coefficient **C** can only be determined experimentally because it varies with the materials and the polish of the surfaces in contact.

Amontons' Laws.—1°. The force of friction is independent of the extent of the surfaces in contact. 2°. It is proportional to the pressure on the resisting surface.

Length, Base, Height of an Inclined Plane (Fig 195). — If we draw through the center of gravity of a body a plane perpendicular to the inclined plane the intersection with the last plane will be a line of greatest slope. Considering that intersection as the hypothenuse of a right angled triangle the two other sides of which are a horizontal line and a vertical line the hypothenuse **l** is called the **length of the inclined plane,** the horizontal **b** is the **base** and the vertical **h** is the **height.**

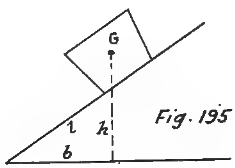
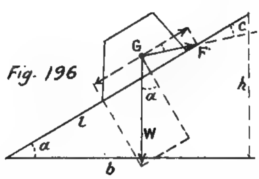


Fig. 195

Equilibrium of a Body of Weight W Resting on an Inclined Plane (Fig. 196).—Let **F** be a force making an angle **c** with the inclined plane and maintaining the body in equilibrium. **F** may be resolved into two forces, one normal to the plane and whose value is **F sin c,** the other parallel to the plane, whose value is **F cos c.** The weight **W** may also be resolved into two forces of same directions as the others; the one normal to the plane has a value of **W cos a,** the one parallel to the plane has a value of **W sin a.** The components parallel to the plane, of opposite directions, are equal because the body is in equilibrium, so that **W sin a=F cos c (1)** is the equation of equilibrium.

Fig. 196



Normal Pressure of a Body of Weight W Resting on an Inclined Plane.— The normal pressure is **N=W cos a—F sin c; (2).**

Case When Equilibrium is Due to a Force F Parallel with the Inclined Plane.

In that case $c = 0$ and $\begin{cases} \cos c = 1 \\ \sin c = 0 \end{cases}$

The equation of equilibrium is then $F = W \sin \alpha$; or $F = W\frac{h}{l}$; or $\frac{F}{W} = \frac{h}{l}$.
The normal pressure is $N = W \cos \alpha$; or $N = W\frac{b}{l}$; or $\frac{N}{W} = \frac{b}{l}$

1°. The force parallel to the inclined plane is to the weight of the body which it balances as the height of the inclined plane is to its length.

2°. The normal pressure exerted by a body upon an inclined plane is to the weight of the body as the base of the inclined plane is to its length.

When the force **F** is horizontal **c=a** and

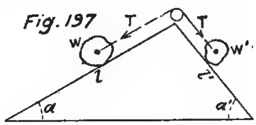
$$F \cos a = W \sin a \ ; \text{ or } \frac{F}{W} = \frac{\sin a}{\cos a} = tg\, a \ ; \text{ or } \frac{F}{W} = \frac{h}{b}$$

3°. The horizontal force is to the weight of the body which it balances as the height of the inclined plane is to its base. These results are modified by friction.

Equilibrium of Two Bodies W and W' Resting on Opposite Inclined Planes of Common Apex and of Different Angles a and a' held Together by a Rope Passing Over a Pulley (Fig. 197).—Being in equilibrium the tension or pull on each branch of the rope will be the same.

$$T = W \sin a = W' \sin a' \ ; \text{ or } \frac{W}{W'} \quad \frac{\sin a'}{\sin a} = \frac{\frac{h}{l'}}{\frac{h}{l}} \ ; \text{ or } \frac{W}{W'} = \frac{l}{l'}$$

Fig. 197



The condition of equilibrium is that the weights of the bodies shall be proportional to the lengths of the inclined planes upon which they rest.

KINEMATICS.

Kinematics.—Treats of motions of bodies as limited by the properties of space and without reference to the action of forces.

Dynamics.—Treats of motion as an effect of forces.

Unit of Time.—Is the second or 1-60 of a minute, 1-3600 of an hour or 1-86400 of a day.

Motion.—Is displacement from one point to another.

Rectilinear Motion.—Displacement along a straight line.

Uniform Motion.—Displacement of equal distances in equal times.

Velocity.—Length of displacement during the unit of time, or displacement per second.

Equation of Uniform Motion—x being the displacement, during the time **t**, of the point (center of gravity of the body) moving with a velocity **v**, and **c** the space already travelled by th[illegible]ginning of **t**, we have the **general equation of uniform motion: x=c+**

vt (1) in which **x** and **c** may be positive if they are on the same side of the point where the motion began, and negative if on opposite sides, in which also **t** may be negative when counted before the time specified; and in which **v** may also be negative when in an opposite direction to the one originally adopted as representing positive velocities. **x**, **c** and **v** are generally expressed in feet and **t** in seconds.

Varied Motion.—When the ratio between the displacement and the time required to produce it, changes continuously.

Mean Velocity.—If **x** be the displacement during the time **t** and

x' that during the time t' such that $t+h=t'$ (h being an increment), $x'-x$ will be the displacement during h and the ratio $\frac{x'-x}{h}$

is the **mean velocity** during the interval **h**; it is also the velocity of the uniform motion which would carry the point the distance **x'—x** during the time **h.**

Acquired Velocity.—Suppose in the ratio

$\frac{x'-x}{h}$ that h tends toward *zero*, we have $v = limit\frac{x'-x}{h}$, and that limit of the ratio of the increment of a function to the increment of a variable t when this last tends toward *zero* is the *derivate* of the function: $v = d_t x$ expresses that

The acquired velocity at the end of a given time t is the derivate of the displacement considered as a function of the time.

Curve of Velocities.—If we have a table of times and distances furnished by experiment, we may draw the **curve of velocities.**

Draw two axes, one horizontal, the other vertical. Lay off to a scale, on the horizontal axis and from the intersection, the times given in the table; they will be the abscissas; draw an ordinate at each point of a length equal to the corresponding distance given in the table. Finally connect the extremities of the ordinates with a continuous curve. This diagram will give

1°. The displacement for any time not given in table.

2°. The time required to produce a certain displacement.

3°. The velocity at any given time. It is equal to the ratio between the ordinate and the sub-tangent, or is equal to the trigonometrical tangent of the angle made by the tangent to the curve with the horizontal line.

Direct Motion.—When the displacement increases with the time.

Retrograde or Contrary Motion.—When the displacement decreases with the time.

Curvilinear Motion.—When taking place along any curve. In that case the displacements are lengths of arcs.

Circular Motion.—When taking place along the cirf. of a circle.

Direction of Velocity in Curvilinear Motion.—The element of direction is introduced in curvilinear motion.

The direction of velocity at a point is that of the tangent to the curve at that point.

Motion Uniformly Varied.—When the velocity varies by equal quantities in equal times.

Acceleration.—The constant quantity that volocity varies in the unit of time.

Motion Uniformly Accelerated.—When the acceleration is positive or additive.

Motion Uniformly Retarded.—When the acceleration is negative or subtractive.

Equations of Uniformly Varied Motion.—Let a be the acceleration, positive or negative,

v_0 the initial velocity, v the velocity at the end of time t; then $v = v_0 + at$ (1).

Let x_0 be the displacement at the origin of Time t, x the displacement at the end of time t; then

$$x = x_0 + v_0 t + \frac{at^2}{2} \qquad (2)$$

Displacement during time t is $x - x_0 = v_0 t + \frac{at^2}{2}$; and if the origin of displacement is at the beginning of time t, (2) becomes

$$x = v_0 t + \frac{at^2}{2} \quad (3)$$

Eliminating a between (1) and (3) we get: $x = \frac{v + v_0}{2} t$ (4); which shows that

The displacement during a certain time is the same as if the point had moved uniformly during that time with a uniform velocity equal to the mean of the initial and final velocities.

Uniformly Accelerated Motion.—Without initial velocity $v_0 = 0$, and a is positive

Equations $\left\{ \begin{matrix} (1) \\ (2) \end{matrix} \right\}$ become $\left. \begin{matrix} v = at & (5) \\ x = \frac{at^2}{2} & (6) \end{matrix} \right\}$; therefore,

1°. Velocity is proportional to time.

2°. Displacement is proportional to the square of the time.

Applying (6) to the t^{th} and the $(t+1)^{th}$ seconds of time we have $x' = \frac{a}{2}(t+1)^2$ and $x = \frac{a}{2}t^2$; and by subtraction $x' - x = \frac{a}{2}(2t+1)$ which proves that **the displacements during the successive units of time increase as the series of odd numbers, 1. 3, 5, 7, etc.,**

Make $t = 1$ in (5) and (6), they become $v = a$ and $x = \frac{a}{2}$.

which proves that **at the end of the first unit of time the velocity is double the displacement during that time.**

Eliminating t between (5) and (6) we obtain $v = \sqrt{2ax}$ (7),

which proves that **the velocity at any moment is a geometrical means between the displacement and double the acceleration.**

Fall of Bodies in a Vacuum.—Experiment proves that 1° **all the points of a body falling in a vacuum follow vertical lines with equal velocities** and 2° **the displacements are proportional to the squares of the times,** whence it follows that 3° **the velocities are proportional to the times.**

The acceleration is represented by **g**, it increases with the latitude and diminishes with the altitude or elevation. At sea level it is **32.173—0.082 cos. 2 lat.**

A body falling from a height **h**

acquires a velocity of $v = \sqrt{2gh}$

from which tables may be made for any values of **g** and **h**.

Uniformly Retarded Motion.—Equations **(1)** and **(2)** become

$v = v_0 - at$ and $x = v_0 t - \frac{at^2}{2}$. Motion will be direct as long as $v_0 > at$; it will be null when $v_0 = at$ which corresponds to the time $t = \frac{v_0}{a}$ and to the *maximum* value of $x = \frac{v_0^2}{2a}$, and motion will be retrograde when $v_0 < at$.

A body thrown upwards in a vertical direction and in a vacuum will assume a uniformly retarded motion.

Its initial velocity v_0 decreases after each second by the constant quantity g, generally taken as 32.16 ft. After a time $t = \frac{v_0}{g}$ the velocity is null and the moving point has reached its *maximum* vertical displacement $\frac{v_0^2}{2g}$.

The body then falls with a uniformly accelerated motion reaching the starting point after the time

$\frac{2v_0}{g}$ for which $x = 0$; at the same time $v = -v_0$

Therefore, **the time of the ascension equals the time of the descent and the velocity at the arrival is equal and of contrary sign to that of the departure.**

Parallelogram of Velocities.—When a point is subjected to two or more motions, that is to say, to motions which would carry it in different directions and at different distances if acting alone, that point will follow a different path than any of these directions.

Case of Two Simultaneous Uniform Rectilinear Motions.—Assume a point as origin of motions. From that point draw two straight lines in direction and length equal to the two component motions soliciting the body. Draw a parallelogram on these two lines, the diagonal of origin will be the path or trajectory followed by the **resultant motion** of the body. This figure is the **parallelogram of velocities.**

In case of more than two velocities, two are first composed, then the resultant obtained is composed with another velocity and the new resultant treated in like manner until all the velocities have been taken, thus forming a **polygon of velocities.**

Case of Two Simultaneous Rectilinear Motions, One Uniform the Other Uniformly Varied.—The diagonal of the parallelogram of

velocities is no more a straight line, because the ratio of the side is not constant.

The equations of the two motions are $v = at$ and $v' = \frac{a't^2}{2}$ for the same time. By eliminating t we obtain $v^2 = \frac{2a^2v'}{a'}$ which is the equation of a *parabola*.

In this case the resultant motion will follow a parabolic path or trajectory.

Rotation About a Fixed Axis.—The simplest is the circular motions when the paths described become arcs of circles. The velocity is given either in degrees and fractions which can be reduced to length of arc, or in revolutions per second or minute.

The formulas are $v = \frac{\pi Rn}{180}$ in the first case, and $v = 2\pi Rm$ in the second.

In these formulas **n** is the number of degrees moved over in the unit of time, and **m** is the number of revolutions per second or minute.

Laws of Inertia.—1°. A material point being at rest will continue in that state as long as no force solicits it.

2°: A material point in motion but not actually solicited by a force is in a state of uniform rectilinear motion.

Other Theorems.—(a) There is no force whose effect is instantaneous because the action of a force has always a certain duration when it may be considered as varying continuously.

(b) A material point in motion being solicited by a force of same direction, the velocity after a time **t** is the algebraic sum of the initial velocity plus the velocity which the force would impart to the point during the time **t** were the point at rest.

(c) A constant force acting in the same direction as the initial velocity of a body imparts to it a uniformly varied motion—and conversely.

(d) Gravity is a constant force.

(e) The weight of a body is the same when at rest as when in the state of motion.

(f) Two constant forces are proportional to the accelerations which they produce when acting upon the same material point whether at rest

or in a motion of the same direction : $\frac{F}{F'} = \frac{a}{a'}$

(g) The intensity of a constant force which, acting alone upon a material point, imparts to it a given acceleration is equal to the product of that acceleration by the ratio of the

weight of the point to $g = 32.16$. $F = a\frac{W}{g}$.

Mass of a body.—Some define it as the quantity of matter in the body.

Equality of mass.—Two material points have an equal mass when two equal forces applied during the same time to these points at rest impart the same motion to them.

If two, three or more material points of equal mass be invariably united together we have new material points of a mass double, treble, etc., that of the first points.

The **mass of a body** is the **sum** of **the masses of all its material points.**

Sum of masses of bodies.—The mass of the body formed with several other bodies invariably united together.

Measure of the mass of a body.—The mass **M** of a body is the ratio

of its weight W to $g = 32.16$ $\qquad M = \frac{W}{g}$.

The mass of a body is constant.—Because **W** and g vary always in the same ratio with the latitude and the altitude.

In the same place masses are proportional to weights.

From $M = \frac{W}{g}$ and $M' = \frac{W'}{g}$ we conclude $\frac{M}{M'} = \frac{W}{W'}$

Unit of mass.—The value of **M** in the equation

$M = \frac{W}{g}$ becomes 1 when $W = g$;

we are therefore led to assume that **the unit of mass is g lbs.** (32.16 lbs): say **m=g.**

Unit of force.—Force which applied to the unit of mass imparts to it in the unit of time a velocity equal to the unit of length.

Theorems.—1. Two constant forces imparting equal accelerations to two different masses are proportional to these masses.

2. When a constant force is successively applied to different masses, the accelerations of motion it produces are inversely proportional to the masses.

3. Two constant forces are to each other as the products of the masses to the center of which they are applied, by the accelerations which they impart to them; and as accelerations are proportional to velocities, if we call

Quantity of motion.—The product of the mass of a body by the velocity, **(3)** may be expressed:

4. Two constant forces are to each other as the quantities of motion which they produce in the same time.

Mechanical work of a force.—1°. The product of the force **F** by the length **d** of displacement **(Fd),** when the point of application of the force moves along the direction of the force.

2°. The product of the force **F** by the projection of the displacement upon the direction of the force: **F l cos a,** when the point of application moves in a straight line but in a direction different from that of the force; or, because **F l cos a=l (F cos a)**

3°. The product of the displacement by the projection of the force upon the direction of displacement. If the force **F** is a power we have a **motive power**; if the force **F** is a resistance we have a **resistance work.**

Work is null in three cases.—1°. When the force is zero; as when a material point is already in motion without being solicited.

2°. When the displacement is zero; as when motion is prevented by an obstacle.

3°. When the direction of the path is perpendicular to the direction of the force.

Unit of work.—Foot pound—Effort or force necessary to raise 1 lb. 1 ft.

Work of the resultant of several constant forces applied to a point whose displacement is rectilinear.—That work is the sum of the works of the components.

Work of gravity in the motion of a heavy body or a system of heavy bodies.—It is the same as if all the masses were concentrated at the center of gravity.

Machines in motion.—The object of machines is not only to equilibrate resistances but to overcome them and displace their points of application. Then only a machine produces work. Two kinds of forces are to be considered:

Moving forces.—Those imparting motion to a machine as manpower, steam, wind, water-head, compressed air, etc. Displacement is in the direction of the forces and the work is positive or **moving** and the force considered is a **motor.**

Resisting forces.—Those which the machine has to overcome. Their points of application move in contrary direction to the forces and their work is negative or **resisting.**

Useful resistance.—A resistance for the overcoming of which the machine was built, as for instance the cohesion of bodies to be reduced in a crusher or pulverized in a mill, or the weight of materials to be hoisted or carried.

Passive resistance.—A resistance which the machine has first to overcome before producing the effective work for which it was designed, as for instance friction of the parts of the machine, resistance to rolling, stiffness of ropes, resistance of the medium (air, water, etc.). These resistances cannot be eliminated and the skill of the mechanical engineer consists in reducing them to a minimum.

The corresponding work will be accordingly **useful resistant work** or **passive resistant work.**

Friction during motion.—It is constant and produces uniformly retarded motion.

Coefficient of friction.—The ratio of friction **F** to pressure **P**

$$f = \frac{F}{P}.$$

That coefficient varies with the substances in contact; it diminishes with the lubricant used.

FROM MORIN'S TABLE (1831)				
Plane Surfaces in contact.	*Ratio of Friction to Pressure — or value of f.*			
	at the start after a certain contact.		*During Motion:*	
	Lubricant.		*Lubricant.*	
	without	*with*	*without*	*with*
Oak on Oak Parallel fibres	0.62	0.44 dry soap	0.48	0.16 dry soap
do. do. Perpendicular "	.54	.44 " "	.34	.16 " "
Fibers on end on flat fibres	.48	.44 " "	.19	.16 " "
Oak on Iron (Wrought)	.62	.15 lard	.62	.08 lard
Wrought Iron on Cast Iron	.19	.10 "	.18	.08 "

Friction of axles upon bearings.—Follow same laws as plane surfaces in contact but coefficient f is smaller being 0.07 to 0.10 with ordinary lubricant and 0.05 to 0.06 with constant lubricant.

Resistance to rolling.—It is directly proportional to pressure and inversely proportional to diameters of wheels. On a good road bed it is practically independent of the width of the tire, but on a soft road bed it increases with such width.

Canal, dead water	0.001
Railroad	0.005
Wood pavement	0.022
Good pavement	0.03
Medium pavement	0.04
Hard ground	0.06
Bad pavement	0.08

Machines in uniform motion.—Under the action of a continuous motive power, machines would tend to assume uniformly accelerated motions after overcoming resistances including the maximum work to be performed. When that maximum is attained, restricting and regulating forces such as fly wheels and brakes are applied to maintain that maximum. The machine will then move uniformly and all the forces working and resisting are in equilibrium; therefore,

Motive work equals resisting work.

Windlass (Fig. 198).—**F** being the force applied to end of crank arm of radius **R, W** the weight at end of rope wound around cylinder of radius **r.** The equation of equilibrium is (1) **FR=Wr** at rest or in motion. In a given time, if **W** ascends a distance **d,** the rope will be wound also a length **d** on the cylinder. If we represent by **a** the arc simultaneously described by a point at a distance **1** from the center,

we have **aR**= length of arc described by the point of application in the direction of the force, and **ar=d,** length of arc wound up around cylinder. The resisting work will be **Wra** and the motive work **FRa,** and by **(1)**: **FRa=Wra.**. The works are therefore equal.

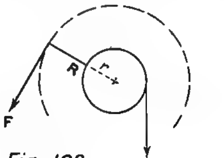


Fig. 198

If passive resistances are taken into account such as the stiffness of ropes or the friction of journals upon bearings, we must have

$$FRa > Wra \; ; \text{ or } \; F > \frac{Wr}{R}$$

Friction is reduced by reducing the size of the journals and lubricating the bearings.

Movable pulley—(Fig. 199).—Supposing the two ends of the rope parallel, when the weight moves up **d,** the diameter of the pulley also moves up **d**; the fixed end is shortened by **d**; therefore any point on the power end has moved up **2d.** In equilibrium we have **W=2P,** therefore **Wd=2Pd=P2d** showing the equality between the motive work and the resisting work.

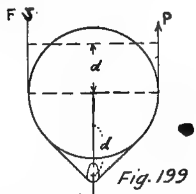


Fig. 199

Inclined plane—(Fig. 200).—Equation of equilibrium

(1) W sin. a=F cos. c, a being the angle of the plane with the horizon and **c** the angle of the force **F** with the inclined plane. If the body is displaced by **d,** so is the pt. of application; the projections of the latter are **d sin. a** and **d cos. c**, and as from **(1)** we have

W sin a. d=F cos c.d, the motive work equals the resisting work.

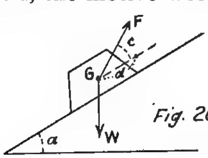


Fig. 200

General equation of work in a machine in uniform motion.—Suppose a machine having attained a state of uniform motion and doing

work. Let **W** be the sum of moving work developed, **Wu** the sum of useful resisting work and **Wp** the sum of passive resisting work; the sum of these must be null because the forces are in equilibrium therefore $W - Wu - Wp = 0$ or $W = Wu + Wp$ **(1)** note that **Wu** and **Wp** are of contrary signs to **W.** This is the general equation of work in a machine in uniform motion.

Efficiency of a machine.—From **(1)** is as follows

that $W > Wu$ and $\frac{Wu}{W} < 1$. This ratio $\frac{Wu}{W}$, always less than 1,

is called the efficiency of a machine.

Horse power.—

550 foot-pounds of work in one second.
33000 foot-pounds in one minute.
1980000 foot-pounds in one hour.

Work necessary to raise a weight **W** a distance f feet is **Wf** foot-pounds.

Kinetic Energy.

$= \frac{Wv^2}{2g}$; W = weight, v = velocity in ft. per sec., g = gravity.

HYDROSTATICS.

Hydrostatics.—Treats of water at rest under the action of forces (fluids act like water).

Hydraulics.—Treats of water in motion through conduits or channels.

Water transmits pressure.—Water transmits any pressure it receives on unit of area, in all directions, undiminished and normal to any unit of area.

Kinds of pressure: Downward pressure, upward pressure, lateral pressure, oblique pressure.

Downward pressure.—Or pressure upon the bottom of the vessel. The downward pressure is composed: 1° of the weight of water; 2° of any additional pressure which the water may receive. $P = W + P'$.

Downward pressure due to water alone (Fig. 201).—The downward pressure is equal to the weight of a prism of water of the same base **A** as the bottom of the vessel, and a height **h** equal to the vertical distance from the bottom to the level of the water; $W = Ah \times 62.5$.

Note that this pressure on the bottom is independent of the shape of the vessel.

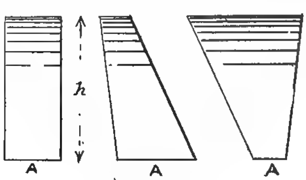


Fig. 201

Downward additional pressure due to pressure received by the water on its surface.—Downward additional pressure is equal to the additional pressure **p** per unit of area on the water surface, multiplied by the area of the bottom

$$P_1' = pA.$$

Total downward pressure.—

$$P = Ah \times 62.5 + pA\ ; \quad \text{or} \quad P = A(62.5h + p)$$

in which **A** = area of bottom of vessel;

h = height of water above bottom;

p = additional pressure per unit area on surface of water;

P = total pressure on bottom of vessel.

Upward pressure (Fig. 202).—Upward pressure on a horizontal immersed surface is equal to the weight of a prism of water of the same base as the surface, and a height equal to the vertical distance from the surface to the water level; and (if the water sustains an additional pressure) plus the additional pressure **p** per unit of area multiplied by the area of the surface. It is therefore the same as the downward pressure would be if the immersed surface were the bottom of the vessel.

$$P = A(62.5h + p).$$

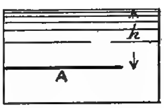


Fig. 202

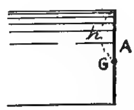


Fig. 203

Lateral pressure (Fig. 203.). — On a vertical immersed surface. Lateral pressure is equal to the weight of a prism of water of the same base as the vertical surface, and a height equal to the vertical distance from the center of gravity of the surface to the level of the water; plus a similar increase as before for any additional pressure. The formula is the same as before

$$P = A(62.5h + p).$$

Normal pressure on a surface immersed with heads of water on both sides equal to **H** and **h.**

$$P = 62.5A(H - h).$$

Oblique pressure (Fig. 204).—When an immersed surface is oblique to a given direction of pressure. The pressure is equal to the weight of a prism of water of the same base as the projection of the surface on the given direction, and a height equal to the vertical distance from the center of gravity of the surface to the level of the water. Plus a similar increase for any additional pressure. The same formula

P = (62.5 h + p) A may be applied if we agree that **A** is the projection of the oblique surface upon the direction of pressure.

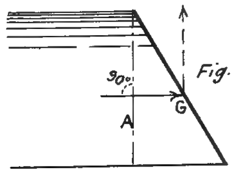


Fig. 204

Case of a cylinder (Fig. 205).—The half shell may be considered as an aggregate of rectangles of same length as the cylinder and of infinitely small width; the sum of their projections on the axis of the cylinder is a rectangular section through the axis. Therefore the total pressure upon the half cylinder is

$$P = (62.5\,\frac{L}{2} + p)\,DL \;;\ \text{in which} \begin{cases} D = \text{Diameter of cylinder in ft.} \\ L = \text{Length of cylinder in ft.} \end{cases}$$

or $$P = (.036\,\frac{l}{2} + p)\,dl\ ,\ \text{in which } d \text{ and } l \text{ are expressed in inches.}$$

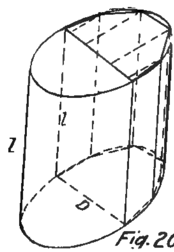


Fig. 205

Examples.—1°. Find the pressure on a valve set in the opening of a 2 inch pipe at the bottom of a basin 18 feet deep.

$$\text{Formula } P = \frac{\pi d^2}{4} \times h \times .036 = \pi \times 18 \times 12 \times .036 = 24.43 \text{ lbs.}$$

2°. Find the pressure on a 2 x 8 gate placed in the vertical wall of a gate-house. The top of the gate is 84 ft. below surface of water.

Data:

width of gate = 2 ft
height " " = 8 "
Value of $h = 84 + \frac{8}{2} = 88$ ft.

Formula:

$P = 2 \times 8 \times 88 \times 62.5 = 88000$ lbs. = 44 tons
or $2\frac{3}{4}$ tons per sq. ft.

Center of pressure.—The center of pressure on a face of a reservoir is that point to which an opposite force should be applied to produce equilibrium.

Center of pressure on a rectangular face.—1°. When the top of the water and the upper side of the face coincide. It is at ⅔ the depth of the wet face below the water surface. If the face considered is the side of a reservoir of depth **d**, the water level being **S** and the center of pressure **P**, we have: **SP = ⅔ d.**

2°. When the top of the rectangular face is below the surface of the water by a height **h** (case of a gate) we have:

$$SP = \frac{2\,(d^3 - h^3)}{3\,(d^2 - h^2)}\ .$$

Center of pressure on a rectangular face just immersed on one side (D being the depth of water) **and partly immersed on the other (d** = depth of water).

$$SP = \frac{D \times \frac{D^2}{3} - d \times \frac{d^2}{3} - (D-d)\,d \times \frac{d}{2}}{D \times \frac{D}{2} - d \times \frac{d}{2}} = \frac{1}{3}\left(2D - \frac{d^2}{D+d}\right)$$

Center of pressure of a completely immersed face with different heads of water on either side is always at the center of gravity of the face, whatever its figure and its position.

Velocity of flow.—The velocity of flow is the distance traveled by a point of the liquid mass in the unit of time—generally a second.

All the points of a mass do not move with the same velocity.—In conduits or channels the bottom and sides prevent the even flow of the liquid strata adjacent to them, which have to overcome the friction encountered.

Conduits.—May be pipes or aqueducts.

Channels.—May be weirs, rivers or canals.

Mean Velocity.—The mean velocity is a certain average velocity **V** which, multiplied by the area **A** of the liquid cross-section, equals the total quantity **Q** of water discharged during the unit of time.

General Formulas. $Q = VA$; $V = \frac{Q}{A}$, $A = \frac{Q}{V}$

Flow of Water in Pipes. Formulas:

(Prony) $V = 97.05\sqrt{RI} - 0.08$;

(Hawksley) $V = 48\sqrt{\frac{dh}{l + 54d}}$;

(Eytelwein) $V = 50\sqrt{\frac{dh}{l + 50d}}$;

(Darcy) $V = C\sqrt{RI}$.

VALUES OF C

Diam. in.	C	Diam. in.	C	Diam. in.	C
.5	65	6	105	14	110
1	80	7	106	16	110.5
2	93	8	107	18	110.7
3	99	9	108	20	111.
4	102	10	109	22	111.5
5	103	12	109.5	24	111.5

Maximum Value $C = 113.3$

Weir.—A weir is a wall built across a stream or on the side of a reservoir to regulate and measure its flow or overflow.

The approaches (sides and bottom) to a weir must be sloped regularly towards the weir to insure better uniformity of flow.

Flow of Water over Weirs (Fig. 206).

1° Weir not contracted (a) : $Q = \frac{10 l}{3}\sqrt{h^3}$;

2° „ contracted at one end (b) : $Q = \frac{10\left(l-\frac{h}{10}\right)}{3}\sqrt{h^3}$;

3° „ contracted at both ends (c) . $Q \quad \frac{10\left(l-\frac{h}{5}\right)}{3}\sqrt{h^3}$.

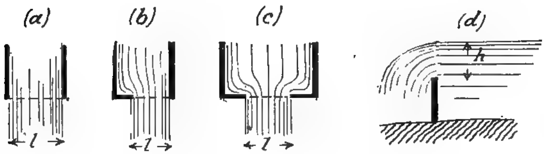


Fig. 206

Flow of Water in Channels. *Chézy's* Formula : $V = \sqrt{RI}$

In the above formulas, d = diameter in feet ;
h = head of water in feet ;
l = length in feet ;
R = hydraulic radius in feet ;
V = velocity in ft. per second ;
Q = discharge in cu. ft. per second ;
I = slope of inclination *(ratio of V^{al} to Length)* ;
C = a coefficient of roughness.

Hydraulic Radius $R = \frac{\textit{Area}}{\textit{Wet Perimeter}} = \frac{A}{P}$.

Slope or Sine of Inclination : $I = \frac{\textit{Total Fall}}{\textit{Total Length}} = \frac{h}{l}$.

Chézy's formula may then be written · $V = C\sqrt{\frac{Ah}{Pl}}$.

The general formula of discharge becomes . $Q = CA\sqrt{RI} = C\sqrt{\frac{A^3 h}{Pl}}$.

Note. The mean velocity of flow in the New Croton Aqueduct was ascertained for variable depths of water by lowering a specially constructed water gauge to certain definite points taken as the centers of small and practically equal areas into which the section of the aqueduct was divided. With this mean velocity a table of values for the factor **C** was computed. From this table, the author evolved the following formula:

$V = 124 R^{.56} I^{.5}$ which corresponds to $C = 124 R^{.06}$

A similar formula for the Boston Aqueduct is $V = 120 R^{.6} I^{.5}$

Ganguillet and Kutter's Formula.—

In the formula $V = C\sqrt{RI}$,

Ganguillet and *Kutter* make $C = \dfrac{41.6 + \frac{.00281}{I} + \frac{1.811}{n}}{1 + \dfrac{\left(41.6 + \frac{.00281}{I}\right) n}{\sqrt{R}}}$; *it is also applicable to pipes*

n is a coefficient of roughness;
I is the slope or sine of inclination;
R is the hydraulic radius.

Values of n. For Pipes—
- in very good condition $n = .01$
- " less " " $n = .012$

For Channels of uniform Section—
- in well planed timber $n = .009$
- " neat cement $n = .010$
- " concrete—Sand 1, Cement 3 $n = .011$
- " rough timber $n = .012$
- " ashlar and brick $n = .013$
- " rubble $n = .017$

Opening in Side of Reservoir. $V = \sqrt{2gh}$; in which $g = 32.2$.

Hydraulic Grade.—The slope which a conduit of uniform section should have in order that the outlet velocity of the water be the same as the inlet velocity. Or the difference of elevation between the inlet and outlet divided by the length of the conduit when the water velocities of inlet and outlet are equal.

Example. For a pipe we must have $\sqrt{2gh} = 48\sqrt{\frac{dH}{l+54d}}$

for which $h = \frac{35.78\,dH}{l+54d}$

difference of elevation $H-h = \frac{H(l+18.22d)}{l+54d}$

and hydraulic grade $\frac{H-h}{l} = \frac{H(l+18.22d)}{l(l+54d)}$

PNEUMATICS.

Pneumatics.—Treats of the mechanical properties and effects of air and aeriform fluids.

Two Kinds of Fluids.—Elastic fluids are of two kinds: **(a) Gases**—which require a great force to reduce to liquid form; **(b) Vapors**—which are more easily reduced to liquid form.

Fluids Behave Alike.—All gases and vapors behave like air and are subject to the same laws.

Air is Elastic.—Air is very elastic and can be greatly compressed and expanded.

Atmosphere.—Air surrounds the earth extending upward to over 45 miles. That mass is called **atmosphere.**

Air is Heavier Near the Earth's Surface.—The strata of air are the more compressed and therefore the heavier as they are nearer the earth's surface. This is due to the weight of the superimposed strata.

Weight of a Column of Air.—The pressure of the atmosphere, caused by its weight is about 15 lbs. per sq. in. at the surface of the earth; it is exerted in all directions.

Volume Varies Directly With Temperature.—An increase of temperature expands air, a decrease condenses it.

Volume Varies Inversely With Pressure.—An increase of pressure condenses air; the pressure removed the air returns to its former state.

Mariotte's Law.—The elasticity or pressure of air is in direct proportion to its density; or, what is the same, inversely proportional to the space it occupies—or again—the density of air (or any gas) is in direct proportion to the pressure which it sustains.

Vacuum.—A space from which air and other substances have been removed.

How to Produce a Vacuum.—Torricelli filled a tube with mercury and reversed it in a basin of the same liquid where it descended, stopping, however, when the column of mercury in the tube was about 30 inches, leaving a space on the top containing no air, which is a vacuum.

Atmospheric Pressure.—The weight of a column of mercury about 30 in. high at the surface of the earth or of water about 32 feet high.

Barometer.—Instrument used to measure pressure or weight of the atmosphere. It is a Torricelli tube, the upper portion of which is divided in inches and tenths.

Thermometer.—Instrument used to measure temperature. It is constructed on the principle that heat expands liquids and cold contracts them. It is a capillary tube filled with mercury or colored alcohol closed at both ends, the lower end expanded to form a reservoir and the upper end being a vacuum.

Centigrade Thermometer (C).—The level of the liquid at freezing point of water is marked zero. The level of the liquid at boiling point of water is marked 100. The interval is divided into 100 degrees, and the graduation is continued above 100 and below zero.

Reaumur Thermometer (R). — Freezing point is marked zero. Boiling point is marked 80°. Interval divided into 80°.

Fahrenheit Thermometer (F).—Freezing point is marked 32. Boiling point is marked 212. Interval divided into 180°.

Convert Centigrade into Reaumur, multiply by 4-5.

Convert Reaumur into Centigrades, multiply by 5-4.

Convert Centigrades into Fahrenheit, multiply by 9-5 and add 32.

Convert Fahrenheit into Centigrades, subtract 32 and multiply by 5-9.

Convert Reaumur into Fahrenheit, multiply by 9-4 and add 32.

Convert Fahrenheit into Reaumur, subtract 32 and multiply by 4-9,

Below zero is a minus quantity.

Formulas :
$$\begin{cases} R^\circ = \frac{4}{5}C^\circ & ; \quad C^\circ = \frac{5}{4}R^\circ \ , \\ F^\circ = \frac{9}{5}C^\circ + 32^\circ & ; \quad C^\circ = \frac{5}{9}(F^\circ - 32^\circ) \\ F^\circ = \frac{9}{4}R^\circ + 32^\circ & ; \quad R^\circ = \frac{4}{9}(F^\circ - 32^\circ) \end{cases}$$

TECHNICAL REQUIREMENTS.

SURVEYING.

Composition of a Level Party.—The leveler, the rodman, sometimes an axeman.

Object of Leveling.—Gathering data to determine the relative elevations of points on the ground, along a given direction or within a certain area, in order to locate works under the most advantageous and economical conditions. Also to take elevations at points on the work during construction for the purpose of making periodical estimates.

Kinds of Leveling.—Differential leveling, profile leveling, grading, contouring, cross-sectioning.

Differential Leveling.—Consists in finding how far above or below a given or known point, one or more specified points of the ground are.

Profile Leveling.—Consists in finding the elevations, above a certain datum, of a series of points along a known direction (previously established on the ground by the transit party) in order to enable the draughtsman to make a vertical cross-section, or profile, of the ground along that direction.

Grading.—Consists in placing stakes at points along the center line of a piece of work, or the principal lines thereof, as located on the drawings in office, and marking on them the vertical distance the ground is at those points above or below the proposed finished surface.

Contouring.—Consists in determining on the ground points of equal elevations and recording them so as to enable the draughtsman to draw the contours of those elevations.

Cross-sectioning.—Consists in running short lines at right angle (generally) to a principal direction (as the center line of a road) and showing on them, by stakes (called slope stakes), the points of the ground where a cut must begin (in case of an excavation), or where a fill must stop (in case of an embankment).

Indirect Leveling.—Calculation of elevations by the measurement of certain lines and angles.

Barometric Leveling.—Calculation of elevations by the difference in weight of the portions of atmosphere overlying each point, as determined by barometric readings.

The Level.—Instrument used in determining a horizontal line, or a line lying in a horizontal plane, which is a plane perpendicular to the terrestrial radius of the place where the instrument is placed.

Horizontal Lines at Different Places Lie in Different Planes.—Horizontals at a place are tangent to the surface of the earth (or rather to a spheroid having same center and form as the earth and a radius equal to the terrestrial radius extended up to the point or place where the horizontal is drawn) and therefore lie in the tangent plane at that place. Horizontals at another place will lie in a different tangent plane.

Admission.—It is admitted that, for short distances, horizontals shall be considered as lying in the same plane.

Datum.—Plane or **point** above or below which the vertical heights of other points are referred. It is sometimes taken as mean low water, mean high water or mean tide; or as the surface of water in a reservoir normally full; or simply assumed at a point 100 ft., 1,000 ft. above or below a certain monument of a known location.

How a Datum Should be Selected.—It should be assumed or selected lower (if possible) than any possible point in the survey, so as to avoid the use of negative numbers.

Elevation of a Point.—Vertical height of the point above the datum. That elevation is sometimes called a level.

Principles Governing the Forms of Leveling Instruments.—1°. A perpendicular to a vertical is horizontal.

2°. Two points of a liquid surface at rest are on a horizontal.

3°. Air, lighter than water, and confined in the same vessel, rises at the highest point.

Plumb-bob Level (Fig. 207).—It is a triangular wooden frame having two equal sides and a plumb-bob attached to the vertex. When the bob bisects the base, where a reference mark is made, the base is horizontal.

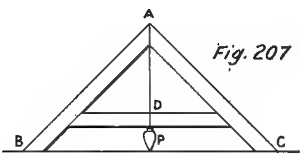


Fig. 207

Mirror (or Reflecting) Level (Fig. 208).—Consists in a small weighted mirror freely held up until the eye is bisected by a mark, when all the points of the ground on the same mark or line are on a level plane with the eye.

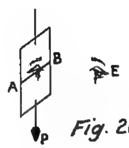


Fig. 208

Water Level (Fig. 209).—It is set on a tripod and consists in two equal open vials the bottoms of which connect with the upturned ends of a bent metallic tube filled with a colored fluid. The free surface of

the fluid in the vials determines a horizontal. The openings of the vials should be corked when the instrument is not used.

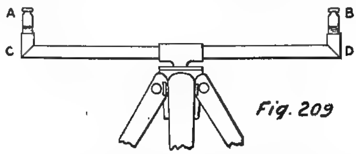


Fig. 209

Spirit Level (Fig. 210).—A sealed glass tube, generally cylindrical on the outside surface but slightly ground convex on the inside is practically filled (except for a small air bubble) with alcohol (so that the instrument may be used in cold weather). The tube is encased in a metallic cylinder resting on a plate or on equal supports. An elongated opening or race is left on the upper side of the case at its middle and through it the air bubble may be observed between graduations of the glass tube extending from the middle as a zero on either side. The bottom plate or the supports are horizontal when the bubble is bisected at the zero.

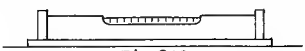


Fig. 210

The Engineers' Levels.— They are of different makes. The one mostly used in the United States is the **Y** level.

Description of the Y Level (Fig. 211).—A circular plate is screwed to a tripod and to this is attached a similar plate parallel to the first and connected with it by a ball and socket joint. Four screws (sometimes only three) called foot screws or plate screws hold these plates apart by resting on the lower one and passing through the other. A vertical spindle in the center of the plates supports a rod bar or beam, and is used to revolve the instrument. The beam is horizontal and carries at its ends two vertical standards or supports of equal size terminated by two forks of the general form of a **Y** from which the level derives its name; the inside of the **Y's** is semi-circular.

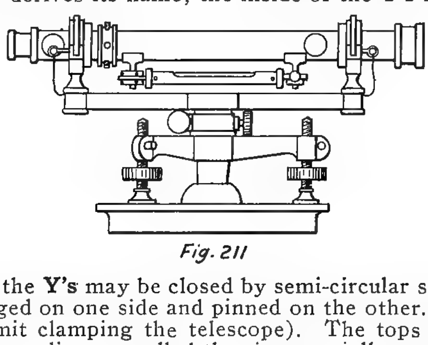


Fig. 211

The top of the **Y's** may be closed by semi-circular straps or bridles called clips hinged on one side and pinned on the other. (The pins are tapered to permit clamping the telescope). The tops of the **Y's** and the corresponding clips are called the rings or collars and should be of equal diameter. On the rings a telescope is placed supporting a

spiril level. A clamping screw just above the upper plate serves to secure the instrument in any position desired. A tangent screw, also above the upper plate, serves to give slow motion to the instrument.

The Telescope.—The **objective** or **object glass** (because facing the object looked at) is a compound lens (made so to correct spherical and chromatic aberrations of single lenses) which gathers light and forms an image at a point in the tube where cross-hairs are placed. The **ocular (eye piece)** is also a compound lens through which the operator looks to see a magnified view of the image. Tangent screws may be used to give motion to the tubes carrying the objective and ocular.

The Cross-Hairs.—Either platinum drawn wires or spider's threads attached to a ring within the telescope at the spot where the image is formed. The ring is secured by four capstan headed screws which pass through the telescope tube. There are commonly two hairs, one supposed to be horizontal and the other vertical, with their intersection in the axis of the telescope.

Bubble Level.—The spirit level attached to the telescope can be raised vertically by means of **altitude screws** at the rear end, and may be slightly moved laterally by means of **azimuth screws** at the forward end.

The Supports.—Form the **Y's** and are supported by the bar to which they are fastened by two nuts one above and one below. These nuts may be moved by means of an **adjusting pin.**

Principal Lines of a Level.—Vertical axis, bubble line and line of collimation.

Vertical Axis.—The vertical which passes through the center of the spindle.

Bubble Line.—The metallic supports of the spirit level are equal and the tangent at their top or bottom is horizontal when the bubble is centered. This tangent is the bubble line.

Line of Collimation.—The line which would connect the intersection of the cross-hairs with the optical center of the objective.

Relations Between the Lines of a Level.—1°. The bubble line and the line of collimation must be parallel.

2°. The plane described by the bubble line should be horizontal, that is to say, perpendicular to the vertical axis. These conditions are generally satisfied in a new level. But exposure and usage may alter these relations; hence the necessity of adjusting the instrument occasionally.

Adjustments of the Y Level.—The first relation cannot be established directly and is divided in two stages:

1°. Making the line of collimation parallel to the bottom element of the collars;

2°. Making the bubble line parrallel to the bottom element of the collars.

The second relation is established by making the bubble line stay in the center of the graduation during a complete revolution of the instrument around its spindle.

First Adjustment.—1°. Making the line of collimation parallel to the bottom element of the collars, or collimating the instrument.— Clamp the instrument. Unclip the collars. Sight at a distant point(as far as distinct) bringing the horizontal cross-hair on it. Carefully turn the telescope in the collars by half a revolution around its axis and sight again. If the horizontal cross-hair is still on the sighted point, the telescope is collimated with regard to that cross-hair; if it is off the point, bring it half-way back by means of the capstan-headed screws and the rest of the way by the plate screws. Repeat the operation over another point till satisfactory. Collimate it with regard to the other cross-hair. Leave screws at a snug bearing.

Second Adjustment.—2°...(a) Setting the bubble line in a plane with the bottom element of the collars. Unclip telescope, clamp the instrument over a pair of plate screws. Center the bubble by means of plate screws. Carefully and slowly turn the telescope in the collars by a small arc to the right, then to the left. If bubble moves from center bring it back by means of the azimuth or side screws.

(b) Making the bubble line parallel to the bottom element of the collars.—Unclip telescope. Clamp the instrument over a pair of plate screws. Center the bubble by means of plate screws. Carefully take telescope up, replacing it as carefully in the **Y's** in the opposite direction, that is to say, the objective sighting in the direction where the eyepiece first was. If the bubble has moved bring it back half way by means of the altitude or foot screws of the spirit level and the rest of the way by the plate screws. Repeat in another direction till satisfactory.

Third Adjustment.—(a) Making the axis of the instrument (not of the telescope) **vertical.** Pin the clips, clamp and center bubble over a pair of plate screws. Reverse telescope over same pair of plate screws; bring bubble half-way back (if it has moved) by means of plate screws.

(b) Making the bubble remain centered during a full revolution of instrument. Center bubble and revolve instrument horizontally by a half revolution. If bubble moves, correct half way by means of the support screws (at the foot of the **Y's**).

Remark.—Should the rings become worn and unequal, use two Two-Peg Method of the Dumpy Level.

Dumpy Level (so named from its compactness).—Mostly used in England, but beginning to claim recognition in the United States on account of the better stability of its adjustments over the **Y** Level from which it mainly differs in that the telescope is permanently attached to the supports or uprights; but these uprights are adjustable.

Dumpy Level Lines.—The principal lines of the **Dumpy** are the same as in the **Y,** and their relations to each other are also the same.

Two-Peg Adjustment.—Drive two stakes (pegs) several hundred feet apart. Set instrument about half way between them. Level up and sight the rod held in succession on each stake. The difference of

the readings is the true difference of the elevation of the stakes, even if instrument is not in adjustment.

To test the instrument, set it over one of the stakes (the highest one for instance); level up and sight the rod held on the other stake. Subtract the height of the instrument from the reading, the difference should be equal to the difference of elevation of the stakes as previously determined; if not, set the target half way between these readings, sight on it and center bubble by means of altitude screws. Repeat operation till satisfied.

Centering the Objective and Ocular.—These adjustments are made permanently by the maker. Four screws hold the tubes carrying the glasses, their heads passing through the outside tube, where, after permanent adjustment, they are covered by a metallic ring.

Parallax.—Apparent motion of the cross-hairs on the object sighted when the eye is moved slightly. This shows imperfect focusing of the ocular over the cross-hairs. To correct, hold a white surface as that of a book a little in front of the objective and move the ocular tube to and fro until the cross-hairs are perfectly defined.

How to Carry the Level.—The instrument should be properly clamped and screwed down on the tripod which is carried generally on the shoulder, except when going over rough ground and fences or obstructions. Carry it upright on rough surface. Then crossing a fence, carefully pass it over on the other side spreading the legs; the rodman will help if necessary.

The Tripod.—See that the ferrules or shoes are not loose. Tighten the legs securely but not too much. Generally place two shoes in the direction of the line. On side hill work, place one shoe up hill. Don't knock the legs when stepping from side to side.

How to Use the Level.—1°. Center the bubble over a pair of plate screws, then over the other pair. Plate screws should have a snug bearing. When looking at the bubble or at the cross-hairs the eyes should look natural, that is, without strain. (Try to observe with both eyes open).

2° Adjust the eye piece to the cross-hairs for parallax.

3°. Turn instrument towards target (it is better to level up facing the target).

4° Look again at bubble.

5°. Sight the target and have it set right by motions according to a prearranged code with the rodman.

6°. Look again at bubble.

7°. Read rod or direct target from the intersection of cross-hairs only.

8°. Approve target when absolutely sure.

9° Have the height of target called out by rodman.

10°. Enter it in level book.

11°. Quickly make needed calculations and

12°. Briskly motion rodman to new station or to stay for a turning-point **T. P.** and backsight and move yourself to another position.

Guarding Against the Sun.—Draw the telescope shade, or use an umbrella or a hat.

Length of Sights.—Avoid sights too short and too long. 250 to 350 ft. should be the limits of sights.

Equal Sights.—Length of backsight should practically balance length of foresight; this may be approximated by pacing, or by the stadia cross-hairs in the telescope.

Long Sights.—When sights longer than the maximum allowable are unavoidable, correction should be made for curvature.

Leveling Up or Down a Steep Slope.—The leveler after some practice will place his instrument so as to take a reading near the top or the bottom of the rod (as the case may be), thus gaining vertical distance; but this produces unequal sights. He may also follow a zig-zag course.

Leveling Across a Large Body of Water.—1°. A running stream.

Drive a stake to the water surface on each side of the stream and in a direction normal to the flow (although the line may not run so). Take a **F. S.** on the first, a **B. S.** on the second and continue to and along the line. The elevations of the two stakes may be assumed equal.

2°. Across a Pond. If a pond or lake is too wide to insure a good sight across, do as for a stream. Drive stakes on each side and to the water surface; take a **F. S.** on the first and a **B. S.** on the second.

Across a Wall.—Take a **F. S.** on the rod set on a stake, driven to the natural surface on the first side of the wall. Measure the height of the wall above stake and enter it as a **B. S.** Drive a stake also to the natural surface on the second side of the wall, measure the height of the wall on that side above the stake and enter it as a **F. S.** Set the rod on the stake and take a sight on it which will be a **B. S.** and continue.

In Underbrush.—If it cannot be cut down on the line of sights, find a high place or provide one by piling logs, rocks, etc., to set the instrument upon.

Through Swamp.—Push legs of tripod down as far as possible. The leveler lies on his side. Two men may be necessary at the level. If still unsafe drive stakes or piles for supporting instrument.

Elevations to be Taken at Road Crossings.—Take elevations both ways for some distance.

Elevations to be Taken at River Crossings.—Take elevations of high water marks, flood marks with dates of same. Question residents for the purpose; also for dates and data of extreme low water.

Carrying Levels Across Large Stream.—Two parties may work simultaneously, one on each side of the stream.

(1). Set up levels.

(2). Sight near target.

(3). Sight target on opposite shore.

(4). Repeat (2) and (3) a number of times; average will be accepted reading.

(5). Parties(with their equipment) change sides and repeat the same operations.

Correction for Curvature (Fig. 212).—A sight through a level is tangent to the surface of the earth and is only an apparent level, the real level being the curved surface itself. The correction **c** is the difference between the radii of the earth at the point sighted on and the point sighted from, and is given by the formula

$c = \frac{d^2}{2R}$ derived from $d^2 = (2R + c)c$ in which c being very small with regard to $2R$ is discarded in the parenthesis. If we suppose $d = 1$ mile, then

$$c = \frac{1m}{7916} = \frac{5280' \times 12''}{7916} = 8''.$$

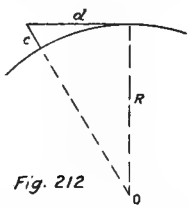


Fig. 212

The curvature then is 8 inches per mile or 1 inch per 660 ft., and may be made proportional for other distances.

How to Start a Line of Levels, or Profile Leveling.—Ascertain the elevation of any **B. M.** near the starting point of the line, which may have been previously established by another party. Have the rod set on the **B. M.** and select a convenient station for the level (it may be half way between the **B. M.** and the first stake). Sight on the rod and enter the reading as a **B. S. or** +.

Motion the rod to the first stake and take on it a **F. S.** or —.

Then **Elev. of B. M. + B. S. = Height of instrument (H. I.)**; **and** H·I. — **F. S. = Elev. of Stake.** Generally,

B. S. is a reading to determine **H. I.**, and

F. S. is a reading to determine **Elev.** of the ground.

Turning Point.—A turning point **(T. P.)** is a stake (rod station) on which a **F. S.** and a **B. S.** are taken, the second after moving the level to another place.

Assumed Elevation.—If no **B. M.** is near the starting point, an approximate elevation of the first stake may be deduced from the study of some map, such as the **Geological Map of the U. S.**, or that elevation may be assumed at a round figure as 100, 1,000.

When the elevation of the first stake or zero of the line is thus established, take on it a **B. S.** and have the rod moved to the next stake on which take a **F. S.**; without changing the position of the level, a **F. S.** may be taken on the next one or two stakes if their distance does not exceed the maximum you have set for yourself. Motion the rodman to stay on the last stake, and carry the level to a convenient point about as far (by pacing) further than the rod that the rod was last from the instrument, and take a **B. S.** on the rod, then motion it to a new station, and so continue to the end of the line, establishing on the way **B. M.'s** not further apart than a half mile (1,500 ft. would be a better distance).

Differential Leveling.—Every rod station is made a **T. P.** and the sights are made the **F. S.** as nearly equal to the **B. S.** as possible to equalize errors of curvature and as nearly the length of the maximum agreed as possible, to save time and labor.

Degree of Accuracy.—In road surveying, intermediate station readings may be taken to 0.1 foot and **T. P.** and **B. M.** readings to 0.01 foot. In more important work, intermediate readings shall be taken to 0.01 foot and **T. P.** and **B. M.** readings to 0.001 foot. For **B. M.** work with wire level and self-reading rod the error **E** is given by equation

$$E = C\sqrt{K}$$

in which **K** = distance in miles between **B. M.'s** to be checked, and **C** may be taken as **0.03.**

Check.—The elevation of the last point minus the elevation of the first point equals the sum of the **B. S.'s** minus the sum of the **F. S.'s on T. P.'s**. This check only shows whether the calculations are correct, not whether the levels are right.

Check Levels.—To ascertain the correctness of a line of levels, a second line of levels must be run between the same points and over a different ground. This second is a random line where the choice of rod stations and level stations is entirely arbitrary. No record is taken of the distance between stations or length of sights. Check levels should always be taken.

Level Notes.—Division of the Book. The pages of a level book are divided as follows: when open, the left hand half page (two sides constitute a page) is divided vertically into 5 and sometimes 6 equal columns by red lines, and the two sides are divided horizontally into equal spaces (closer together) by blue lines.

Top Notes.—On top of each page should be written the name or number of the line leveled, the names of the leveler and rodman, the date and the weather.

Use of the Several Columns.—From left to right, the vertical columns are generally appropriated as follows:

Stations.—In the first column the rod stations (generally the same as the line stations) are written reading from top to bottom, beginning about the third line; that column may properly be headed **Sta.**

Back Sights.—In the second column are entered the **B. S.'s** opposite the stations on which they were taken; it may be headed **B. S.**

Heights of Instrument.—In the third column is the calculated height of instrument at every **T. P.**; it equals previous elevation of rod sta. or **B. M.** plus **B. S.** on the same; it may be headed **H. I.**

Foresights.—In the fourth column are entered the **F. S.'s** opposite the stations on which they were taken; it may be headed **F. S.**

Elevations.—In the fifth column are the calculated elevations of the rod stations and **B. M.'s**; they equal the **H. I.** minus the **F. S.**; it may be headed **El.** or **E.**

Distances.—The sixth column (if any) is reserved for distances, that is, length of **F. S.** and **B. S.** Their sum must be equal. That column, which may be headed **Dist.** or **D.** is useful only on very accurate leveling when irregular sights are taken and for purposes of correction for curvature.

Bench Marks.—The right hand half pages, which may be headed **Remarks**, is reserved for **B. M.'s**, their designation, description and location. No station is entered in the first column on the same line as a **B. M.** if the same is not a station of the line itself. It is also used to note any remarks, such as elevation and dates of high and low water in streams, etc.

Page of a (TARGET ROD) *Level Book.*

Line C_3			Leveler Rodman	A. B. C. M. N. P.		March 16/05 Cloudy but clear
STA.	B.S.	H.I.	F.S.	Elev.	Dist.	Remarks.
	7.61	191.33		183.72		B.M. 46'L from Sta. 0 and 76'L from Sta. 1 {Bridge Coping (previous party)
0			8.03	183.30		
1			4.32	187.01		
+62			10.18	181.15		R. bank
+78			11.55	179.78		C. Mountain Brook; H.W. 184.96 Ap. 7-1864.
+93			10.19	181.14		L. bank
2			5.10	186.23		
3	10.28	196.98	4.63	186.70		⊙ T. P.
4			9.27	187.71		
5			8.54	188.44		
6	9.94	203.73	3.19	193.79		⊙ T. P.
7			7.71	196.02		
+86			8.18	195.55		B.M. 16'R. Stump

Data Should All Be Entered and Calculations Made in the Field.—The level book should all be written out in the field with a moderately hard pencil, say a **No. 3** or **No. 4.** The **H. I.'s** and **elevations** all calculated, and the column of remarks duly filled in. Don't trust to memory for details, but write them down at once.

LEVELS FOR B.M'S. (3-WIRE INSTRUMENT & SELF READING ROD.)

Page 64 *Level No 1792* *Leveler A.B.* *Rodman M.N.*
Date: Ap. 27-1908.

STA. No	Readings B.S.	Aver.	Intercepts	Sum of Interc's	Rod No	Readings F.S.	Aver.	Intercepts	Sum of Interc's.	Remarks.
					From	B.M. No 32			El.	261.057
	(a)	(b)	(c)	(d)		(e)	(f)	(g)	(h)	
1	1224 1740 2258	1741	516 518	1034	No 6 S.R.	3412 4003 4589	4001	591 586	1177	
2	1333 1781 2232	1782	448 451	1933		2791 3257 3725	3258	466 468	2111	
3	2041 2512 2980	2511	471 468	2872		5628 6124 6625	6126	496 501	3108	
4	2750 3282 3817	3283	532 535	3939		4003 4553 5103	4553	550 550	4208	
5	4404 4985 5571	4987	581 586	5106		6608 7215 7819	7214	607 604	5419	
6	2175 2782 3388	2782	607 606	6319		7146 7558 7969	7558	412 411	6242	
	51 255 17 085	17 086				98 128 32 709	32 710			(i) 15.624
					to	B.M. No 33			El.	245.433

(a) Rod readings of each of the 3 wires, on B.S.'s.—(b) Average of (a)—(c) Intercepts betw. middle wire and each of the others.—(d) Sums of (c) from top of page.—
(e) (f) (g)(h) same as (a) (b) (c) (d) but for F.S.'s.—Sum of (a) ÷ 3 = sum of (b);—Same for sums of (e) and (f).
(i) Differ. of Elev. bet. B.M's (or of the page). It is = 17 086 (sum of (b)) − 32 710 (sum of (f)).

Determination of a Grade Line.—If the level line was that of the center line of a road, the grades are first determined in the office after the profile has been plotted by the draughtsman.

Maximum Grade.—A ruling or maximum grade is generally agreed upon which varies according to the purposes of the road. It may be 1, 2, even 3 per cent. (in exceptional cases and for short distances) for a railroad; 8 per cent. for a highway on which an electric railroad is contemplated; 10 or 12 per cent. for a country road, etc.

Grade Determined by Means of a String.—Stretch a thread over the profile with a slope not exceeding the ruling grade and cutting of the ground projections approximately as much as is left of hollows. Fix the principal points, such as the changes of grade, on the paper, and draw a pencil line from point to point.

Vertical Curves.—May be drawn on the profile the better to merge the grades into one another.

This is always done in railroad work.

When a grade line is thus finally determined upon, calculate the grade at every station and at every **plus** point.

Laying Out a Grade Line.—Drive a reference stake near each station marking on it the cut or fill as calculated in the office.

Cut.—When the elevation of the ground is higher than the grade (as established and calculated in the office), subtract the grade from the elevation for the depth of cut which you mark on the reference stake prefixing the sign + to it. Example: **At Sta. 18 Elev.=164.92; grade = 159.84; 164.92 — 159.84 = 5.08.** Write on stake: **18+00+5.08.**

Fill.—When the **Elev.** is lower than the **Gr.**, subtract the elevation from the **Gr.** for height of fill and mark it on the reference stake prefixing the sign — to it. Example: **At Sta. 19, E = 152.68; G = 158.39; 152.68 — 158.39 = — (158.39 — 152.68) = — 5.71.** Write on stake **19+00—5.71.**

The first indicates a **cut** of **5.08 ft.** and the second a **fill** of **5.71 ft.**

Grade Stakes.—When the ground is pretty regular and when the cuts and fills are very small, such as in street grading, stakes are sometimes driven with their tops to grade. To grade a line between two points, drive a stake at each point, the top of which shall be at the required grade. Set the level on one of them and measure the distance between the cross-hairs and the top of the stakes; set the target at that figure on the rod and order it set over the second point. Sight the target with the telescope in an inclined position. At any intermediate station have a stake driven with its top at such a height that the line of sight shall meet the target as set, when the rod stands on it.

Contouring.—In order to determine on the ground points of a given elevation, as for instance the **flow-line** of a reservoir, set a stake the top of which shall be at the required elevation and have the rod stand on it. Level up and motion the target up or down till bisected by the cross-hairs and let it be made fast at that point. The rodman now moves a certain distance away up or down hill as directed until the target is bisected by the cross-hairs of the telescope. A stake is then driven flush with the ground. A succession of points is thus determined as far as the maximum limit of sights allows. The level is then moved to a new position about the same distance further. There it is again leveled up and sighted (as for a back sight) on the rod still held at the last point of the ground and the target of which is set anew to a bisection with the cross-hairs, and a new series of points is determined. The position of the stakes thus set will be surveyed later on by another party who will reference them to a transit line.

Cross-Sections.—Are lines of levels run at right angles to the transit line. These may be run as the main line with rod and level.

Cross-sections are often run with **optical square** and **cross-section rods.**

Optical Square.—A box with 4 apertures giving lines of sights at right angles. One aperture is a slit and the opposite a rectangular opening with a vertical cross-hair. At the center of the instrument is a plate of glass the lower part of which is silvered and serves as a

mirror; it is set at 45° with one of the lines of sights so that when the instrument is set over a point and in the direction of the line, the rodman or topographer may be directed along a perpendicular to the line which is the case when his figure or rod as seen in the lower portion of the mirror is bisected by the cross-hair seen in the upper portion.

Cross-Section Rods.—A more general way is the use of cross-section rods. One of them is to be held horizontal and is provided with a bubble level at each end for that purpose, it is 10 or 12 ft. long. It is graduated to 0.01 ft. The other is a graduated rod to be held vertical. They are used in two ways: 1°. For determining the distance from the line to points of certain elevations as every 2 or 5 ft. vertical. 2°. For determining the elevation of the ground at every rod's length, the last is more general because quicker in the field. Here are forms of cross-section notes:

Cross Section Notes.

23+50 A4 - 90°L		
0		E. 116.82
6	+.82	116
13		115
22.5		114
30.8		113
38.2	+.36	112.64
		bottom gully

9+13.5 FL8 - 90°R.F.S.		
0		E.212.91
10	-2.3	210.6
20	-1.4	209.2
30	-0.7	208.5
40	-2.0	206.5
50	-2.6	203.9
52	-0.8 Stream	203.1
64	+4.3	207.4

STA. 84.

Top			L		R			
$\frac{189.}{36}$	$\frac{186.2}{30}$	$\frac{184.5}{20}$	$\frac{181.}{10}$	$\frac{182.5}{0}$	$\frac{182.}{10}$	$\frac{179.1}{20}$	$\frac{176.}{30}$	$\frac{174.3}{40}$
								Brook
					$\frac{174.}{50}$	$\frac{175.4}{60}$	$\frac{173.4}{70}$	$\frac{171.2}{77}$

Slope Staking.—When a piece of work, such as a road, has been determined upon, the builder or contractor must be guided in order not to excavate or fill more than necessary and enough for the purpose intended. Limits must be set to his work.

The center stakes tell him how deep the excavation (**5.85)** or how high the fill**(—3.21)** is at these points and that is all.

The points of the ground on either side of the center line where excavation is to begin or embankment cease are to be shown by **slope stakes.**

Distance from Center Line of Slope Stakes in Excavation.—That distance is given by the formulas:

$\frac{S(hs+d)}{S-s}$ for the up-hill stake and $\frac{S(hs+d)}{S+s}$ for the down-hill stake.

Height of Stakes Above Grade—in Excavation.—The height above grade is given by formulas:

$\frac{hS+d}{S-s}$ at the up-hill stake and $\frac{hS-d}{S+s}$ at the down-hill stake.

Distance from Center Line of Slope Stakes in Embankment.—That distance is given by formulas.

$\frac{S(hs+d)}{S+s}$ for the up-hill stake and $\frac{S(hs+d)}{S-s}$ for the down-hill stake.

Height of Slope Stakes Above Grade—in Embankment.—The height above is given by formulas.

$\frac{hS-d}{S+s}$ at the up-hill stake and $\frac{hS+d}{S-s}$ at the down-hill stake.

In these formulas h = cut or fill at center stake.

d = half the width of roadway.

S = slope of the ground

s = slope of the excavation or embankment.

These values of S and s are the ratio $\frac{\text{horizontal}}{\text{vertical}}$; as for instance: $S=\frac{25}{6}$; $s=\frac{2}{1}$

Indirect Leveling.—Determination of heights by the measurement of vertical angles and horizontal distances.

Instruments Used to Measure Vertical Angles.—Besides the **transit** and **theodolite,** the principal are the **clinometers** of different models

The heights are deduced from the observed data by trigonometrical computations. When the sights are long, a correction for curvature must be added to the calculated height.

Barometric Leveling.—Two stations are selected where observations should be made simultaneously and the following data obtained: height of barometric column **B** and **b**; temperature of mercury **M** and **m** as given by attached thermometer; temperature of atmosphere **T** and **t** as given by detached thermometer. The capital letters represent observation at the lower station.

Formula for Corrected Barometer at Upper Station, Due to Mercury Temperature.—

$$b' = b + .00009(M-m)b = b[1 + .00009(M-m)]$$

Formula for Approximate Difference of Elevation.—

$$d = 60159 \; (\log B - \log b').$$

Corrected Difference of Elevation Due to Difference of Atmospheric Temperature.—

$$d' = 60159 \; (\log B - \log b')(1 + \frac{T+t-64}{900})$$

Corrected Difference of Elevation Due to Latitude L.—

$$d'' = 60159 \; (\log B - \log b')(1 + \frac{T+t-64}{900})(1 + .00265 \cos 2L).$$

Estimate Work.—At stated periods, generally monthly, the leveler runs lines of levels

1°. On a principal line of the work;

2°. On cross sections at right angles to it.

The points to be leveled have been previously surveyed by the transitman. The leveler copies his notes and has the rod placed on the points

marked by a *cross* X or by an *arrow-head* (or *crow-foot*) ←

He thus gets data to calculate the progress or quantity of earth and rock excavation; concrete in foundation pits or footings; block masonry in lower and (afterward) buried courses; coursed rubble, etc., and the progress of refill and embankment, etc.

TOPOGRAPHER.

Topographer.—He who takes notes of the topography of the country, on each side of the line of a survey.

The topographer is under the orders of the assistant engineer.

What He Carries.—A drawing board, sheets of cross-section paper, small protractor, drawing scale, compasses, pencil and eraser, pocket compass, hand level and note book.

Composition of a Topographical Party.—Topographer, rodman and two chainmen.

Scope of the Work.—The topographical party sometimes does the cross-sectioning. They locate, within 400 ft. more or less on either side of the line and as accurately as consistent with speed and importance of information: property lines, buildings, fences, highways, streams, springs, limits of woods, rock outcrops, and contours generally 5 ft. apart.

Property Lines.—Portions of properties to be condemned are to be carefully surveyed; this is done by a transit party. Other property lines are measured by the topographical party with pocket compass and chain, or by intersections or simply with the chain. They are plotted as fast as surveyed.

Buildings.—Are located by intersections of two adjacent faces with lines of survey, distances to these lines and dimensions of sides. Or by right angle offsets to two corners, distances of same and size of sides. Or by such lines ran to a cross-section line or to an angular line. Or by offsets to a located property line.

The character and purpose of each building should be noted as: 2½ story brick (or frame) Dwelling with Name of owner; barn, stable, shed, M. E. church, Dist. School No. 3, P. O., etc.

Fences.—Locate the corners, changes in direction; specify the kinds as **S. W.** (stone wall), **R. F.** (rail fence), **W. F.** (wire fence), **P. F.** (picket fence), **w. F.** (worm fence), **H.** (hedge), etc.

Conventional lines may be adopted to represent the different classes of fences, as indicated in **"Drawing,"** but these abbreviations seem necessary in cases where a property line, or a civil division line (as a Town or a County line) are shown by fences on the ground and are represented by special lines on the map.

Roads and Highways.—Show their directions, intersections, levels on roadway and sides, top of banks or foot of slopes. Indicate their nature: lane, county or State road; their geographical direction: "from **Albion** to **Mertel,**" or "to **Albion**" at one end and arrow showing direction and "to **Mertel**" at the other end and another arrow.

Locate bridges carefully, size of culvert or arch, distance to water or to roadbed (if a road crossing).

Streams.—Locate their principal bends, note their size and depth at various places, stages and marks of high water with dates, velocity and direction of current, their names as known in the locality; also quality of water as available for water supply, etc.

Springs.—Locate them precisely; note if they are protected or confined by a construction; also their flow as affecting water supply.

Limits of Woods, Cultivation.—Sketch the wooded areas after locating some points in their boundaries; note the principal species of trees. Note also the various crops grown in the plots. Locate the outlines of marshes and swamps; indicate sand pits and sand hills, clay beds, etc. These notes may be useful for purposes of adjustment of interests as well as for locating works.

Rock Outcrops.—Show outcrops of rock wherever found. These notes may be valuable in the selection of a location for a structure, as a bridge, a dam, etc.

Contours.—Are determined as explained in leveling, elevations may be taken and plotted by means of the plane table.

Plane Table.—A table which can be placed horizontal on a tripod by means of foot or plate screws and plate levels. It may revolve in a circle.

Alidade.—Small telescope mounted on a standard the foot of which is a long ruler. The telescope may move along a graduated vertical arc and has objective, eye piece and stadia cross-hairs.

Skeleton Drawing.—A sheet of strong drawing paper on which the line as laid by the transit party has been previously carefully drawn in the office is fixed to the table by rollers, springs or thumb-tacks.

Orienting the Table.—Consists in placing the instrument over a transit point or a known station on the line in such a manner that the corresponding point or station on the paper shall be on the same vertical and that another point of the line on the paper shall be in the same direction as the same point on the ground.

Bent Plummet.—In connection with the plate table a bent plummet is used which consists in two bars forming an angle, generally one side and the hypothenuse of a right angled triangle. The free end of the hypothenuse carries a bob or plummet so that when the ruler forming the side of the triangle in level, the direction of the other (missing) side is vertical or in a line with the bob.

Centering the Table for a Point.—Level it up over the station. Set the side of the bent plummet on the table with its free end touch-

ing the point; the bob will direct what displacement the table should be given in order to have the stake under the point. Try again to make sure.

Setting the Table for a Line.—After centering the table for a point place the edge of the alidade along the station on the drawing where the instrument is and the other station which is to determine the alignment. Revolve the table until this other station may be seen at the intersection of the cross-hairs of the telescope. This setting may disturb the first centering which must be checked. It is therefore important, when making the centering, to keep the table as near as possible with a line on the paper in the direction of the line on the ground.

How to Use the Plane Table.—When oriented, place the edge of the alidade on the station and take a sight at any point you wish to plot, chain the distance, draw a line along the edge making it to a scale, equal to that chained. As many points as are desired may thus be plotted and the topography filled in.

Stadia Work With Plane Table.—Elevations may be taken of ruling points, within a certain radius, from the several angles of a traverse as table stations. Two or three rodmen may thus be kept busy. Using a telemeter rod, the difference of readings, or intercept, between the stadia cross-hairs of the telescope gives the distance; the elevation of the station plus the height of instrument and less the reading of the middle cross-hair gives elevation of the point. For additional notes on stadia work, see "TRANSITMAN." ....

Office Work.—Plots his notes on skeleton map drawn by the transitman.

Plots profiles and cross-sections.

Calculates excavation and embankment.

Calculates cu. yds. of masonry, etc.

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THE ASSISTANT ENGINEER

BY

PROF. JEAN P. GENTHON
Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK V

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THE TRANSITMAN

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PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with the most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3° The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examination in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON.

July 1, 1907. New York.

PRESS OF THE CHIEF NEW YORK

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1°. Aptitude for mathematics.

2°. Habit of observation.

3° Good memory.

4°. System.

5°. Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION.—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials or new appliances and processes are used for old or new purposes. Notice the several kinds of labor, and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory, which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results; principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books, always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figures and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations, in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around you will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public works have a Chief Engineer who prepares the work and directs its

construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

Chief Engineer.			Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist
	Division Engineers.		Inspectors.	
		Assistant Engineers.	Transitmen.	Chainmen. Axemen.
			Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen.

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.	Memphis, Tenn.	St. Louis, Mo.
Boston, Mass.	Milwaukee, Wis.	St. Paul, Minn.
Buffalo, N. Y.	Mobile, Ala.	San Francisco, Cal.
Charleston, S. C.	Nashville, Tenn.	Savannah, Ga.
Chattanooga, Tenn.	New London, Conn.	Seattle, Wash.
Chicago, Ill.	New Orleans, La.	Sioux City, Iowa.
Cincinnati, Ohio.	Newport, R. I.	Tampa, Fla.
Cleveland, Ohio.	New York, N. Y.	Vicksburg, Miss.
Detroit, Mich.	Norfolk, Va.	Washington, D. C.
Galveston, Tex.	Philadelphia, Pa.	Wheeling, W. Va.
Grand Rapids, Mich.	Pittsburg, Pa.	Wilmington, Del.
Jacksonville, Fla.	Portland, Me.	Wilmington, N. C.
Little Rock, Ark.	Portland, Oreg.	Yellowstone Park, Wyo.
Louisville, Ky.	Rock Island, Ill.	

where examinations may be taken except that of Assistant Engineer, which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above-named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken at

Albany.	Ithaca.	Ogdensburg.
Amsterdam.	Jamestown.	Olean.
Auburn.	Kingston.	Plattsburg.
Binghampton.	Lockport.	Poughkeepsie.
Buffalo.	Malone.	Rochester.
Elmira.	Newburg.	Utica.
Hornelsville.	New York.	Syracuse.
		Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewer construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman; topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps.

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service, appear regularly in "THE CHIEF."

BOOK V
THE TRANSITMAN

Transitman.—He who carries and uses the transit, runs the lines (preliminary or location), keeps the notes necessary for drawings and calculations, and directs the party in the absence of the assistant engineer.

Who his Superiors are.—The transitman is under the orders of the **assistant engineer** or of the **chief of party** if there be one.

What he carries.—Transit, magnifying glass, adjusting pin, pocket tape, transit book, pad (for calculations), and list of party.

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service—Panama.

Title—Instrument man; also transit man.

Age Limits—Instrument man, 25 to 50 years old at examination.

Transitman, 21 to 50 years old at examination.

Salary—Instrument man, $175 per month.
Transit man, $150 " "

Examination.

Subjects.	Weights.
1. Pure and applied mathematics (elementary problems in mensuration, solution of plane triangles, and theoretical and applied mechanics, involving a fair knowledge of pure mathematics to and including calculus)	15
2. Construction and care of instruments (comprising transit, including stadia work, level, plane table, rods, chain, tape, current meters, etc.)	20
3. Theory and practice of surveying (comprising surveying, leveling, and other field work required in engineering and not covered in subjects 1 and 2)	20
4. Design and construction (involving elementary knowledge of designing and constructing highways, railroads, dams, retaining walls, foundation work, trusses, etc.)	10

5. Training and experience	35
Total	100

Five years practical experience. Two years for graduate in engineering.

After one year's satisfactory service, the transitman is eligible for promotion as assistant engineer without further examination.

N. Y. State and County Civil Service.

The manual is silent about this position.

New York City Municipal Civil Service.

Transitman or Computer—Schedule **B.** Grade 3.

Age Limit—Not less than 18 years of age.

Salary—$1,350 to $1,950.

Scope of Examination.

Subjects.	Relative Weights.
1. Handwriting (as shown in examination papers)	1
2. Arithmetic	1
3. Technical knowledge	6
4. Experience	2
Total of weights	10

QUESTIONS GIVEN AT CIVIL SERVICE EXAMINATIONS.

Mathematics.

1. The inscribed circle in a regular octagon has a radius of 23 ft. What is the area of the figure?

2. Extract by logarithms the cube root of .0075218.

3. Multiply by logarithms the cube root of .0075218 by the square root of 176.923.

4. The side of a right angled triangle is 27 and the base is 13.

(a) What is the natural sine of the angle opposite the base?

(b) What is the natural tangent of the angle opposite the side?

Note: Do this by arithmetic.

5. The radius of a circle is 50; what is the length of an arc of 28 degrees? (Do this by arithmetic).

6. Solve $\sqrt{\dfrac{6\frac{2}{7} - 4\frac{5}{9} + 8\frac{2}{10} + 678.^{792}}{4\frac{4}{12}}}$.

7. Value of x and y in $2x+3y=33$
$4x-y=17.$

8. Find x in $x2-x-40=110$.

9. Find x in $\dfrac{a}{x} + \dfrac{\sqrt{a^2+x^2}}{x^2} = \dfrac{x}{6}$.

10. Explain $a^{\frac{3}{4}} \times b^{\frac{1}{2}}$.

11. Area of a circle.

12. Divide .0000459 by 346.75 by logs.

13. Solve $\dfrac{x}{x+1} - \dfrac{x+3}{2(x+4)} = -\dfrac{1}{18}$.

14. Numerical values were given for the bases and slant side of a right angled trapezium—find its area.

15. Wall of masonry 12' 4" thick at bottom, 8' on top, height 10' 8", length 104' 8"—Find the cubic contents.

16. Compute the number of square yards of pavement on a curve of which the radius of the center line is 350 feet, the angle at the center is 47 degrees, and the width of the pavement 40 feet.

17. Compute the natural sine of an angle in a right-angled triangle of which the hypothenuse is 107 feet, and the side "opposite" the angle is 83 feet.

18. Compute the natural tangent of the "adjacent" angle in the same triangle.

19. Find the contents of the frustrum of a cone of which the radii of the bases are respectively 7 feet and 4 feet 7 inches and the height 8 feet 5 inches. The prismoidal formula may be used, if it is so desired.

20. The side of a hexagon circumscribing a circle is 12 feet. What is the area of the circle?

Technical.

1. State the duties a transitman may be called upon to perform in the City of New York.

2. Describe a party organized for transit work and the duties of each person.

3. When the best and most accurate work with a transit is required, describe every precaution requisite on the part of the transitman in adjusting, handling, protecting and sighting his instrument.

4. How are errors in reading the instrument and in graduation of the limb guarded against or minimized?

5. (a) What causes are there for error in long sights? (b) Can these be guarded against in any way, and if so, how?

6. Describe the operation of making an accurate survey of a city block, both exterior and interior, where every side is filled with buildings.

7. Give a sample of notes, as you would record them in your note book, of a survey of a new street half a mile long, having several angles, and crossing other streets and a stream.

8. Describe the adjustment, for leveling purposes of the long bubble which is attached to some transits.

9. Name all the other adjustments of a transit in the order in which they should be made.

10. Suppose you had to measure the distance between two objects on the opposite side of a stream without crossing it. (a) Describe the instrumental work necessary. (b) Make a sketch and describe the method of making the computation of the distance.

11 & 12. A trapezoidal piece of ground has one end at right angles to the parallel sides. The sides measure respectively 110 ft. and 175 ft. and the inclined side has a length of 97 ft. Required first, to compute the perpendicular distance between the sides; and second to cut off ⅓ of the area from the widest portion by a line parallel to the sides, computing its perpendicular distance from the widest side (show all figures.)

13. What is the limit of error allowed in chaining in measurements on city streets?

14. How can you compute a very small angle without the use of tables having given the distance run and the side opposite the angle required?

15. An area of ground is to be used as a borrow pit. It has been laid out by lines at right angles to each other at suitable distances and the cuts at each intersection determined. How would you determine the cubical contents?

16. What is refraction and how does it affect the sight?

17. Does it make any difference what time of day should be chosen for accurate transit work? If so why.

18. After setting up transit and before giving line, what should the transitman be sure of?

19. What errors may arise in handling transit?

20. Do two persons always read vernier alike? If not why is the difference?

21. How can you avoid error due to faulty graduation?

22. In running a random line it was found that the two end points do not coincide. What easy method of finding angle by proportion.

23. Find area after making necessary corrections:

20 chains N 15° 15' E; 10 ch. N 37° 30' E; 7.6 ch. E; 12.5 ch. S 11° E; 13.5 ch. S; 10 ch. W; 9.9 ch. S 36° 30' W; 8.5 ch. N 36° 15' W.

24. Give difference between Traverse Table and Table of Natural Sines and Cosines.

25. What way of finding error in a survey.

26. Given the sides of a quadrilateral as 75; 100; 110 and 130 feet in rotation, and the diagonal from the first to the last points as 105 feet, find the area.

27. The length of a chord of a R. R. curve is 478 feet; the angle of the curve is 71° 06', find the radius.

28: Solve $\frac{90}{x} - \frac{90}{x+1} - \frac{.27}{x+2} = 0$

29. N. Sine=0.4694716
N. Cosine=0.8829476
Find Natural Tangent.

30. What are the duties of a Transitman?

31. Describe sextant.

32. How is a party on Railway or Aqueduct made up?

33. Give duties of each man.

34. Give adjustments of Transit and state the true order.

35. Describe Transit.

36. When is the magnetic needle used?

37. Describe transit vernier and how would you read angles to minutes—to ½ minutes.

38. How would you find the area of a parcel of land from transit notes of its boundaries?

39. What knowledge of mathematics is required by transitman?

40. How would you produce a straight line when the instrument is out of adjustment?

41. What precautions are to be taken in running a long straight line.

42. Define and illustrate sine, cosine, tangent and cotangent.

43. Give Transit notes for a 2° curve 10 sta. long—3 st. up—at 2+63; 3+91 and 6+42.

44. How would you pass an obstacle in running a line.

45. Vertex inaccessible how would you find angle of intersection and position of **P. C.** and **P. T.**

46. Two sides of a triangle are 40.61 ft. and 39.80 ft. and the angle opposite the first side is 12°32'. Find remaining parts of triangle and its area.

47. The sides of a triangle are 32.61 ft.; 42.31 ft. and 98.75 ft.; find the angles.

48. One side of a right angled triangle is 46.38 ft. and the adjacent angle is 18° 36'. Find the remaining parts.

49 Meeting a river 800' broad how would your leveler project line across without having recourse to surface of water?

50. (a) Describe the adjustment of the Level on the horizontal plate of the Transit.

(b) How will it affect the angles if out of adjustment?

51. (a) Explain how the line of collimation is to be adjusted; (b) will all angles be affected if the line of collimation is not in adadjustment?

52. If the horizontal axis of the telescope is not at **R.** angle to the vertical axis of the instrument how will this affect the angles? Describe its adjustment.

53. If a Transit has a vertical limb, does it need to be adjusted; if so how may this be done?

54. What causes of error are there in reading the vernier.

55. What other causes of error besides those already mentioned are there in Transit work?

56. Show the field notes of a large farm crossed by a R. R. and a stream, having at least 12 angles and showing the location of farm buildings, etc.

57. How would you arrange your party for such a survey?

58. Explain the Tables used for balancing such a survey—describe fully.

59. If there is any large error in one of the courses of the survey, do you know of any way to find this error without doing the whole work over again?

60. What do you think would be the error per angle in a number of angles observed with a good Transit of ordinary description?

61. Suppose there are obstructions on one line of your survey requiring the angle to be turned to a small offset; by what simple proportion can you tell the angle without the use of tables?

62. You are running along the center line of a street which deflects **x°** to the right — the width of street is **b** find an algebraical

expression for the distance from the angle point on the center line to either angle point on the side line.

63. A line **ab**, parallel to the bases, divides a trapezoid into 2 equal parts—Find the distances, along the slant lines from the lower base at which line **ab** intersects the slant lines. The bases are 275 ft. and 470 ft.; and the slant sides 300 ft. and 350 ft.

64. One side of a regular hexagon=30' find its area.

65. Declination is 5° 10' West; required the true bearings of the following magnetic bearings: N. 7° 20' W.; N. 45° 00' E.; S. 15° 20' E.; S. 2° 30' W.

66. What is the ratio of the chord deflection to the tangent deflection?

67. The level rod held 300 ft. from instrument reads 6.81. After causing the bubble to move over one division of the scale, the reading is 6.84. What is the radius of bubble tube?

68. What is meant by **Power** in Definition of a Telescope.

69. What basis of tides is used for soundings?

70. What condition of rainfall will most severely tax the storm-water sewer capacity?

71. What general form of equation will express the rate of rainfall of a storm of any duration?

72. What is the relation between rate of rainfall per hour and the number of cu. ft. per second falling upon the acre?

73. What rates between the rainfall and storm water is most important to ascertain in designing storm-water sewers

74. What rate of rainfall will give the maximum storm-water flow at any point in a sewer?

75, What is the very first adjustment of a transit that should always be made, and how is it done?

76. (a) What planes should a transit instrument describe or revolve in truly, if in perfect adjustment? (b) In case it describes a cone when revolved on its horizontal axis, what adjustment must be made and how is it done?

77. If the plane described when revolved vertically is inclined to the vertical, what adjustment must be made and how is it done?.

78. Describe fully the making of a topographical survey with a transit, and a stadia rod.

79. If an error of 1 minute be made in reading an angle, what would be the effect from the true line at a dist. of 3,000 ft? Show all your figures. No tables required.

80. In running a line with a transit instrument by foresights and backsights with several set-ups, would you or would you not get a

straight line if you kept foresights and backsights equal; in other words, would instrumental errors be compensated by so doing?

81. Describe 2 ways of prolonging a straight line accurately.

82. Describe all the ways you know of repeating the readings of an angle for the purpose of accuracy and state the objections, if any, to each; or else, the reason why it will be the most accurate method.

83. Describe clearly the method of measuring a base line for an accurate triangulation in the surveys of the city.

84. (a) In what ways are accurate sights more difficult in doing city work than in work in the open country?

(b) State how you overcome these difficulties in each case.

85. State how (that is, describe the operation by which) you would carry a line of survey through a city block covered with buildings and locate and measure all lot intersections as well as external lot measurements; in other words, make a complete and accurate map of the block.

86. (a) Suppose you have the courses and lengths of all the sides of a piece of ground but one. Could you compute the area, and how would you do it?

(b) In so doing, what assumption would be made?

87. Suppose you wish to determine the area of a piece of ground with a number of sides, which is entirely unobstructed and you have only a chain or tape; how can you do it? Make a sketch.

88. Why is a very small angle undesirable in trigonometric measurements?

89. State a simple arithmetical rule for determining the angle subtended by a very small offset from a transit line at any given distance; state also the explanation of the rule.

90. (a) Name and describe two systems of co-ordinates used in geodetic work? Give an illustration of each.

(b) What is the result of adding the logarithms of numbers together? What of subtracting them?

(c) What does a logarithm represent? In other words upon what is a table of logarithms based?

(d) What numerical operations are simplified by the use of logarithms.

91. The co-ordinates of two points are respectively the 1st. South 237.831 feet and East 1307.005 feet; the second: North 53.762 feet and West 831.773 feet. Compute the bearing between the points to the nearest second and the distance to the nearest thousandths of a foot.

92. Having the following notes of a survey, determine each of the interior angles, and give a proof of their correctness:

A B North 19 Degrees East; B C South 77 degrees East; C D South 27 degrees East; D E South 52 degrees West; E F South 15½ degrees East; F G West; G H North 36 degrees West; H I—North; I A North 62 degrees West.

93. The following survey has been made, and the latitudes and departures computed as given:

Line A B course 274 ft.; South 0 degrees 11 minutes 10 seconds West South 274; West 0.89.

Line B C course 589.8 ft.; South 29 degrees 59 minutes 10 seconds West South 510.87; West 294.75.

Line C D course 374 ft.; South 12 degrees 21 minutes 40 seconds East South 365.33; East 80.06.

Line D F course 995.30 ft.; South 33 degrees 19 minutes 50 seconds East South 831.59; East 546.89.

Line E E course 549 ft.; South 82 degrees 13 minutes East; South 74.35; East 543.94.

Line F G course 627.60 ft.; South 51 degrees, 11 minutes 40 seconds East South 393.50; East 487.07.

Line G H course 574.50 ft.; North 65 degrees 8 minutes 10 seconds East North 213.95; East 533.18.

Line H A course 2934.90 ft.; North 40 degrees 19 minutes 00 seconds West North 2257.80; West 1898.91.

1st. Correct the latitudes and departures.

2d. From the figures so corrected assuming the course A B to remain unchanged, compute the corrected length of the courses BC CD; lastly compute the corrected bearings of the same. Note: Give results to the nearest second and the nearest hundredth of a foot.

94. The correct bearing of station D, as sighted from Station A is North 29 degrees 44 minutes 50 seconds. The points were joined by a traverse line of three courses as follows: A B North 29 degrees 40 minutes 30 seconds East 911.462 ft.; B C North 29 degrees 47 minutes 50 seconds East; 873.50 ft.; and North 29 degrees 46 minutes 55 seconds East, 640.707 ft.

Compute the corrections to be made, practically at right angles to B C, to place these points on the line. Also state in which direction (right or left) when facing D, the points are to be moved.

95. Correct the following field notes of a survey for temperature: Taking the standard at 62 degrees F and the co-efficient of ex-

pansion at .0000063—the sides are as follows: 256.435 ft. measured at 27 degrees, 698.842 ft. at 104 degrees; 569.874 ft. at 31 degrees; and 33894.756 ft. at 28 degrees.

96. Having an instrument which is in perfect adjustment, which reads to 20 seconds of arc, and knowing the distance to a certain point to be established to be 700 ft., how would you proceed to lay off an angle at 79 degrees 00 minutes 15 seconds accurately to the nearest second, and fix the point?

97. The accepted elevations of two B. M's are A equals 100 ft., B equals 120.75 ft., or the true difference is 20.75 ft. By a subsequent set of levels, the following results were obtained: B at 2500' distance was 17.31 ft. above A; C at 1600' ft. from B was 15.65 below B; and D 900 ft. further was 18.93 above B. Adjust the elevations at B and C.

98. One side of a regular hexagon has a length of 38½ ft. Find the difference in area between the hexagon and the inscribed circle.

99. A piece of land has six sides. The angles A, F, C and D are each 90 degrees. The re-entrant angle B is 194 degrees 34 minutes 27 seconds. The side F A is 60 ft., A B is 200 ft., B C is 300 ft., and C D is 50 ft. To find D E and E F

SCIENTIFIC REQUIREMENTS.

What precedes (See Book I. The Axeman; Book II. The Chainman; Book III. The Rodman; Book IV. The Leveler).

Also Conic Sections, Mechanism, Strength of Materials, Slide Rule and its uses, Surveying.

CONIC SECTIONS.

So called because they may be obtained by cutting a cone with a plane. (Fig. 213).

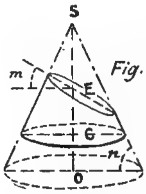


Fig. 213

Circle.—Conic section of a right cone (also of a right cylinder) when the secant plane **C** is perpendicular to the axis.

Ellipse.—When the secant plane **E** is oblique to the axis of the cone and cuts all the elements on the same side of the vertex **S** (It is also the oblique section of a cylinder).

Hyperbola.—(Fig. 214). When the secant plane **H** cuts all the ele-

ments of the cone (or their prolongations beyond the vertex S) but on both sides of the vertex **S.**

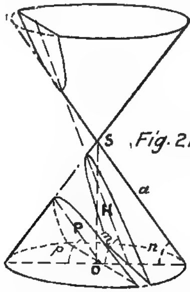


Fig. 214

Parabola.—When the secant plane **P** is parallel to one element **a** of the cone.

Other Definitions—Ellipse (Fig. 215.)—Plane closed curve the sum of the distance **m** and **n** from any point of it to two given points **F-F'** called **focii** situated in its plane is constant **2a** (it equals the major axis) **m+n=2 a.**

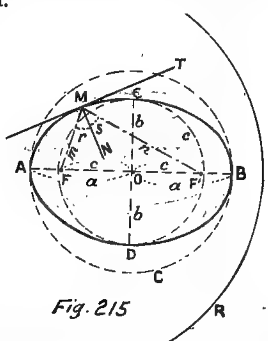


Fig. 215

Focal Distance.—The focal distance **F F'** is generally given and is represented by **2 c.**

Major Axis.—The line connecting the focii is the **major axis=2a.**

Minor Axis.—The perpendicular bisecting the major axis is an other axis called **minor axis=2b.**

Center.—The point of intersection of the axes is the **center o** of the curve.

Radii Vectors.—The lines connecting any point of the curve to the focii are called the **radii vectors m** and **n** of that point.

Tangent.—The **tangent** to the ellipse at a point makes equal angles with the radii vectors of that point **P=q.**

Normal.—The normal (perpendicular to the tangent at the point of contact) to an ellipse at a point bisects the angle of the radii vectors **r=s.**

Major Circle—The **major circle** is that described on the major axis as a diameter **R=a.**

Minor Circle.—**The minor circle** is that described on the minor axis as a diameter **r=b.**

Circles Directors.—**Circles directors** are two circles described from either focus as a center and the major axis as a radius **R=2a.**

Ordinates.—Are perpendiculars to the major axis and they measure the distance from any point **P** in the curve to said major axis. They are generally represented by **y.**

Abscissas.—Perpendiculars to the minor axis; they measure the distance from any point **P** in the curve to said minor axis.—They are often considered simply as the distance between the center **O** and the foot of the ordinates **y.** They are generally represented by **x.**

Co-ordinates.—Ordinates and their corresponding abscissas are called **co-ordinates.**

Drawing of an Ellipse.—Given the Focii and major Axis 2a.

(a) By continuous Motion (Fig. 216). Take a flexible line as a cord, a thread, a light wire, etc., of a free length of **2a** (exclusive of fastenings). Fasten the ends of it to the focii and run a pen, pencil cr marking instrument along the line, keeping it taut.

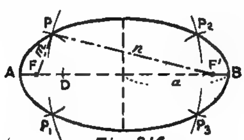


Fig. 216

(b) By Points.—Draw a line through the focii **F—F'**; bisect it by a perpend. and lay off on the major axis the length **a** on either side of the perpendicular, let **A** and **B** be the points so obtained. Mark any point on **AB** such as **D.** With **AD** describe two arcs from each focus as a center; cut these arcs with other arcs described from the opposite focus as a center with a radius equal **DB;** this will give 4 points of the curve **P-P1-P2-P3.** The selection of more other points on the major axis and a like operation will give as many times 4 new points of the curve.

Given the two Axes.—1st Method (Fig. 217).—From one end **C** of the minor axis **CD** as a center and with the semi major axis **OA** as a radius **r=a,** cut the major axis in two points **F** and **F'** which are the focii of the curve and the case reverts to the preceding.

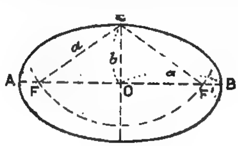


Fig. 217

2nd Method (Fig. 218).—Draw the major and minor circles on the given axes. Draw any number of radii **OR** to the major circle. From the end of each radius on the major circle draw a perpend. **RP** to the major axis (or a parallel to the minor); cut these ordinates by parallels **QP** to the major axis drawn from the points **Q** where these radii cut the minor circle. The intersections are points of the required ellipse.

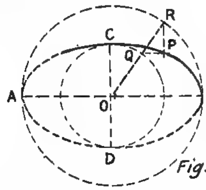


Fig. 218

3rd Method (Fig. 219)—From one focus **F** as a center draw the circle director with **R=2a.** Draw any radius **F C** of that circle director and connect its extremity **C** with the other focus **F'**. Bisect **CF'** with a perpend. which will be tangent to the ellipse at the point **M** where it intersects the radius **FC.** This pt. **M** is a pt. of the curve and any number of other pts. may be obtained in like manner.

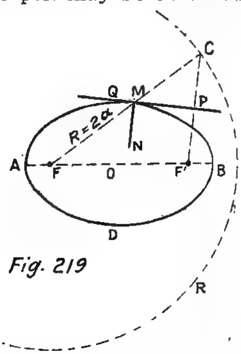


Fig. 219

The advantage of this method is that it gives at the same time a pt. and a tangent.

With a Slip of Paper. (Fig. 220.)—Lay off the semi-axes **a** and **b** from a point **A** on the straight edge of a slip of paper to **B** and **C** in the same direction so that **AB=a AC=b.** Apply this slip to the drawn axes of the ellipse to be determined holding it in such a man-

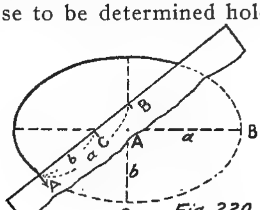


Fig. 220

ner that the end B of the semi-major axis shall be on the minor axis and the end C of the minor axis shall be on the major axis. Mark points at A.

Tangent to the Ellipse (Fig. 221).—1° **Through a pt. on the curve.**—Bisect the angle of the radii vectors; it will be a normal. Erect at the point a perpend. upon that normal, it will be the required tangent.

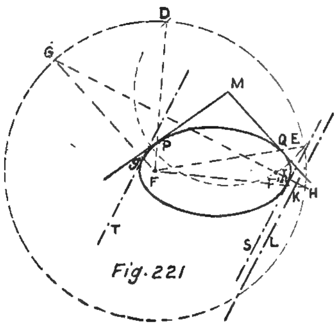


Fig. 221

2°. Through a Point without.—From focus **F** draw a circle director **R=2a.** From given pt. **M** as a center with radius **r=MF'** draw a circle which will cut circle director in two points **D** and **E.** Join **DF, EF,** also **DF', EF'** which last two lines bisect. The lines will be tangents to the curve at the points where they cut **DF** and **EF** and they will pass through **M.**

3°. Parallel to a given Line.—Draw circle director from focus **F.** Through other focus **F'** draw a perpend. to given line and let **G** and **H** be the points where said perp. cuts circle director; join **GF** and **HF.** Bisect **GF'** and **HF'** these will be tangents to the curve at the points where they intersect **CF** and **DF.**

Area of Ellipse.—Product by π of the semi-major by semi-minor axis: **A=π a b.**

Equation of the Ellipse. *With regard to its two axes:*

$$a^2y^2 + b^2x^2 = a^2b^2 \quad \text{or} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 .$$

Volume of the Ellipsoid.

Prolate *(Elongated)-Revolving about the major axis* : $V = \frac{4}{3}\pi ab^2$

Oblate *(Flat)* " " " *minor* " : $V = \frac{4}{3}\pi a^2b$.

Hyperbola (Fig. 222)—A plane curve such that the difference of

the distances **m** and **n** from any point of it **M** to two given points **F** and **F'** called **focii** is constant **2a : m—n=2a.**

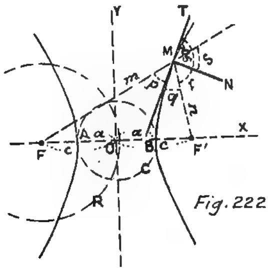


Fig. 222

Focal Distance.—The focal distance is generally given and is represented by **2c.**

Axes of Symmetry.—The line connecting the **focii** and the perpend. bisecting it are **axes of symmetry** and the first is the **transverse axis**; the portion between the curve is = **2a.**

Center.—The point of intersect. of the axes **O** is the **center** of the curve.

Radii Vectors.—The lines connecting any pt. **M** of the curve to the focii are the **radii vectors m** and **n** of that point.

Tangent.—The **tangent** to the hyperbola at a point **M** bisects the angle formed by the radii vectors **m-n** of the point of contact; **p=q.**

Normal.—The **normal MN** to a hyperbola bisects the exterior angle made by the radii vectors; **r=s.**

Principal Circle.—The **principal circle C** is that described from the center of the curve with **R=a.**

Circles Directors.—They are two circles **R** described from either focus **F** as a center and the transverse axis **2a** as a radius; **R=2a.**

Ordinates.—Perpendiculars to the major axis, as **y.**

Abscissas.—Dist. from the center of the curve to the feet of the ordinates, as **x.**.

Drawing of a Hyperbola.—Given the focii F and F' and transverse axis 2a—

By continuous Motion (Fig. 223).—Fix the extremity **A** of a ruler to one of the focii **F**; fix one end of a flexible line as a cord, etc. equal in length to that of the ruler less **2a** to the other extremity **B** of the ruler, and the other end of the line to the other focus **F'** and run a pen, pencil, etc. along the edge of the ruler and along the line keeping it taught. This will give one of the branches; the other

branch will be obtained in the same manner by operating on the second focus **F'** as was done on the first **F**.

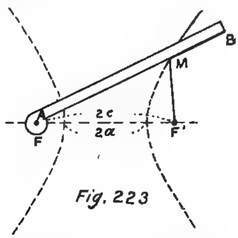


Fig. 223

By Points—First Method (Fig. 224).—Draw the transverse axis indefinite; bisect the focal dist. with a perpend.; lay off **a** from the center **O** and on either side on the transverse axis, **A** and **B** are the points thus obtained. Mark any point **D** on **AB** outside of **FF'**. With **AD** as a radius describe two arcs from each focus as a center; cut these arcs with two other arcs described from the opposite focus as a center with a radius equal **DB**; this will give 4 points like **M-M'** of the curve. The selection of more other points on the transverse axis and a like operation will give as many times 4 new points of the curve.

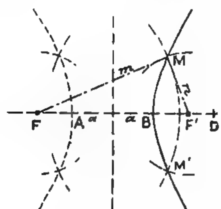


Fig. 224

Second Method (Fig. 225).—Draw from one focus **F** as center the circle director with **R=2a.** Draw any radius **FC** of that circle director and connect its extremity **C** with the other focus **F'**. Bisect **CF'** with a perpend., which will be tangt. to the hyperbola at the pt. **M** where it intersects the radius **FC** produced. This pt. **M** is a point of the curve and any number of other points may be obtained in like manner.

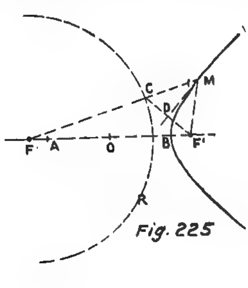
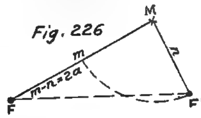


Fig. 225

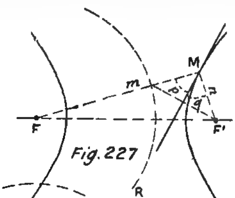
This method determines the curve better by giving at the same time a point and a tangent.

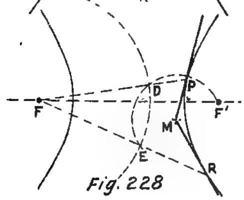
Given the focal Distance 2c and a Point M (Fig. 226).—Join **MF =m** and **MF'=n.** Get **m-n** which is the transverse axis **2a.** The case is reverted to the preceding one.



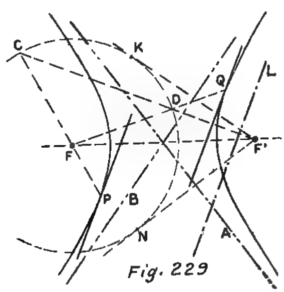
Tangent to the Hyperbola.—1° Through a point M on the curve (Fig. 227).—Bisect the angle **FMF'** made by the radii vectors, it will be the required tangent.

2° **Through a Point without** (Fig. 228).—From focus **F** draw a circle director **R=2a.** From given point **M** as a center with radius **r=MF'** draw a circle which will cut circle director in two points **D** and **E**; join **DF, EF,** also **DF', EF'**; bisect the last two. The lines will be tangent to the curve and they will pass through **M.**





3° **Parallel to a given Line** (Fig. 229).—Draw circle director from focus **F.** Through other focus **F'** draw a perp. **F'C** to given



line **L**; **C** and **D** are the points where said perp. cuts circle director. Join **CF** and **DF**. Bisect **CF'** and **DF'**; these will be the tangents to the curve at the points where they intersect **CF** and **DF** produced.

Asymptotes.—The perpendiculars **A** and **B**, bisecting the tangents **F'K, F'N** drawn from a focus **F'** to the circle director described from the other focus **F**, are extreme tangents to the curve and their points of contact are at an infinite dist. from the center **O.** Their angles are bisected by the axes. These extreme tangents are called **asymptotes.**

Equation of the Hyperbola

$$a^2y^2 - b^2x^2 = -a^2b^2 \quad \text{or} \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Volume of the Hyperboloid with two Nappes—Obtained by revolution about the minor axis.

$$V = \pi y \left(a^2 + \frac{y^2}{3}\right); \quad \text{or} \quad V = \pi y \left(\frac{2a^2 + x^2}{3}\right).$$

Volume of the Hyperboloid with one Nappe.—Obtained by revolution about the major axis.

$$V = \pi h \left(\frac{3y^2 + x^2}{6}\right)$$

Parabola (Fig. 230).—A plane curve all the points of which are equally distant from a point **F** called **focus** and a straight line **D** called **directrix.**

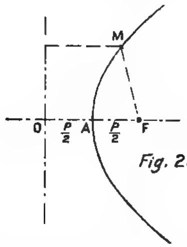


Fig. 230

Axis.—The perpend. **FO** from the focus **F** to the directrix is an **axis**

Parameter.—The dist. **FO** is called **parameter=p,** and its middle point **A** is the **apex** of the curve.

Radius Vector.—A line **MF,** drawn from a point **M** of the curve to the focus **F,** is the **radius vector** of that point.

Drawing of a Parabola—Given the directrix and focus.

(a) **By continuous motion** (Fig. 231). Place a ruler **R** along the directrix and a triangle **T** with its short side along it. Attach a flexible line at the corner **D** of the triangle—its length should equal **DC;** the other end of the line is attached to focus **F.** Move the triangle

along the ruler and, with a tracing point run the line along the side **DC** of the triangle keeping it taught.

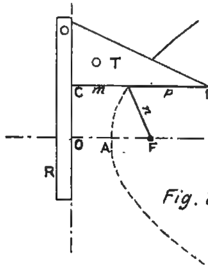


Fig. 231.

(b) **By Points** (Fig. 232).—Draw the axis **OF=p** and bisect it; the parameter **OA=AF=½p**. Draw any perp. (ordinate) to the axis, on the side of the focus and at a greater distance from **O** than **½p**. Let **D** be the foot of the perp. Take **OD** as a radius and describe with it arcs from **F** as a center. The points **M** and **M'**, where they intersect the ordinate, belong to the curve. Other points may be obtained in like manner for any number of ordinates.

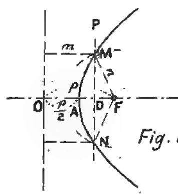


Fig. 232

Tangent to the Parabola. 1°. Through a pt. M on the Curve (Fig. 233).—Draw the radius-vector **MF** and the parallel to the axis **MD**. Bisect the angle **FMD** thus formed; the line will be the required tangent. Or join **M** to the middle point **B** of the line **FD**. The point **B** is always upon the perp. **AK** to the axis at the vertex **A** of the curve

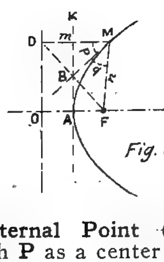


Fig. 233

2° **Through an external Point** (Fig. 234).—Join the point **P** to the focus **F** and with **P** as a center and **PF** as a radius, describe a circle that will cut the directrix in two points **B** and **C**. Bisect lines

BF and **CF** with perpendiculars which will be the required tangents and will pass through **P.**

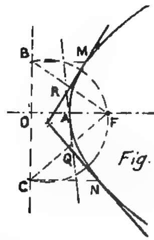


Fig. 234

3° **Parallel to a given Line** (Fig. 235).—Draw from focus **F** a perpend. **FB** to given line **L.**; it will cut directrix at a point **B**; bisect line **FB** with a perpend. which is the required tangent.

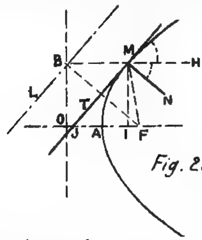


Fig. 235

Theorems.—The line bisecting two tangents issuing from a common point is also a tangent to the parabola.

The **sub-tangent** is bisected by the vertex of the curve.

The normal bisects the external angle formed by the radius vector and the parallel to the axis.

Area of a parabolic Segment.—1° A Segment limited by the curve, an abscissa and its ordinate:

$$A = \tfrac{2}{3}\,xy.$$

2° A Segment limited by the Curve and an ordinate:

$$A' = 2 \times \tfrac{2}{3}xy = 4xy \div 3.$$

Equation of the Parabola:

$$y^2 = 2px.$$

Volume of a Paraboloid.—Generated about the axis and limited by the curve and an ordinate. $V = \frac{1}{2}\pi\,xy^2$ which is half the cylinder having same base $R = y$ and same height $h = x$.

OTHER CURVES.

Cycloid. (Fig. 236).—Curve generated by a point **D** of a circle **O** rolling on a straight line **AB.** The straight line is a diam; its extremities **A-B** vertices of the curve and middle **C** the center of the

curve. One full revolution of the circle determines on the diameter a length equal to the circumference of the generating circle.

Rectification of a Circle.—(Specht.) (Fig. 237)—Draw a diameter

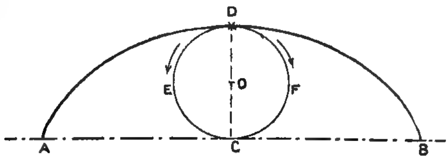


Fig. 236

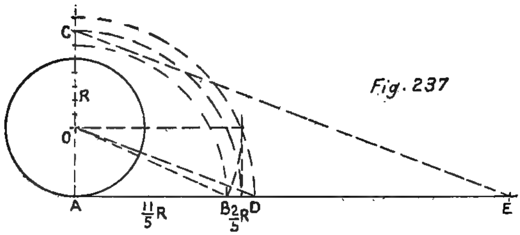


Fig. 237

and a tangent at one end **A** of it. On that tangent lay off a distance

$$AB = \frac{11R}{5} \quad \text{or} \quad AB = 2R + \frac{1}{5}R$$

connect **B** to the center **O** of the circle. On the diameter produced, lay off **AC=OB**; on the tangent take

$$AD = \frac{13R}{5} \quad \text{or} \quad AD = 2R + \frac{3}{5}R \text{ and connect } D \text{ to } O\,;$$

finally, draw **CE** parallel to **OD**; **AE** will be very nearly equal to the circumference.

In the figure thus formed, the similar triangles **AOD** and **ACE** give

$$\frac{AO}{AD} = \frac{AC}{AE} \quad \text{or} \quad AE = \frac{AD \times AC}{AO}\text{, in which } AO = R;\ AD = \frac{13R}{5} \text{ and } AC = \sqrt{\overline{OA}^2 + \overline{AB}^2};$$

$$AC = \sqrt{R^2 + \frac{125R^2}{25}} = \frac{R}{5}\sqrt{146}\,;$$

$$AE = \frac{\frac{13R}{5} \times \frac{R}{5}\sqrt{146}}{R} = \frac{13R}{25}\sqrt{146} = 2R \times \frac{6.5}{25}\sqrt{146} = 2R \times 3.14159193\ldots.$$

$$AE = 2\pi R \text{ very closely.}$$

Drawing of the Cycloid. (Fig. 238).—At one extremity **A** of a diameter **BA** of the generating circle **O** draw a tangent **MC** on which lay off on either side a distance **AC=AM=**π**R**; the total tangent **MC** will be the cirf. rectified. Divide the circle into a number of equal parts as 8, 16, 32....and draw the chords from the point of contact **A.** Divide the tangent into the same number of equal parts. Through the points of division of the circle draw parallels to the tangent. The extremity **B** of the diam. opposite the point of con-

tact **A** is the middle point of the curve. The other points are obtained by drawing, through each point of division of the tangent, a parallel to the corresponding chord in the circle. Where the chords intersect the parallels to the tangent are points of the cycloid. The extremities **M—C** of the tangent are also vertices, one at the origin and the other at the end of the curve.

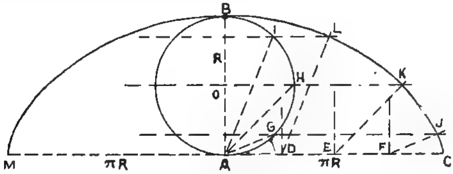


Fig. 238

Tangent to the Cycloïd. (Fig. 239).1°—**Through a point P on the curve**—Draw through **P** a parallel **PM** to the directrix **AC**; draw the cord **MA** from the intersection of this parallel with the circle to the pt. of contact **A** and from **P** draw a parallel **PD** to that cord; this will be a normal; finally draw at **P** a perpend. **T** to that normal, it will be the required tangent.

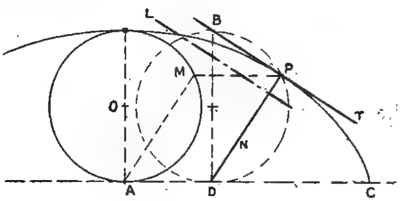


Fig. 239

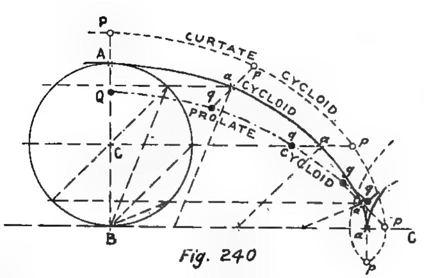


Fig. 240

2° **Parallel to a given Line.**—Through the center **A** of the curve draw a perpend. **AM** to the given line **L**; then a parallel **MP** to the directrix **AC** from the intersection **M** of that perpendicular **AM** with the circle **O**; the point **P**, where that parallel meets the curve is the point of contact of the required tangent **T**.

Properties.—Development of the Cycloid (Involute).—The involute of a cycloid is an equal cycloid (Huyghens).

Length of the Cycloid.—It equals 4 diameters of the generating circle: **$l=8R$.**

Area of the Cycloid.—Three times that of the generating circle. **$A=3\pi R^2$.**

Curtate Cycloid (Fig. 240).—Curve generated by a point beyond the revolving circle and tied to it.

Prolate Cycloid.—Curve generated by a point within the revolving circle and tied to it.

Epicycloid (Fig. 241).—Curve generated by a point of a circle rolling on the convex circumference of another circle. In the drawing of the epicycloid the parallels to the directrix become arcs of circles.

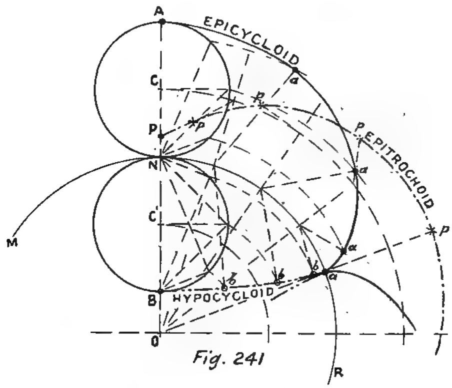


Fig. 241

Hypocycloid.—Curve generated by a point of a circle rolling on the concave circumference of another circle.

Epitrochoid.—Curve generated by a point beyond or within a circle rolling on another circle.

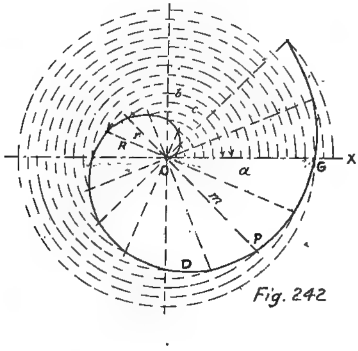


Fig. 242

Spiral of Archimedes (Fig. 242).—Line described by a point moving uniformly along a straight line while this line revolves uniformly about one of its points. The fixed point **O** is the **pole.** A **radius vector m** is the distance from a point **P** of the curve to the pole **O.**

Spire.—A **spire** is the position **OPDG** of the curve drawn during a complete revolution (360°) of the line.

Pitch.—The **pitch a** is the length traveled on the fixed line by the moving point during a complete revolution.

Drawing of a Spiral.—Draw from the pole **O** as a center a circle with **(R=a),** which divide into a number of equal parts, as 8, 16, 32, etc., and draw the radii to these points. Divide also **a** into the same number of equal parts and through each point draw a circle from the center **O.** The pole **O** is the first point (origin) of the spiral; other points are given in succession by the intersection of the first radius with the first circle, the second radius with the second circle, etc.

Parameter of the Spiral.—The constant

$$\frac{a}{2\pi} = p \quad \text{is called the } \textit{parameter}$$

The equation, being written under the form **a=2pπ** shows that the parameter equals the radius of the circle whose circumference equals the pitch **a.**

Sub-Tangent.—Projection of the tangent upon a perpendicular to the radius-vector of the point of contact.

Sub—Normal.—Projection of the normal upon a perpendicular to the radius-vector of the point of contact.

The sub-normal is constant and equals the parameter $\frac{a}{2\pi}$.

Drawing of a Tangent to the Spiral at a given Pt. on the Curve (Fig. 243.)—Draw the radius-vector **OP.** and a perpend. to it through the pole **O,** on which line lay off the parameter

$$OA = \frac{a}{2\pi} \;;$$

Join **A-P** and at **P** erect **PT** perpend. to **AP;** it will be the required perp. to the spiral.

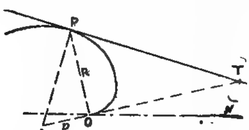


Fig. 243

Area of the Spiral—Between the curve and a radius vector v.—It equals ⅓ the corresponding circular sector.

Call n the angle described: $A = \frac{1}{3}\,\frac{\pi v n}{180} \times \frac{v}{2} = \frac{\pi v^2 n}{1080}$

Involute of a Circle (Fig. 244).—Curve described by the end of a string, supposed wrapped around the circle, while unwinding it and keeping it taught. In any position the string is tangent to the circle and its length is equal to the arc between the origin **A** and the point of contact.

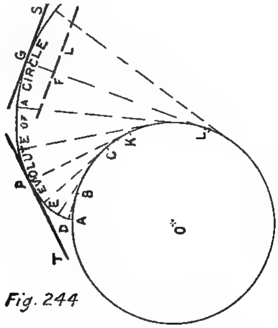


Fig. 244

Tangent to the Involute of a Circle.—1° Through a Pt. P on the Curve.—Draw a tangent **PK** from **P** to the circle, then a perpendicular to that tangent at **P,** it will be the required tangent to the curve.

2° Parallel to a given Line.—Draw a tangent **FL** to the circle perpendicular to the given line; then a perpendicular **GS** to that tangent at the pt. **G,** where it intersects the circle.

MECHANISM.

Parts of a Machine.—Frame, mechanism.

Frame.—Fixed parts or supports. Their arrangement, form and size depend upon the arrangement, form and motions of the mechanism and the most severe loads or pressures transmitted to them by the mechanism.

Working of a Machine.—Three requisites: 1° Source of energy or power; 2°, Transmission of motion and power; 3°, Working parts.

Source of Power.—It may be animal force, or steam pressure, electric current, hydraulic head, pneumatic force, etc., which communicate force and motion to one or more pieces.

Prime Mover.—The pieces which receive power from the source and whose duty it is to transmit that energy and the motion received through it.

Train of Mechanism.—Pieces which transmit force and motion to the working pieces. During that transmission the same energy and motion are more or less changed in direction and amount so as to use them to the best advantage, according to the purpose of the machine.

Working Pieces.—Those pieces in a machine which produce a useful effect.

Mechanism.—Treats of motion and its modifications.

Direction of Motion.—When a point follows a straight line, that line is the direction of the motion.

When a point follows a curve, the direction of the motion is, at every instant, that of the tangent to the curve. That curve may or not be a plane figure.

Acceleration.—Additive velocity.

Retardation—Subtractive velocity.

Acceleration and retardation constitute **direct deviation.**

Deflexion.—Lateral deviation from a plane (even elementary) to another.

Revolution.—Path described by a point from the time it passes a point to the time it returns to that point.

Motions, velocities, deviations, may be compounded or resolved like forces by a **triangle of forces** or a **polygon of forces.**

Transmission of Force by a Fluid.—Force may be transmitted by air, gas, water or steam, admitted into a cylinder where they are either compressed or allowed to expand, in either case pressing a piston attached to the end of a rod, and giving motion and power to both.

Change in Capacity of Cylinder.—**A**=area of cross section of cylinder; **V**=velocity of motion.

In the unit of time the capacity of the cylinder is altered by **AV.**

Comparative Motion of two Pistons.—Of cylinder areas **A** and **a** and velocities **V** and **v.**

It is expressed by the equation $\frac{V}{v} = \frac{a}{A}$.

The velocities are opposite in direction.

The velocities are inversely proportional to sections of cylinders.

Motion of a rigid Body.—That motion may be; 1°, a translation; 2°, a rotation; 3°, a combination.

Translation—May be: 1°, in a straight line; 2°, in a circle.

Regulation of straight Translation.—1°, Fixed straight guides; 2°, Parallel motion or link work.

Reciprocating Motion.—The returning to the starting point of a piece that has traveled in a straight line.

Regulation of circular Translation.—1°, Cranks attached to points of the moving piece; 2°, Wheel-work.

Example: 1°, The piston rod has a reciprocating motion; it is directed by parallel guides. 2°, The coupling rod of a locomotive. Its

position remains parallel to itself and each point describes a circle with regard to the frame of the engine.

Fixed Axis.—A line immovable with regard to a rotating piece to which it is attached. As the center of a shaft, of a wheel or arc.

Rotation about a fixed Axis.—Angular velocity of rotation

$$a = \frac{d\theta}{dt}\ .$$

Linear velocity =**ar**; **r** being the distance from the moving point to the fixed axis.

Lever.—A rigid body turning about a fixed axis called **fulcrum,** one point of which receives motion and another receives the same and transmits it.

Wheel and Axle.—Two concentric cylindrical surfaces, one of which receives motion and the other receives and transmits it. The motion is about the fixed axis, and the velocity ratio of the surfaces is that of their radii, **R÷r.**

Rolling Cylinders (Fig. 245).—When the point **P** of the cylinder **o** rolling on cylinder **O,** is within the circle, that point describes a curve of the genus **epitrochoid.** When **P** is on the circle, its path is an **epicycloid.** When **P** is at the center **o** it describes a circle.

Rolling Cones (Fig. 246).—The point **P** on the surface of the cone **c** revolving on cone **C** describes a special **epitrochoid**—on sphere of radius **OP** and center **O.**

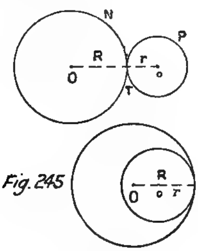
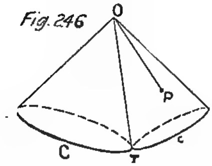


Fig. 245

Fig. 246



Helical Motion.—Combination of rotation and translation. This translation is perpendicular to the plane of rotation and parallel to the axis. Any point in the moving body describes a **helix** or screw about the axis. To make a body assume that motion, it is shaped like that curve and made to follow guides of the same form.

Right or left-handed Screws.—When the body moves to the right to produce advance, the screw is **right-handed.** When it moves to the left to produce advance, it is **left-handed.**

Pitch.—The advance per turn. ,

Thread.—Helical projection of a screw.

Elementary Combinations.—Two pieces connected with each other and with the frame. ,

Their connection with the frame determines their kind of motion.

Their connection with each other determines their comparative motion.

Driver.—Piece receiving motion from source.

Follower or Driven.—Piece receiving motion from driver.

Smooth Wheels.—Turning pieces in rolling contact (Toothless wheels).

Smooth Rack.—A piece shifting when in contact with a turning piece.

Centers.—Points of intersection of the axis with a plane perpend. to them.

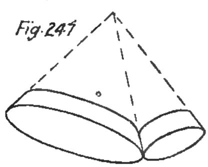
Pitch-Point.—Intersection of that plane with the line of contact.

Shapes of Wheels.—They are circular, elliptical, etc., when their section by that plane is a circle, an ellipse, etc.

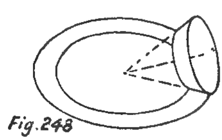
Outside Gearing.—When the pitch-point (point of contact) is between the centers.

Inside Gearing.—When the centers are on the same side from the pitch-point.

Bevel Wheels (Fig. 247.)—Each is a frustrum of a circular cone.



Disc Wheel (Fig. 248).—One of the rolling surfaces is a portion of plane contained between concentric circles.



Disadvantage of smooth Racks and Wheels.—They slip easily, not having sufficient adhesion.

Means of increasing Adhesion.—Alternate ridges and grooves are made in the surfaces in contact, a ridge of one piece to fit into the cavity of the other.

Teeth.—The ridges in the surface of contact of a sliding piece, when made regular in width and depth. The teeth are cast in one piece with the wheel.

Spaces.—The regular intervals (or hollows) between the teeth.

Pitch-Surface.—Cylindrical or conical surface half way between the top of the teeth and the bottom of the spaces.

Pitch-Circle.—In circular cylindrical or conical toothed-wheels, the intersection of the pitch-surface by a plane perpendicular to the axis.

Pitch-Point.—Point of contact of the pitch-circles.

Face.—Part of the acting surface of a tooth outside the pitch-circle.

Flank.—Part of the acting surface of a tooth inside the pitch-circle.

Cogs.—Movable teeth fitting into mortises made in the periphery of a wheel.

Pinion.—Small toothed-wheel.

Trundle.—A pinion the teeth of which are cylindrical (generally wooden staves).

Geometrical Radius.—The radius of the pitch-circle.

Addendum Circle.—Circle tangent to the tips of the teeth.

Real Radius.—Radius of the addendum circle.

Addendum.—Portion of the teeth between the addendum and the pitch-circles.

Pitch.—Distance from the face of a tooth to the face of the next, measured on the pitch-circle.

Relations between the Pitch and the number of Teeth.—1°. The pitch should be an aliquot part of the pitch-circle—That condition is not necessary in reciprocating arcs or **sectors.**

2° The pitch should be the same in pieces (wheels or racks) working together.

3°. (a) In a couple of wheels (circular) the number of teeth in the peripheries are directly proportional to the radii; (b) they are inversely proportional to the angular velocities; (c) the ratio of the number of teeth must be a whole number, and so must be its equal the inverse ratio of angular velocities.

Formulas.—a=number of teeth that pass the pitch-point from the moment that a certain pair of teeth work together and that when they again come in contact.

b=number of different teeth in the large wheel with which a certain tooth in the small wheel will work.

c=number of teeth in the small wheel with which a certain tooth in the large wheel will work.

N=number of teeth in large wheel.

n=number of teeth in small wheel.

M and m represent whole numbers and are read **Multiple of.**

When $N = Mn$: $a = N$; $b = \frac{N}{n}$; $c = 1$

When $N = Md$ and $n = md$: $a = mN = Mn = Mmd$; $b = M$; $c = m$

When N and n are prime to each other : $a = nN$; $b = N$; $c = n$

Forms of Teeth.—In Wheels.—External epicycloid for the face. Internal epicycloid for the flank.

In Rack.—Cycloids for both face and flank.

Diameter of the describing Circle.—The circle describing these curves should have a diameter not greater than the radius of the pitch-circle. That radius should be ½ the diam. of the pitch-circle for the smallest wheel in a set, in order to have straight or radial flanks.

Circumference of the describing Circle.—That circumference is generally six times the pitch.

Number of Teeth in a Pinion.—The smallest pinion of a set should have 12 teeth.

Thickness of Teeth and Width of Spaces.—In order to allow for slight defects in the castings, spaces are made a little wider than the teeth. The rule that prevails is;

Thickness of a tooth $= \frac{5}{11}$ pitch ;
Width of a space $= \frac{6}{11}$ pitch .

Back-Lash.—Excess of the width of a space over the thickness of a tooth.

Clearance.—The bottom of the spaces is made deeper than the path of the point of the teeth by about 1-10 of the pitch.

Backs.—The backs (non-bearing surfaces) of the teeth and spaces are made like the fronts, so that the wheels may be reversible.

Teeth of Bevel-Wheels.—Like teeth of spur-wheels, except that the problems involved relate to a sphere instead of a plane. A center becomes a pole, and a straight line becomes a great circle.

Cam (Fig. 249).—A single tooth. It is meant to either rotate or oscillate, thus driving a sliding or a turning piece either continuously or intermittently.

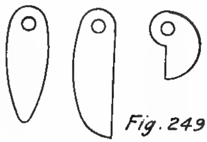


Fig. 249

Screw.—A cylinder with continuous helical ridges or projections called **threads.**

Male Screw.—The threads are on the outer surface of the cylinder.

Female Screw.—The threads are on the inner surface of a hollow cylinder.

Nut.—A short female screw.

Pitch.—Distance, measured on an element of the cylinder, between a point of the thread and the corresponding point after a complete revolution.

Shaft.—A long cylinder, to which drums, geared-wheels or drums are attached at intervals and by means of which power and motion are transmitted at a distance and at several points.

Coupling.—1°. Two shafts on the same axial line are coupled by means of collars and keys.

2°. Two parallel shafts when close to each other are generally coupled by means of geared-wheels.

3° When at a certain distance, the coupling is made by belts and pulleys.

Wrapping Connectors.—Belts, cords and chains passing over drums or pulleys fixed one in each shaft. Belts are made of leather or rubber; cords are made of hemp or catgut.

Tendency of Belts moving on Drums.—Belts tend to move around those portions of drums of greatest diameter; therefore, the periphery of drums is generally made slightly convex so that belts will remain in the center.

Shifting of a Belt.—To take off a belt from a drum when in motion or to shift it from one drum to another is done by means of a forked lever, which pushes that portion of the belt which moves toward the drum.

Rimmed Drums.—Drums with rims on the edges are used when cords are the connectors. Grooved pulleys are also used for the same purpose.

Chains.—When chains are the connectors, the drums or pulleys used must be properly grooved and toothed so as to accommodate the links.

Endless Connectors.—Are used when the motion they transmit is to be continuous.

Secured Connectors.—When intended to transmit reciprocating motion, the belts or chains have their ends secured to the drums or pulleys, which are then only sectors.

Line of Connection.—The center line of the connector.

Pitch-Line of Drum or Pulley.—The theoretical curve to which the line of connection is tangent. It is distant from the bearing surface of the pulley or drum by ½ the thickness of the belt, cord or chain, and is parallel to that surface.

Circular Pulleys and Drums.—Are those used for transmitting a constant velocity.

Effective Radius.—The radius of the pitch-circle which equals the radius of the drum or pulley plus ½ the thickness of the connector.

Angular Velocities of a couple of pulleys or drums connected by belt or pulley are inversely as the effective radii.

Reversion of Direction of Motion.—To reverse the direction of motion of two parallel shafts, cross the belts.

Length of a Belt (Fig. 250).

1° *Crossed belt* : $l = 2[d^2 - (R+r)^2]^{\frac{1}{2}} + (R+r)(\pi - 2\sin^{-1}\frac{R+r}{d})$.

2° *Uncrossed belt* : $l = 2[d^2 - (R-r)^2]^{\frac{1}{2}} + \pi(R+r) + 2(R-r)\sin^{-1}\frac{R-r}{d}$

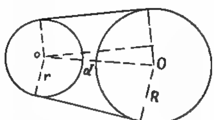
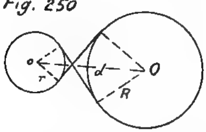


Fig. 250



Modification of Rotation of Axes.—Increase or decrease of speed in a shaft is obtained by means of stepped pulleys (Fig. 251), or speed-cones (Fig. 252). The belt is shifted from one pulley to another while in motion, or from side to side on the speed-cones (or conoids). In the cones the speed may vary gradually.

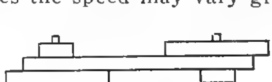


Fig. 251

Link.—A rigid rod connecting pieces to one another.

Rods have different names according to their purpose; eccentric-rod, crank-rod, coupling-rod, connecting-rod.

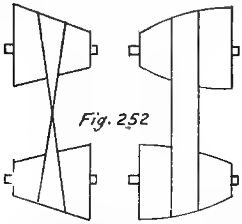


Fig. 252

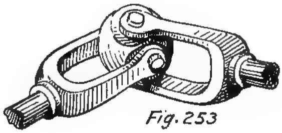


Fig. 253

Straight Link.—A rod should be straight, unless a particular reason exists requiring another form.

Connected Pieces.—When they rotate: cranks, excentries.

When they oscillate: beams, levers.

How a Link is attached.—A rod or link is attached to the pieces by pins around which it is free to revolve.

Effect of the Link.—It maintains constant the distance between the center of the pins. These centers may be considered as the connected points.

Line of Connection.—The perpendicular common to the axes.

Crank-Arm.—Perpendicular from the connected point to the axis of rotation.

How Links cause Shafts to revolve with equal Velocities.—At the ends of parallel shafts, equal cranks are connected by a rod which keeps these cranks constantly parallel. Example: Connecting-rods of a locomotive. The length of a connecting-rod is the distance between the extreme shafts.

Eccentric.—A disc attached to a shaft at a point different from its center. It revolves like a crank and performs the function of a crank. A link is attached either to a point of its surface or to a briddle surrounding it.

Use of the Excentric.—It transforms a circular into a reciprocating motion.

When used.—When an ordinary crank in the shaft might weaken it.

Center of the Excentric.—Its connected point.

Excentricity.—Distance between the connected point and the center of the shaft; it is the length of the crank-arm, which it replaces.

Excentricity may be Changed.—By means of a radial groove and clamping screw.

Stroke.—In a reciprocating piece, the distance between the ends of the path traveled by the connected point. Example. The piston of a locomotive.

Dead-Points.—When a reciprocating piece is connected with a revolving piece, the dead-points are the ends of the stroke, where the connecting line is on a line with the crank-arm.

Coupling of converging Shafts (Fig. 253).—Each shaft is terminated by an arm bent in the form of a horseshoe, with holes near the extremities of each branch, and into these fit freely the ends of a link shaped like a cross.

Intermittent Motion produced by continuous Motion.—This is produced by click and ratchet.

Ratchet-Wheel (Fig. 254).—A toothed wheel made to advance at

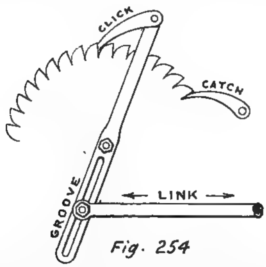


Fig. 254

regular intervals by the same angle. The back of the teeth, on which the motion is communicated by the mover, is straight and radial; the front is an arc of a small circle or a parabolic curve, and on these the mover slides back.

Click.—An arm jointed at one end to the extremity of a lever free to move, in an arc of a circle, around a point where it is pivoted. That lever is jointed to the end of a reciprocating link or rod. The free end of the click pushes (or pulls) the ratchet-wheel a certain distance and returns to its original position.

Catch.—An arm jointed to the frame and whose action consists in preventing the ratchet-wheel from moving back.

Train.—A series of pieces, of which the intermediary ones are driven by the preceding one and drive the following one.

Example :

A train of wheels on axes .... A_1 A_2 A_3 .... A_n
with angular velocities .... v_1 v_2 v_3 .... v_n
carrying wheels with the number of teeth .... N_1 N_2 N_3 .... N_n
driving wheels with the number of teeth .... n_1 n_2 .... n_{n-1}

then

$$\frac{v_n}{v_1} = \frac{a_2}{a_1} \times \frac{a_3}{a_2} \times \frac{a_4}{a_3} \times \cdots \times \frac{a_n}{a_{n-1}} = \frac{N_1 N_2 N_3 \quad N_n}{n_1 n_2 n_3 \quad n_{n-1}}.$$

Rule.—The velocity ratio of the last axis to the first one is the ratio of the product of the number of teeth in the drivers to the number of teeth in the followers.

Converging Train.—When their motions are all transmitted to the same piece. Example: When several engines actuate the same shaft.

Diverging Train.—When the motion of one piece is transmitted to several other pieces whose action is independent of each other.

Example: When the motion of a shaft is transmitted, through several pulleys and belts, to different machines.

Aggregations.—Assemblage of pieces in which the motion of a piece is the resultant of the motions imparted to it by several drivers.

Differential Pulleys (Fig. 255).—The axis **O** carries two concentric solidary pulleys of different diameters (pulleys revolving together), revolving with angular velocity **a** in the direction of the arrow. The suspended pulley **p**, called **sheave,** supports a weight, **W,** suspended at its center **o.** One end of the cord is fixed to **P** and wound around it; the other end is fixed to **Q** and also wound around it.

a_1 is the angular velocity of p and is the aggregate velocity produced by the joint action of the drivers P and Q.

Velocity of point $C = a \times R$.

Velocity of point $D = -a \times R'$ (motion upward is $-$).

Angular velocity of p; $\quad a_1 = \dfrac{a(R+R')}{2r}$

Velocity of o; $\quad v_1 = a_1 \dfrac{r}{2} \times \dfrac{R-R'}{R+R'}$; $\quad v_1 = \dfrac{a(R-R')}{2}$.

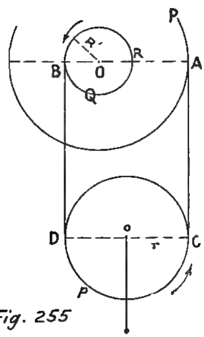


Fig. 255

The velocity of the sheave is the average velocity of the vertical parts of the cord.

Block.—An axis carrying several pulleys.

Fall-Block.—A block attached to a fixed point.

Running-Block.—A movable block suspended from a fall-block by several parts of a cord.

Parallel Motion (Fig. 256).—The arm **OC** rotates around center **O.** It is jointed at **C** to the middle of a beam **BA** of double its length **(AC=CB=OC).** The extremity **B** of the beam slides between radial parallel guides **G.** In any position **ABO** is always a right-angled tri-

angle at **O**, because of **OC=CB=CA**; therefore **OB**, being fixed in position, **OA** is perpendicular to it and fixed in direction.

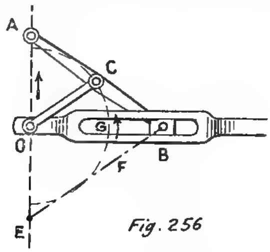


Fig. 256

STRENGTH OF MATERIALS.

Strength of Materials.—Capability of materials to resist the action of external forces.

Tension.—State of a piece of material pulled by forces acting in opposite directions (the directions are diverging).

Compression.—State of a piece of material pushed by forces acting in opposite directions (the directions are converging).

Stress.—State of a piece of material subjected to either tension or compression.

Shear.—Tendency to the sliding of the molecules of a piece of material, along a certain line, over the molecules of the next stratum, due to the action of extraneous forces generally applied crosswise.

Formulas: *Tension, Compression, Shear:* $P = \frac{aS}{f}$; *For breaking stress* make $f = 1$; $P = aS$ — in which:

P = *Stress in lbs.*
a = *Area of X Sec. of specimen,*
S = *Ultimate strength of the material in lbs. (see Table A).*
f = *Factor of safety (see Table B).*

TABLE A				Materials.	TABLE B			TABLE C	
Ultimate Strength S *in lbs.*					*Factor of Safety* f			*lbs. per sq. in.*	
Tension	Compression	Shear	Flexure		Steady	Varying	Shock	Coefficient of Elasticity E	Elastic limit for Tension
20 000	80 000	20 000	36 000	Cast Iron	6	15	20	15 000 000	6 000
50 000	50 000	47 000	50 000	Wrought „	4	6	10	25 000 000	25 000
100 000	150 000	70 000	120 000	Steel	5	7	15	30 000 000	50 000
10 000	8 000	600 to 3 000	9 000	Wood	8	10	15	1 500 000	3 000
	6 000		2 000	Stone	15	25	30		
200	2 500			Brick					

Elongation (in tension), *Shortening* (in compression): $e = \frac{Pl}{aE}$; in which e = elongation (or shortening);
l = length of piece in inches;
E = coefficient of elasticity in *lbs.* per *sq. in.* (See *Table C*).

Beam.—Breaking strength; or bending moment. M=SR; in which
M=Bending moment, lbs. per in. (Table D).
R=Moment of resistance. (Table E).

TABLE D

Kind of Load and Support.	M Bending Moment	S Set or Deflection
Supported at one end, load at the other;	Wl	$\frac{Wl^3}{3EI}$
do. do. uniformly loaded;	$\frac{Wl}{2}$	$\frac{Wl^3}{8EI}$
Simply supported at ends, load at center;	$\frac{Wl}{4}$	$\frac{Wl^3}{48EI}$
do. do. uniformly loaded;	$\frac{Wl}{8}$	$\frac{5Wl^3}{384EI}$
Fixed at both ends, load at center;	$\frac{Wl}{8}$	$\frac{Wl^3}{192EI}$
do. do. uniformly loaded;	$\frac{Wl}{12}$	$\frac{Wl^3}{384EI}$

in which;
W= Load in lbs.
I = Moment of Inertia (Table E) depending on cross-section.

TABLE E

Section.	I	R	G²
Circular	$\frac{\pi d^4}{64}$	$\frac{\pi d^3}{32}$	$\frac{d^2}{16}$
Hollow Circular	$\frac{\pi(d^4-d'^4)}{64}$	$\frac{\pi(d^4-d'^4)}{32d}$	$\frac{d^2+d'^2}{16}$
Square	$\frac{d^4}{12}$	$\frac{d^3}{6}$	$\frac{d^2}{12}$
Hollow Square	$\frac{d^4-d'^4}{12}$	$\frac{d^4-d'^4}{6d}$	$\frac{d^2+d'^2}{12}$
Rectangular	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{b^2}{12}$
Hollow Rectangular	$\frac{bd^3-b'd'^3}{12}$	$\frac{bd^3-b'd'^3}{6d}$	$\frac{b^3d-b'^3d'}{12(bd-b'd')}$
Eye (I) Beam	$\frac{bd^3-b'd'^3}{12}$	$\frac{bd^3-b'd'^3}{6d}$	$\frac{b^3d-b'^3d'}{12(bd-b'd')}$
Tee (T) Beam	$\frac{bd^3+b'd'^3-(b'-b)d''^3}{3}$		

in which
I = *Moment of Inertia* about axis of rotation through center of gravity;
R = *Moment of Resistance*;
G^2 = *Square of least Radius of Gyration*.

Strength of Beams—Hypothesis.—It is supposed that the material of the portion of a beam above the neutral axis (supposed to pass through the center of gravity of the section) is in compression, and the material of the portion below that axis is in tension.

Results of Experiment.—1°. The strain varies as the square of the depth **d**; the length **L** and the breadth **b** being the same.

2° The strain varies as the breadth **b**; the length **L** and the depth **d** being the same.

Strength of a Beam.

General Formula: $W = \frac{bd^2}{L} \times C$ in which $\begin{cases} W = \text{distributed load in tons;} \\ b = \text{breadth in inches;} \\ d = \text{depth " "} \\ L \text{ length " feet;} \\ C = \text{a constant.} \end{cases}$

Noting that $bd = a$ = area of cross sect. in sq. in. we can write

$$W - \frac{adC}{L}$$

Beam	
Fixed at one end, load at the other;	$W = \frac{bd^2C}{L} = \frac{adc}{L}$
do. uniformly loaded;	$W = 2\frac{bd^2C}{L} = \frac{2adC}{L}$
Supported at both ends, load at center;	$W = 4\frac{bd^2C}{L} = \frac{4adC}{L}$
do. uniformly loaded;	$W = 8\frac{bd^2C}{L} = \frac{8adC}{L}$

Constant. The constant C = *breaking load in lbs. of a bar 1"x 1"x 12"* between supports is determined by experiment, then $\frac{1}{3}$ or $\frac{1}{4}$ or $\frac{1}{6}$ of it is taken as the safe load.

Safe Load constant for wrought Iron Beams. The safe load per sq. in. of effective cross section is taken as 12,000 lbs.=6 tons.

Effective Cross Section of a Beam.—Area **a** of one flange, plus ⅙ the area ⅙ **a'** of the web.

Effective Depth of a Beam.—Distance between the centers of gravity of the two flanges.

Safe Load on rolled Beams.—For equally distributed load on floors, the general formula for safe load is

$$W = \frac{8D(a + \frac{a'}{6})S}{L} \quad \text{in which} \begin{cases} D = \text{Effective depth in ft.} \\ (a + \frac{a'}{6}) = \text{effective section in sq. in.} \\ S = \text{strain in tons (2000 lbs) per sq. in. of section;} \\ W = \text{safe load in tons.} \end{cases}$$

(For effective depth and effective section see Table.)

Stiffness.—Property of resisting the action of bending forces, but with little flexibility or pliancy.

Maximum Deflexion allowable.—About 1/30 in. per foot of clear span is the maximum permissible.

Relation between Depth of Beam and Clear Span.—Under ordinary loads the maximum deflexion is obtained when the span is about 26 times the depth of a beam.

General Formula for Deflection : $s = \dfrac{0.004\,WL^3}{(a+\frac{a'}{6})d^2}$

Approximate " " " $s = \dfrac{L^2}{62\,d}$

in which $\begin{cases} d = \text{depth in in.} \\ L = \text{span in ft.} \\ W = \text{Equal}^{y}\text{ distr. load in net tons.} \end{cases}$

TABLE FOR BEAMS
(from Phœnix Iron Company's Book)

Beam		Effective Area in sq. in.			Effective Depth		Factors for S=6 tons	
Size in.	Weight lbs.	a flange	a' stem	$a+\frac{a'}{6}$	D ft.	d in.	Load $8D(a+\frac{a'}{6})\times 6$	Deflection $(a+\frac{a'}{6})d^2$
15	200	6.142	7.715	7.428	1.150	13.80	410	1415
15	150	4.330	6.340	5.386	1.170	14.04	302	1062
12	170	5.777	5.446	6.684	.910	10.92	292	797
12	125	3.810	4.880	4.623	.930	11.16	208	576
10½	135	4.375	4.750	5.166	.800	9.62	178	478
10½	105	3.353	3.793	3.986	.812	9.74	155	378
9	150	5.586	3.828	6.224	.658	7.90	197	388
9	84	2.800	2.800	3.261	.691	8.30	108	225
9	70	2.381	2.238	2.754	.698	8.38	92	193
8	81	2.812	2.476	3.225	.610	7.37	94	175
8	65	2.109	2.282	2.489	.618	7.42	74	137
7	69	2.500	1.900	2.816	.530	6.37	72	114
7	55	1.775	1.949	2.100	.537	6.44	54	87
6	50	1.858	1.284	2.072	.456	5.47	45	62
6	40	1.421	1.158	1.614	.458	5.50	35	49
5	36	1.200	1.200	1.400	.383	4.60	25	30
5	30	1.000	1.000	1.166	.385	4.62	21	25
4	30	1.135	.730	1.257	.298	3.58	18	16
4	18	.562	.682	.676	.304	3.65	10	9

Formula for Floor Beams uniformly loaded.

$W = wLB$ in which $\begin{cases} W = \text{Safe load for each beam,} \\ w = \text{load per sq. ft.} \\ L = \text{Clear span in ft.} \\ D = \text{distance between centers of beams.} \end{cases}$

From formula $W = \dfrac{8D(a+\frac{a'}{6})S}{L}$, we have: $wLB = \dfrac{8D(a+\frac{a'}{6})S}{L}$, from which:

$$B = \frac{8D(a+\frac{a'}{6})S}{wL^2}$$

a formula giving the distance between centers of beams in a floor designed to carry **w** lbs. per square foot, when the span and size of beam (determining **D** and **a**+⅙**a'**) are given. **S** may be assumed as 6 tons.

Cheapest Beams.—For a given span it is more economical to select the deepest beam.

Stiffness of a Floor.—The stiffness is increased by using deeper beams, preferably to a greater number of smaller sizes.

Columns.—The strongest form is a circular section. The weakest form is a square section.

General Formula $\qquad W = \dfrac{Sa}{1 + q\frac{l^2}{G^2}}$

in which $\begin{cases} q = \text{a constant varying with the material;} \\ G = \text{least radius of gyration} \end{cases}$

Formulas for circular columns: $\begin{cases} \text{Wrought Iron:} & W = \dfrac{Sa}{1+\frac{1}{3000}\left(\frac{l}{d}\right)^2}; \\ \text{Cast} \quad \text{,,} \quad : & W = \dfrac{Sa}{1+\frac{1}{400}\left(\frac{l}{d}\right)^2}; \end{cases}$

in which
- a area of section in sq. in.
- S = ultimate strength $\begin{cases} 50\,000 \text{ lbs. for wrought iron,} \\ 80\,000 \text{ ,, ,, cast ,,} \end{cases}$
- W = breaking load in lbs.
- $\frac{l}{d}$ = ratio $\frac{\text{length}}{\text{diameter}}$,
- $\frac{1}{3000}$ and $\frac{1}{400}$ factors of safety for W.I. and C.I.

For $\begin{cases} \text{wrought iron} \\ \text{cast} \quad \text{,,} \end{cases}$ Safe load $= \begin{cases} \frac{W}{4} \\ \frac{W}{6} \end{cases}$

Specific Gravity and Weight of various substances.

Substance.	Weight in lbs. per cu. ft.	Weight in lbs. per sq. ft. 1" thick.	Weight in lbs. per cu. in.	Specific Gravity.
Water pure	62.3	5.19	.036	1.
,, - sea	64.3	5.36	.037	1.03
Wrought Iron	480	40	.277	7.70
Cast ,,	450	37.50	.260	7.20
Steel	490	40.84	.283	7.84
Lead	710	59.16	.410	11.36
Copper - rolled	548	45.66	.317	8.80
Brass - rolled	524	43.66	.302	8.40
Sand	98	8.23	.057	1.57
Clay	120	10	.069	1.92
Brickwork - ordinary	120	10	.069	1.92
,, - close joint	140	11.66	.081	2.22
Limestone	168	18.	.124	2.68
Glass	156	13	.090	2.49
Pine - white	30	2.50	.017	.48
,, - yellow	35	2.91	.019	.56
Hemlock	25	2.08	.015	.40
Maple	49	4.08	.028	.78
Oak - white	50	4.16	.030	.80
Walnut	41	3.41	.023	.65

Shafting. $d = \sqrt[4]{\frac{H}{N}}$; in which $\begin{cases} d = \text{diam. of shaft in in.} \\ H = \text{horsepower transmitted;} \\ N = \text{no. of revolutions per min.} \\ c = \text{constant.} \end{cases}$

Values of c.

	Shafts very long requiring great stiffness.	When strength only is required.
Cast Iron	5.26	4.02
Wrought "	4.75	3.63
Steel	3.96	3.03

Pipes

Thin Pipes; $pdf = 2tS$

Thick Pipes and Cylinders; $pf = \frac{St}{r+t}$

in which $\begin{cases} d = \text{diam. of pipe in in.} \\ f = \text{factor of Safety} \begin{cases} \text{Cast iron} = 12 \\ \text{Wrought ,,} = 6 \end{cases} \\ p = \text{pressure in lbs. per sq. in.} \\ r = \text{inside radius in in.} \\ t = \text{thickness in in.} \end{cases}$

Ropes, Chains.

$W = cd^2$; in which $\begin{cases} c = \text{constant;} \\ d = \text{diam. in in of rope, or iron in link.} \end{cases}$

Values of c: Rope $\begin{cases} \text{hemp } \frac{1}{3}; \\ \text{iron wire } \frac{8}{3}; \\ \text{steel ,, } \frac{14}{3}; \end{cases}$ Chain $\begin{cases} \text{close link W.I. } 6; \\ \text{stud ,, ,, } 9. \end{cases}$

THE SLIDE RULE.

Slide Rule or calculating Rule.—The slide rule is a double rule so arranged that one of the portions slides within the other one (which last should therefore be both wider and thicker), by means of tongues and grooves, so that their flat surfaces are in the same plane.

Four Scales, forming two Sets.—The top line of the rule (along the slide) and the top line of the slide are divided alike.—The bottom line of the slide and the bottom line of the rule (along the slide) are also divided alike, but differently than the top lines. The top scales from a set; the bottom scales form another set. These scales are generally called **Scale A, Scale B, Scale C and Scale D** from top to bottom; scales **A** and **B** form the upper set; scales **C** and **D** form the lower set.

Size of the Instrument.—It might be made of any length, but is most commonly 25 centimeters long.

Lower Set.—Scale D is divided as a Table of Logarithms.

Primary Divisions.—Scale **D** (25 cent. long) is first numbered 1 on the left and 10 on the right. The intermediary numbers 2, 3,.... 8, 9, are determined in the hypothesis that the markings shall represent logarithms and the figures shall represent the numbers corresponding to these logarithms. This hypothesis requires to consider the full length of the rule, or that portion between 1 and 10, as the unit. In which case we see that log. 1=0, which is a point of the line, and that log. 10=1, the full line. In order to interpolate the other numbers it was ascertained what portions of the line were the log. of 2, the log. of 3, etc., in the following manner: Log. 2=0.3010300 (by tables); this multiplied by 25 cm. gave 7.52575 cm. as the length of log. of 2, beginning at the left hand; likewise, log. 3=0.4771213, and its length on the scale is 0.4771213×25=11.928 cm., etc. Markings are made in the rule at the distances 7.52575 centimeters, 11.928 centimeters, etc., and these are numbered 2, 3, etc. This accounts for the unequal distances between 1 and 2, 2 and 3, 3 and 4, etc.

Further similarity of the Rule to a Table of Logarithms.—Remembering that the logs. of two numbers composed of the same figures placed in the same order differ only by their index, the decimal parts being the same, the index being supposed omitted in the rule, as it is omitted in the tables, we are able to find

not only the logs. of	1 ;	2 ;	3 ;........	10 ;
but those of	10 ;	20 ;	30 ;.......	100 ;
and generally those of	1×10^n;	2×10^n;	3×10^n;.....	10×10^n.

at just the same points of the rule.

Secondary Divisions.—As logs. of 1 and 10 correspond, and also those of 2 and 20, we can sub-divide the space 1—2 into ten parts by inserting the logs. of 11, 12, 13..... and 19 in the same manner as the markings for 2, 3, 4, ... 9 were made.

These markings are numbered 1.1, 1.2, 1.3,....... 1.9. Likewise spaces 2-3, 3-4, ... 9-10, are also divided into 10 parts, corresponding to logs. of 2.1, 2.2, ... 2.9; 3.1, 3.2 .... 3.9, etc. But these are not numbered any longer except 2.5, 3.5, 4.5, etc., on account of lack of room.

Tertiary Divisions.—In like manner the logs. of 1.01, 1.02, 1.03,.... 1.09 may be inserted between the logs of 1 and 1.1; the logs. of 1.11, 1.12, 1.13, 1.14, ... 1.19 between the logs. of 1.1 and 1.2; and so on, continuing to the right hand of the scale if space would allow. As it is, the ten divisions between 1 and 2 are alone subdivided into 10 parts; the divisions between 2 and 3 and between 3 and 4 are subdivided into 5 parts only, and the divisions of the rest of the scale are sub-divided into 2 parts only. But it is expected that the user of the rule shall estimate the missing sub-divisions as nearly as he can when they are required in the solution of a problem.

Division of Scale C.—Scale **C** of the slide is the exact duplication of Scale **D** of the rule, so that when the slide is closed, the markings on the two scales appear to form one scale only, corresponding as they do throughout the length of the instrument.

Multiplication by means of Scales C and D.—These scales constituting a real table of logs. may be used in its stead, and with them

multiplications may be made by means of additions and divisions by means of subtractions.

Example I. Find 136×63. Add logs. of 136 and 63 as follows: Draw the slide to the left until 1 of the rule is under 136 of the slide (136 is between 1 and 2); look along the rule until you find 63 (between 6 and 7); finally read on the slide the number corresponding to 63; this is 857 as near as can be estimated, and 857 or 8570 is the product sought. All that is left to do is to ascertain the number of figures which the product should have.

Example II. Find 892×76.

If in the working of this example we draw the slide to the left until 1 of the rule is under 892, the other factor 76 on the rule will be found opposite a vacant space. In that case draw the slide to the right until 10 of the rule is under 892 of the slide (between 8 and 9); look along the rule until you find 76 (between 7 and 8); read on the slide the number (678) opposite 76; 678 or 6780 or 67800 is the product sought. Here again we must ascertain how many figures the product should have.

Number of Figures in a Product of two Factors.—The number of figures in a product of two factors is the sum of the numbers of figures in the two factors or one less.

1°. When using the slide rule, if the setting of 1 of the rule under the first factor read on the slide can be effected by drawing the slide to the left, as in the first example, there will be in the product as many figures less one as there are figures in the two factors. This is 4 in the first example and the answer as read is 8570.

2°. If the setting of 1 of the rule under the first factor read on the slide can only be effected by drawing the slide to the right, as in the second example, there will be in the product just as many figures as there are figures in the two factors. This is 5 in the second example, and the answer as read is 67800.

Results of Calculations only approximate.—Theoretically the results of calculations by the use of the slide rule are correct; practically they are only approximate. This is due to the impossibility of estimating intervals between the markings, but these results are approximate enough for all practical purposes. In the first example the error is only 2 units; in the second it is 8 units. Both errors are very small compared with the correct numbers, and they are much less than ½ of 1 per cent., which is generally considered as the limit of permissible error in engineering calculations.

Division.—Division is effected by reversing the operation of multiplication, or subtracting from the log. of the given product (dividend), the log. of the given factor (divisor), the difference being the log. of the unknown factor (quotient).

Example I. Divide 4401 by 27.

The runner may be useful in this as in most operations. Set the runner with its index over the divisor 27 read on the rule and pull the slide so as to bring the dividend 4401, read on the slide, opposite 27

or under the index. Now over 1 of the rule read the quotient 163 on the slide.

Example II. Divide 10290 by 294.

Set the runner with its index over divisor 294 read on the rule and pull the slide so as to bring the dividend 1029 (the last zero being left out) read on the slide opposite 294 or under the index. Over 1 of the rule is an empty space, so the quotient is read over 10 of the rule where 35 is found.

Number of Figures in the Quotient.—By reversing the rule given for multiplication, we see that

1° When the slide is drawn to the left, the difference plus one of the numbers of figures in the dividend and divisor is the number of figures of the quotient.

2°. When the slide is drawn to the right, the exact difference of the numbers of figures of the dividend and divisor is the number of figures of the quotient.

Upper Set.—Division of Scale A.—Scale **A** is also a logarithmic scale, but running from 1 to 100. Bearing in mind that log. 1=0, log. 10=1, log. 100=2, the point 10 will be located just in the middle of the scale. The distance 1-10 is divided by the same process that **D** was divided. The subdivisions, however, could not be as numerous for lack of space, but they are to be understood and mentally interpolated by the operator. The ten divisions of space 1-2 are only divided into 5; those between 2-3 and 3-4 are only divided into 2, and those between 4-5, 5-6, etc., to 10 are not subdivided at all.

From 10 to 100 the spaces 10-20, 20-30, 30-40, ... 90-100, are exactly of the same lengths as spaces 1-2, 2-3, 3-4, ... 9-10; the markings of the right-hand half are the same as those of the first or left-hand half, but the numbers are ten times greater. Scale **A** is in reality a double scale.

Division of Scale B.—Scale **B** of the slide is the exact duplication of Scale **A** of the rule, so that, when the slide is closed, the markings on the two scales appear to form one scale only, corresponding as they do throughout the length of the instrument.

Uses of Scales A and B.—They may be used for multiplications and divisions as Scales **C** and **D**, and in fact they are more generally used for that purpose on account of their greater range.

Use of the Two Sets Combined.—The upper set being a double scale its markings are twice closer than those of the lower set while the corresponding numbers are the squares of those in the lower set.

This is evident from the fact that $\log . a^2 = 2 \log . a$; hence .

Square of a Number.—The square of any number in the lower set is found opposite in the upper set, and conversely.

Square Root of a Number.—The square root of any number in the upper set is found opposite in the lower set.

The index of the runner establishes clearly the coincidence.

Cube of a Number.—Let 242 be a number the cube of which is desired.

Bring the index of the runner over 242 on Scale **D**. Pull the slide until 1 of Scale **B** is under the index.

and there corresponds to $\overline{242}^2$ on scale A. Move the runner to 242 on scale B and take the corresponding reading on scale A; it is 1417 and we have $\overline{242}^3 = 14170000$.

Cube Root of a Number.—Suppose we want the cube root of 42530000. Reverse the operation; bring the index of runner over 4253

on Scale **A.** Pull the slide until the index and 1 of Scale **C** are both covering the same reading, the first on Scale **B** and the second on Scale **D**; this reading, 349, is the cube root of 42530000.

Trigonometrical Scales.—On the back of the runner are scales marked **S** on the top and **T** on the bottom one. These may be used in conjunction with Scales **A** and **D** of the rule by reversing the slide.

Scale of Sines.—The scale on the back of the slide marked **S** is a scale of sines, and is used in conjunction with Scale **A.** To obtain, for instance, $x=672.5$ sin. 16° 45' pull the slide (after reversing it in order to use the **S** scale) to the left until its right-hand line is opposite 672.5; move the runner until the index covers the angle 16° 45' on the **S** scale when the number 194 opposite that angle and on Scale **A** will be the value of **x**.

Scale of Cosines.—To obtain $y=672.5$ cos. 16° 45'.

Write $y=672.5$ sin. 73° 15' which solve as before.

73° 15'=90°—16° 45' and sin. 73" 15'=cos. 16° 45'.

After calculating $x=672.5$ sin. 16" 45', the rule is all set for calculating $y=672.5$ sin. 73" 15'; simply move the runner until it covers angle 73° 15', when 644 will be read on Scale **A.**

Scale of Tangents.—The scale on the back of the slide marked **T** is a scale of tangents; it runs from 0 to 45° only; the tangent for instance of 62° being the cotangent of 28°

and $tg.\,62° = cotg.\,28° = \dfrac{1}{tg.\,28°}$ so that, for instance,

$$z = 138\ tg.\,62° = \frac{138}{tg.\,28°}.$$

will be obtained by moving the runner till the index covers 138 on Scale **D,** pulling the slide until the angle 28° of the **T** scale is under the index; the reading on **D** opposite the right (or 45°) end of the **T** scale will be the value of **z**. This is 259.4.

Scale of Logarithms.—In the center of the back of the slide and lengthwise, between the logarithmic scales **S** and **T** is another scale marked **L.** It is a scale of equal parts and runs from 0 to 10, being

subdivided into 10 equal parts and each subdivision is in turn subdivided into 5 equal parts. This scale is a Logarithmic scale and is used conjointly with Scales **C** and **D.**

Find the Log. of a Number.—The slide should not be reversed. Bring the left index of Scale **C** over the number of which the log. is desired; then without disturbing this coincidence turn the scale over and read the divisions of Scale **L** opposite the reference mark in the notch of the rule. That reading is the mantissa of the log. required, and its index is known by the number less one of figures in the integral part of the given number.

Advice.—Let the student buy a Slide Rule and read carefully the book of instructions which accompanies it, work out the examples given, and practice in order to attain a free and rapid use of the instrument.

SURVEYING.

Composition of a Transit Party.—Besides the chief of party, transitman, two chainmen, axeman, one or two flagmen, sometimes a stakeman.

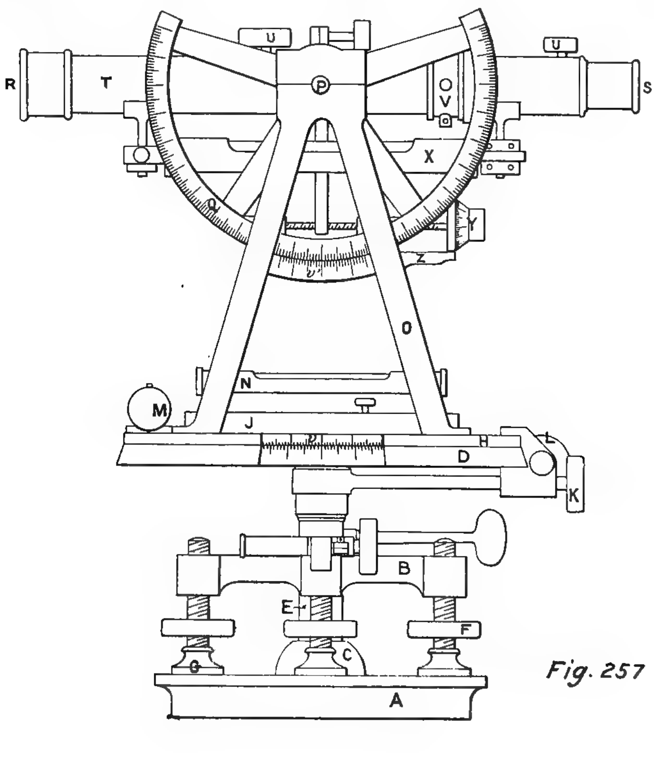


Fig. 257

Object of the Transit Work.—1°. To run lines on the ground upon which to base levels, cross-sections and topography for the intelligent study of a portion of country in which some work is to be built, so as to have the same erected under the most advantageous and economical conditions.

2° To finally lay on the ground the lines (or location) on which the work is to be built, as determined in the office of the engineer.

3°. To ascertain what properties are to be condemned and to survey them.

4° To give points and lines to the contractor before and during construction.

5°. To survey the progress of the work.

The Transit (Fig. 257).—Instrument to measure angles (a goniometer), principally

1°. **Horizontal Angles.**—Or angles with sides lying in a horizontal plane, and

2° **Vertical Angles**—Or angles with sides lying in a vertical plane and one of which is horizontal.

Description of the Transit.—Parallel Plates. There are two parallel plates, **A** and **B**, as in the engineer's level.

Lower Plate A.—The lower plate is formed with two parts. The outside part is a flat ring and is screwed to the tripod head. The inside part is another flat ring, of a diameter larger than the opening in the outside part, and it has a central dome **C** perforated on the top. The inside part is movable and rests on the underside of the outside part.

Upper Plate B.—The upper plate is generally made in the form of a central nut, with four arms at right angles (or three arms at angles of 120°). The upper plate carries an inverted conical shell the lower portion of which passes through the perforation in the dome of the inside part of the lower plate where it expands into a spherical shape and thus forms a **ball joint** with the lower plate. This spherical member is perforated in the center to allow the passage of a plumb-bob string.

Foot Screws.—The two plates are connected by four (sometimes three only) foot screws **F** for the two following purposes:

1°. The foot screws clamp the lower and upper plates, making them solidary with each other and with the inverted shell.

2°. They serve to level up the instrument.

The screws pass through the ends of the arms of the upper plate and are surmounted by dust caps; their feet fit into small cups **G** simply resting on the top surface of the lower plate, this to avoid wear.

Shifting Center.—As these cups may be moved as well as the central part of the lower plate (after slightly loosening the foot screws),

a slight motion may be given to the instrument for better setting it over a given point of the ground. This arrangement is called a **shifting center.**

Outer Spindle.—In the conical shell attached to the upper plate a second conical shell fits and may revolve. It is the **outer spindle** and carries projections to form attachments with other parts of the transit.

Divided Limb.—The upper portion of the outer spindle terminates in a horizontal (it is intended to be so) disc of plate **D** the limb of which is divided into 360° sub-divided into half, or third, or quarter degrees. The degrees are numbered every ten, either from 0° to 360° or from 0° to 180° either way; the degree-marks are a little longer than the sub-divisions, and every fifth degree has a mark a little longer yet.

Lower Motion.—The outer spindle and the divided limb are also called the **lower motion.**

Inner Spindle.—A solid inverted cone fits into the outer spindle and may revolve in it; it is the **inner spindle** and, like the outer one, it is provided with some projections for like purposes.

Vernier Plate.—The upper portion of the inner spindle projects further than the divided limb and carries also a horizontal disc **H** which moves in a plane parallel to the divided limb which it covers, except for two rectangular openings in opposite directions through which the divisions of the limb may be seen. These openings each carry a vernier **v** by means of which the sub-divisions of degrees are again divided. Some verniers read to one minute, others to one-half minute and some to 10 seconds.

To facilitate the reading of the vernier, the openings are sometimes fitted with reflector and magnifying glass.

Upper Motion.—The inner spindle and vernier plate **H** are also called the **upper motion.**

The vernier plate carries a compass-circle **J.**

Compass-Circle.—A circular box the bottom of which carries at its center a sharp pivot of very hard metal (very hard steel or iridium) upon which a magnetic needle about 5 inches long is balanced by an agate cap fixed in the middle of its length. The needle is very strongly magnetized; its north end is distinguished by color or ornamentation and its balance is regulated by a small coil of fine wire wound around one arm and which can be shifted. The limb formed by the edge of the sides of the box is divided into 360° with half degrees shown; they are numbered from two zeros marked at the ends of a diameter to 90° right and left. The bottom of the box is marked with two rectangular diameters corresponding to the graduations 0 and 90° of the vernier and two other diameters at 45° to the first. The forward end of the diameter marked 0 is designated by the letter **N** and the rear end by the letter **S**; they meaning **North** and **South.** The ends of the transverse diameter marked 90° is designated by the letters **E** on the left and **W** on the right; they meaning **East** and **West.** The reason for this reverse arrangement from the true direction of the cardinal points in which the east is to the right and the west to the left of

the north point will be explained in the use of the instrument. The compass-circle may be turned on its center so that, by setting it to the magnetic declination, the bearings as read shall be true bearings When set to an ordinary Surveyor's Compass, the forward end of the frame carries a vernier and a tangent screw to read fractions smaller than ½ degree.

Controlling Clamps.—1° A screw **K** permits to clamp the vernier plate **H** to the divided limb **D.**

2°. Another screw attached to the upper plate permits to clamp the divided limb to the upper plate.

Tangent Screws.—One tangent or slow motion screw **L** accompanies each clamp screw. It is used to complete the clamping at the exact spot where it is to be made.

Spirit Levels.—The spirit levels are attached to the vernier plate, one **M** in front (north point of the box) the other **N** on the side, thus forming an angle of 90°.

Standards.—The vernier plate carries two vertical standards or supports **O** shaped like an inverted **V** and placed one on each side, the center of their legs being just opposite the 90° graduation of the compass-box. They are made equal.

Horizontal Axis.—The standards carry between and on the top of them a movable horizontal axis **P.**

Vertical Circle.—To this axis is attached, by means of a clamp-screw, a vertical circle or arc **Q** divided like the horizontal divided circle, and which, in its vertical motion, just touches a circuler vernier **v'** carried by the left standard together with a slow-motion screw.

Telescope.—In the middle of the horizontal axis and perpendicular to it is solidarily attached a telescope **T** of a description similar to that of the engineer's level with objective **R** and ocular **S,** racks and pinions **U** for their motions, adjustable cross-hairs ring **V** with ordinary and stadia hairs.

Telescope Level.—An adjustable spirit level **X** is also attached to the under part of the telescope as in the engineer's level. This permits of the transit being also used as a leveling instrument if necessary.

Motions of the Telescope.—Being solidary with the vernier plate, the telescope has the full range of the horizon and can measure any horizontal angle. Being on a horizontal axis endowed of free motion, it may move in a vertical plane carrying with it the vertical arc, and it can therefore measure vertical angles.

In the horizontal motion, the vertical cross-hair of the telescope is brought exactly on the point sighted by means of the slow-motion screw **L** carried by the vernier-plate **H.**

In the vertical motion, the horizontal cross-hair of the telescope is brought exactly on the point sighted by means of the slow-motion screw carried on the inside of the left hand support and moving the vertical circle.

Principal Lines of a Transit.—Vertical axis, horizontal axis, plate-level line, attached-level line, line of collimation.

Vertical Axis.—The vertical which passes through the center of the spindle **E.**

Horizontal Axis.—The axis **P** of the shaft by which the telescope rests on the supports. It must be made horizontal.

Plate-Level Line.—The top or bottom lines of the plate-level case **N.** These are level when the bubble is centered.

Attached-Level Line.—The level line of the bubble level **X** attached to the telescope. Employed only when the instrument is used as an engineer's level.

Line of Collimation.—Line determined by the optical center of the objective and the intersection of the cross-hairs.

Relations between the Lines of a Transit.—1°. The plate-levels must be perpendicular to the vertical axis. 2°. The line of collimation must be perpendicular to the horizontal axis; 3° The horizontal axis must be perpendicular to the vertical axis; 4° The attached level line and the line of collimation must be parallel; 5° The zero of the vertical circle must correspond to the zero of the vernier when the telescope is horizontal.

Adjustments of the Transit.—1°. Making the axis of the spindle vertical, and the planes of the plates perpendicular to it; 2° Collimating the telescope; 3° Adjusting the horizontal axis so that the line of collimation will move in a vertical plane; 4°. Making the line of collimation horizontal when the bubble of the attached level is centered; 5°. Making the vernier of the vertical circle read zero when the bubble of the attached level is centered.

1st Adjustment—Making the axis of the spindle vertical, and the planes of the plates perpendicular to it. Set one level over a pair of plate screws; the other level will thus be set over the other pair. Level up both levels by means of the plate screws. Turn the vernier-plate around by ½ a revolution; if the bubbles remains centered during the motion, the vernier-plate is in adjustment; if they have moved, bring them halfway back by means of the adjusting screws and the rest of the way by means of the foot screws. Repeat.

Verify if the bubbles remain centered when revolving the divided circle; if not, the plates are not parallel and the transit must be sent to the maker for repairs.

2nd Adjustment—Collimating the telescope. Set up transit in center of open and practically level ground. Carefully level the instrument. Drive a stake or better a chaining pin some 200 or 300 feet away; chain the distance. Take a sight on that point and clamp the plates. Revolve the telescope vertically (in altitude) by half a revolution, thus reversing the line of sight. Chain in that new direction the same distance as first measured and set a pin. Unclamp and revolve the vernier-plate by half a horizontal revolution. Sight again at the first point and clamp. Again revolve the telescope vertically by half a revolution. If the line of sight falls on the pin, the telescope is

collimated; if not, set a new pin on the last sight and at the same distance as before and another pin at ¼ the distance between the first pin and the second. Move the vertical cross-hair, by means of the capstan headed screw and an adjusting pin, until the intersection of the cross-hairs covers the last pin set. Repeat.

3rd Adjustment.—Adjusting the horizontal axis so that the line of collimation will move in a vertical plane. Level up carefully and sight on a high, well defined point, as a corner of a chimney, and clamp. Slowly move the telescope down till it sights the ground and drive a pin there. Unclamp; revolve the vernier-plate half a revolution and revolve the telescope vertically half a revolution, reversing the line of sight. Look again at the high point and clamp. Slowly move telescope down till it sights the ground. If the intersection of the cross-hairs covers the pin, the horizontal axis is in adjustment; if not, correct halfway by means of a support adjusting screw and the rest of the way by means of the plate screws. Repeat.

4th Adjustment. Making the line of collimation horizontal when the bubble of the attached level is centered. Drive two stakes 300 or 400 feet apart and set up the instrument about halfway between. Level up and take readings on the rod held successively on the two stakes; the difference of the readings is the difference of elevation of the stakes. Next set the transit over one of the stakes, level up and take a reading of the rod held on the other stake; measure the height of the instrument; the difference between this and the last rod reading should equal the difference of elevation as previously determined; if it does not, correct half the error by means of the attached level adjusting screw. Repeat.

5th Adjustment.—Making the vernier of the vertical circle read zero when the bubble of the attached level is centered. Level up the instrument. Sight on a well defined point and take note of the reading on the vertical circle. Turn vernier-plate half a revolution and the telescope also vertically half a revolution and again sight on the same point. Read again the vertical circle. Half the difference of the two readings is the index-error which may be corrected by moving either the vernier or the vertical circle; or the error may be noted and applied as a correction to all measurements of vertical angles.

Adjustments of the Compass.—1st, Straighten the needle; 2d, Place pivot in center of the plate.

1st Adjustment.—Straighten the needle. Examine if the ends of the needle set on opposite divisions; if not, fix pivot so that they will. Revolve the box by half a revolution; if the needle does not set on opposite divisions, bend both ends by half the difference.

2nd Adjustment.—Place pivot in center of the plate. When sure that needle is straight, move pivot till the needle sets on opposite divisions at points such as 0°, 45° and 90°.

Attachments.—Gradienter. Some transits carry, attached to the horizontal axis by means of a clamp-screw and inside of the right-hand support, an attachment called gradienter, **Y** for the determination of grades and distances. It consists in an arm of the shape of an inverted **Y** with curved branches, to the extremities of which are

attached an encased spiral spring and a nut through which moves a micrometer-screw with a graduated head revolving in front of a scaled index Z also carried by the arm. The ends of the screw and of the spring abut on opposite sides of a shoulder carried by the right hand support. The head is divided into tenths and hundredths, and every revolution moves it in front of the scale by one division of the latter; so that the scale gives the number of turns of the screw and the graduated head the fraction of a turn.

Grading.—If one revolution of the screw moves the cross-hair a space of one foot on a rod held 100 feet away, the slope indicated by the telescope is 1 per cent. To establish a grade, level up the telescope, clamp the arm of the gradienter and turn the micrometer-screw as many divisions as required in the grade. For instance: 2.35. Move the head two complete turns plus 35 subdivisions. Measure height of telescope from ground; set rod at that height; then hold the rod at any point of line, raising it till the target is bisected by the cross-hairs; the foot of the rod will then be on the grade.

Measurement of Distances. 1° On level ground. On ground nearly level, set up rod at station, level up telescope and read rod; turn gradienter-screw one revolution and again read rod. Difference of the two readings multiplied by 100 equals distance from station.

generally $d = \frac{100R}{r}$ in which $\begin{cases} d = \text{distance ;} \\ r = \text{rod reading at 100 ft. for 1 revolution;} \\ R = \text{" " " distance } d \text{ " ".} \end{cases}$

2°. **On a Slope.** Measure the vertical angle. Multiply the reading on the rod by 100 and subtract the product of the reading by the number in the accompanying table which is opposite the elevation (vertical) angle.

Gradienter Table.

A	B	A	B
1°	.1	11°	3.8
2	.2	12	4.5
3	.3	13	5.3
4	.5	14	6.1
5	.8	15	7.
6	1.2	16	7.9
7	1.6	17	8.8
8	2.1	18	9.8
9	2.6	19	10.9
10°	3.2	20°	12.

A Vertical Angle ;
B $\sin a\,(100 \sin a + \cos a)$

The formula is $d = 100R - R \sin a (100 \sin a + \cos a)$

in which $\begin{cases} d = \text{distance ;} \\ R = \text{rod reading at distance } d \text{ for 1 revolution ;} \\ a = \text{V}^{al}\text{ angle made by} \begin{cases} \text{bottom of rod when in elevation;} \\ \text{top " " " " depression.} \end{cases} \end{cases}$

Example: Elev. angle 7° ; reading 3.11 ft.

Calculation.

3.11 × 100	=	311.0
− 3.11 × 1.6 (given by Table	=	5.0
Horizontal distance d	=	306.0 ft.

The Stadia or Telemeter.—On the cross-hairs ring of the telescope are stretched two horizontal cross-hairs one above and the other below the central horizontal cross-hair, and at equal distances from it.

Formulas.

1° *On level ground* · $d = kR + (l+f)$

in which
- d = distance ;
- f = focal distance of objective ;
- k a constant $= \frac{f}{w}$; w = distance apart of stadia cross-hairs
- l = distance from objective to horizontal axis of telescope ;
- R rod reading at distance d.

f is given by the formula $f = \dfrac{(100-l)q}{100+(q-l)}$

in which q = distance from cross-hairs to objective.

2° *On a slope.* *(a) Formula for distance:* $d = kR\cos^2 a + (l+f)\cos a$

or $d = kR - kR\sin^2 a + (l+f)\cos a$ (1)

(b) Formula for diff. of elevation: $e = kR\sin a\cos a + (l+f)\sin a$

or $e = \frac{kR}{2}\sin 2a + (l+f)\sin a$ (2)

in which
- a – vertical angle made by line of central cross-hair ;
- e = diff. of elev. between transit station and rod station

Stadia Tables are calculated from equations (1) and (2).

Direct reading Rod. 1°. **Setting the stadia hairs to a given rod.** The cross-hairs may be set at such a distance apart as to intercept 1 foot on the rod held at the distance (100+l+f) feet, so that a rod-reading, for instance, of 3.84 feet will give directly the distance (384+l+f) feet.

2°. **Marking a rod to agree with stadia hairs.** Or again, the cross hairs being already in position; set up a blank rod (without markings) at an even number of 100 feet plus l+f, say (400+l+f) feet; note on the rod the intercept given by the stadia cross-hairs; divide that space into four equal principal parts which you repeat along the rod as many times as its length will allow; divide each space into 10 and each tenth into 10, and the rod is ready to be used as in the first case; an intercept on the rod of 5.61 units will indicate a distance of (561+l+f) feet.

Use of the Transit. The transit is used for: 1st,, Producing lines; 2d, Measuring horizontal angles; 3d, Measuring vertical angles; 4th, Grading; 5th, Leveling.

Producing a Line. Method of double sights.—Suppose line determined by tacks in stakes **A** and **B**, (Fig. 258), **A** being the origin of the line. Set transit over **B**; plumb it and level it carefully. Take a sight on **A** and clamp, revolve telescope vertically (or in attitude) thus reversing the line of sight. Drive a stake **C** about 300 feet further and in the new line of sight and make a light mark on it at the spot sighted. Unclamp. Revolve telescope horizontally (or in azimuth) and again sight **A**. Clamp. Revolve telescope in altitude a second time; sight **C** and make a light second mark on it. A nail driven half-way between the two marks is on the prolongation of **AB**.

Continue in same manner. This is the method of **double sights** and should always be used on important work.

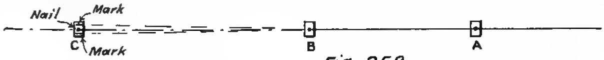


Fig. 258

The more common method, followed in preliminary work, consists in back sighting **A** with the vernier at 180°, revolving the telescope in altitude and locating **C** with the same vernier reading zero.

Running a Line between two Points invisible from each other (Fig. 259). To run a line between **A** and **B** when **B** cannot be seen from **A**, as in a wooded country, run **AC** in the supposed direction and measure it to a perpendicular **BC** from **B**.

Then $\tan A = \frac{m}{l}$ can be calculated by trigonometrical tables and angle A turned to the right or left (as in the fig.) from AC. If no tables are handy, use the following formula.

$$A = \frac{m}{l} \times 57.3$$

which is quite correct for small angles.

(57.3 or more accurately 57.29576 = 57° 17′ 44.75″ is the number of degrees of an arc the length of which equals the radius.)

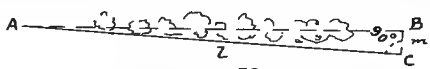


Fig. 259

Measuring horizontal Angles. The sides are determined by stakes or rods set vertical. Set the transit plumb over the vertex and level up. Set vernier to zero and clamp vernier plate; sight the left-hand point, clamp divided circle and complete the sighting by means of the tangent screw. Unclamp vernier plate, sight the right-hand point completing the sighting with the tangent screw. Read the angle. The angle may be measured again and in like manner from right to left; the mean of the two readings is the correct measure of the angle. The readings should not differ more than one minute from each other.

Repeated angles. After measuring the angle as explained above, note the readings of both verniers. Unclamp divided circle, sight at left-hand point, unclamp vernier plate, sight at right-hand point. Repeat this operation. After the last take the mean of the vernier readings and divide by number of operations. Compare result with first measurement. Another set of repetitions may be made from right to left. Then the half-sum of the two averages will be the correct measure of the angle.

Cumulative Angles. Instead of measuring separately adjacent angles (angles having a common side and common vertex), it is better practice to measure them all from the same side and to the right, spanning the horizon from 0° to 360°. Thus adjacent angles the measures of which

are separately	would be measured as
26° 18′ 10″	26° 18′ 10″
36° 03′ 30″	62° 21′ 40″
8° 47′ 20″	71° 09′ 00″
45° 33′ 50″	116° 42′ 50″
87° 21′ 20″	204° 04′ 10″

If all the angles measured are referred to a **N-S** line (whether true, magnetic or assumed) as a common origin, they are azimuth angles.

Azimuth Angle.—An azimuth angle therefore is the horizontal angle made by a line with a **N-S** line, and is measured from the **N** as zero and to the right. Such an angle may have any value between 0° and 360°.

Deflection Angle.—The deflection angle of a line is the horizontal angle which that line makes with the prolongation of the last line run; it is measured from that prolongation as zero and is recorded as to the right (**R**) or to the left (**L**).

Included Angles.—An included angle is the horizontal angle made by a line with the back sight **(B. S.)** on the last line run; it is measured from that back sight as zero and is recorded as to the **R** or **L.** So that an included angle to the **L** of 76° 12′ 20″ for instance, would correspond to a deflection angle to the **R** of 103° 47′ 40″, the two being supplemental.

Vertical Angle.—Angle made by a line of sight, in the vertical plane drawn through it, with the horizontal drawn in that plane at the center of the transit.

Angle of Elevation. A vertical angle in which the line of sight goes up from the instrument.

Angle of Depression.—A vertical angle in which the line of sight goes down from the instrument.

Measuring a vertical Angle.—Set up transit and level the vernier-plate. Level up the attached level and see whether vernier reads zero. Turn telescope in azimuth toward point; raise or lower it in altitude in the direction and complete accurate sighting by tangent screw. Note the reading with + for elevation angle and — for depression angle, correcting any vernier error.

Grading.—A grade line may be shot in as with the level; or rod readings may be calculated for distances with known elevations.

Leveling.—The transit may be used as a level. For that purpose, level up vernier plate and clamp; center bubble of attached level by means of the vertical arc tangent screw and the instrument is ready.

Running a Traverse.—Consists in measuring the elements of a polygonal line (or part of it) on the ground. Two measurements are required for each side: 1st, the bearing and 2d, the distance. The chainmen measure the distance.

Bearing.—Deflection (less than 90°) of a line from the **N** or the **S** toward the **E** or the **W.**

Notation of Bearings.—To indicate or record a bearing, write first the letter **N** if the deflection is from the north, or the letter **S** if the deflection is from the south; follow the letter **N** (or **S**) with the number of degrees, minutes and seconds of the deflection angle and conclude by writing the letter **E** if the deflection is towards the east, or **W** if the deflection is towards the west.

Magnetic Bearings.—When the bearings are based on a magnetic **N-S** line.

True Bearings.—When the bearings are based on a true **N-S** line.

Why the E is marked on the left and the W on the right of the N-S Line in the Compass. To take the bearing of a line lying **N-W,** for instance, set the verniers at zero with the compass needle at **N**; clamp the divided circle. Move the telescope to the left (towards the **W** of the horizon) and to the point. The zero of the vernier is carried along to the left in that motion, whereas the magnetic needle remains stationary and to the right of the telescope. The angle is read on the limb between the zero on the left and the needle on the right. A contrary apparent motion of the needle would take place if the bearing of a **N-E** line was being taken; the zero would move to the right and the needle remain to the left. In all cases the reading on the limb is contrary to the bearing and this is the reason for reversing the letters **E** and **W** in the compass.

Determination of a true Meridian, or **true N-S Line** (Fig. 260).—**1st. By means of Polaris.** Polaris, or the north star, and Alioth, in the constellation of the Great Bear (or Dipper) and the third star of the handle, the one next to the quadrilateral, very nearly determine a true meridian when covered by a plumb line. Ascertain the time when both stars are thus covered and take a sight at Polaris 25 minutes later for a true meridian.

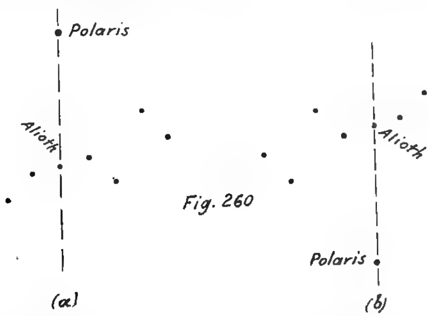


Fig. 260

2d. **By Shadows** (Fig. 261). Select a level piece of ground and drive in it a pole with its upper end sharpened; it is better that it slants slightly towards the south. Drop a plumb-line to the ground

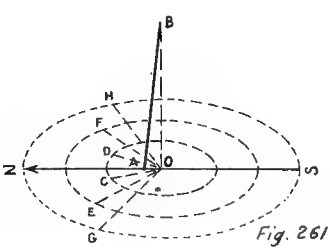


Fig. 261

from the top of the pole, and from the foot of that vertical describe several circles. Observe when the shadow of the point is on each circle and mark the point both before and after noon. Finally bisect the arcs so marked and join the points to the center. Take the average for a true meridian.

In reality the result is absolutely correct only if the observations are made at the solstices, June 21st or December 22d.

Closed Traverse.—The survey of a polygonal line which ties to itself.

Tie.—Line run for checking or calculating purposes between a point of a line and another point of the same or another line. These points are generally angle points.

It is important to tie together lines which come in proximity of each other and to carefully determine the intersections of lines which cross each other, measuring the angles which they form.

It is more important to tie a traverse to two or more triangulation points.

Distribution of Errors.—The tying of traverses to Trig. Points is the best method of distributing errors.

Three kinds of Errors.

1°, **Distance errors** or errors in chaining; **2 , Azimuth errors** or errors in deflection angles; 3°, **Instrumental errors.**

Whence Azimuth Errors arise. Azimuth errors are most to be feared. They come from 1°, The graduation of the instrument; 2°, Shortness of sights; 3°, Thickness of poles used; 4°, Care in centering over station.

Care to be dependent on Object of the Survey. The same care and attention should not be given to all classes of transit work.

Preliminary Survey.—Is intended to rapidly secure necessary and sufficient information as to the character of the topography as will enable the draughtsman to prepare a map the examination and study ot which will permit the engineer to select a location for the work intended. In a preliminary survey it is not required to have the angles exact to a fraction of a minute, nor the distances measured to .01 ft. The primary consideration is rapidity consistent with a fair degree of accuracy. The transitman may also run his lines on more open ground to save time.

Location Survey.—Is intended to fix on the ground the final lines agreed upon in the office by the engineer; these lines may be straight or curves and they are established with more care than was allowable on preliminary work.

Locating a Curve. (Fig. 262).—**P. I.** is the point of intersection of the tangents; **P. C.** is the point of curve, or where the curve begins;

P. T. is the point of tangent, or where the curve ends, and the tangent begins.

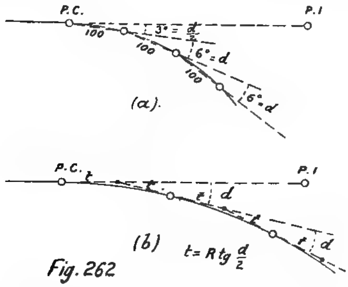


Fig. 262

P. C. C. Point of compound Curve.—Where a second curve of same direction but of different radius is tangent to a first curve.

P. R. C. Point of reverse Curve.—Where two tangent curves reverse curvature.

A curve is located by means of 100 ft. distances (or less) measured on certain deflections.

Degree of a Curve.—The degree of a curve is generally known by the number of degrees which subtend a cord 100 ft. long. Ex.: A 4° curve is one in which an arc (or central angle) of 4° subtends a cord of 100 ft.

Railroad Curves.—Curves given by their degree of curvature are usually known as **Railroad Curves.**

Long Tangent of a Curve. The distance between the **P. I.** and the **P. C.** (or **P. T.**)

That distance is given by formula $T = R \tan. \frac{D}{2}$

in which T = long tangent ; R = radius of curve ; D = deflection angle of the alignments P.C.-P.I. and P.I.-P.T

External Secant.—Distance between the **P. I.** and the curve; it is given by either of the formulas:

$$S = -R + \sqrt{R^2 + T^2} \quad \text{or} \quad S = R\left(\sqrt{1 + \tan^2 \frac{D}{2}} - 1\right)$$

in which S - external secant.

1st. **Location by equal Cords** (Fig. 262).—(a) Set up transit at **P. C.**; sight on **P. I.** and deflect right or left, according as the curve is to the **R** or to the **L** by half the degree of the curve (if a 6° curve, for instance, the first deflection is to be 3°); chain 100 feet on that deflection and drive a stake. Set up transit on stake, back sight on **P. C.** with vernier set at 180°; reverse telescope in altitude for forward direction and deflect to the **R** or **L,** as before, by the degree of the curve (if a 6° curve, this and subsequent deflections are to be 6°); chain 100 feet and drive a stake, and so continue.

2d. **Location by Tangents (Fig. 262).** (b) Chain from **P. C.** toward **P. I.** a distance equal to

$$t = R \tan . \frac{d}{2} \text{ ; in which } \begin{cases} t & \text{a tangent ;} \\ R = & \text{radius of curve ;} \\ d = & \text{degree of } \quad " \end{cases}$$

and drive a stake. Set transit up on stake; foresight on **P. I.** and deflect **d.** On this deflection, chain **t** twice in succession, driving a stake at each end: the first point belongs to the curve, the second is for setting up transit to continue the operation in like manner.

Advices.—1°. Don't let the plate screws become too tight or cramped; they should be snug.

2°. Let the lower parallel plate be naturally as near horizontal as possible.

3° Better back sight with telescope reversed in altitude.

4°. Take magnetic bearings at both ends of all lines to use as checks.

5°. Produce lines by double sightings.

6°. Always check on **B. S.**

7°. Reference all angle plugs, also the **P. I., P. C., P. T., P. C. C.** and **P. R. C.** of curves by 4 stakes forming two intersecting lines and make careful notes of such references.

8°. Let your notes be clear as to meaning; remember they have to be interpreted by the draughtsman in the right way only.

Property Surveys.—If possible have neighboring owners agree as to the common lines and corners. Consult deeds. Find monuments by running old lines and digging at the places where they should be. Note the station where each property line intersects the location line; take deflections to 1-minute of all such lines and chain them to two or three hundred feet from the location, or further if there is a possibility of the location being subsequently changed. Make a survey showing the buildings, outhouses, fences, gardens, orchards, and all that which may help in determining damages. Sometimes it may be necessary to survey the whole farm.

Giving Lines and Points to the Contractors.—When a structure has been determined upon by the engineer, it is located in the field by establishing some principal line or base; as an axis, the face or the center of a wall, the center of a highway, etc. But as this line is liable to be disturbed during construction, one or more parallels to it are carefully run 100, 200 feet or more (or less) to the right and left. These reference bases are known as (for instance) **Line C,** 100′ **R** (or **L**); **Line C,** 200′ **R** (or **L**). They are stationed as the line itself, with like station numbers on a perpendicular.

Lines or points in the structure will be measured from the reference base and marked by stakes.

Points are generally designated by the station and the distance **R** or **L** from the base; as 3+84, 61.3 **L**; 0+31.5, 164.85 **R**; that is to say, by rectangular co-ordinates.

Progress Survey.—The points reached by the excavation, or by the structure as it progresses, are surveyed generally every month. These points are determined by the station and the distance **R** or **L** and are marked with a cross.

x or an arrow-head (*crow-foot*) → to show the rodman

where to set the rod. In excavation these points are marked with stakes. Careful notes are made of these points, clear enough to be understood by the leveler.

Forms or Transit Notes.—The left-hand page is reserved for measurements and calculations of the line. The right-hand page for notes, sketches of features and measurements of the topography.

Top Notes.—On top of each page write the number or name of the line surveyed, the names of the transitman and chainman, the date and the weather.

Line Notes.—The left-hand page is usually divided into six vertical columns in which are written, from left to right:

Stations.—In the first column, the stations running from the bottom of the page up, so as to have on the right-hand page, a correct view of the topography in case a sketch should be needed. That column may be headed: **Sta.**

Distances.—In the second column, the distances between plugs or angle points are entered. That column may be headed: **Dist.**

Deflections.—In the third column, the deflection **R** or **L.** Sometimes two columns are reserved: one for **R** deflections, the other for **L** deflections. That column may be headed: **Defl.**

Double Deflections.—In the fourth column, double deflections (obtained by repetition). That column may be headed: **D. Defl.**

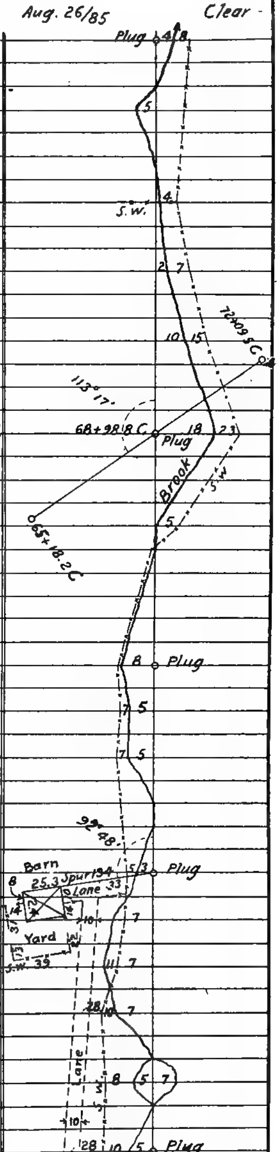
Magnetic Courses.—In the fifth column, the magnetic bearings of both **B. S.** and **F. S.** to be used as a check. That column may be headed: **Mag. C.**

Calculated Courses.—In the sixth column, the calculated bearings (or courses) calculated from the deflections applied to a true or a reference magnetic meridian. That column may be headed: **C. C.**

Topography notes.—Lengthwise and through the center of the right-hand page is a line which represents the transit line as run (except as to angles which are supposed to be extended). On either side of this line neatly sketch the features of the topography which are judged worth plotting on the map, as fences, roads, houses, streams, etc.; also tie-lines and intersections of lines with distances, generally cumulative, clearly written with a N°-4 pencil pressed hard in the paper, opposite the station entered on the left-hand page. Calculations, such as the average of repeated deflections are also entered on that page.

Line B^{12}. A Transitman; B, C } Chainmen. Aug. 26/85. Clear.

10+19.6					
+07					
+02					
10					
+99					
+92					
+57					
9					
8+56.8	162.8	38°50'R.	77°40'	N.27½ W.	N.27°52'25"W.
=68+98.8C				S.66¾ E.	
+26					
+06					
8					
7+76.4	80.4	28°22'50"L	50°46'	N.66¾W	N.66°42'25"W.
				S.41½ E	
+48					
7					
+73					
+60					
6+54.9	121.5	3°18'R.	6°36'	N.41 W.	N.41°19'35"W.
				S.44¾ E.	
+51					
+11					
6					
+88					
+75					
+68					
+57					
5+50.7 B^{12}	104.2	23°16'30"L	46°33'	N.44½W S.23½ E.	N.44°37'35"W



Progress Profile of a Wall (Fig. 263).—On a cross-section paper plot the profile of the ground and the top of the wall. Plot on this sheet every month the average excavation and later the average work done on the center line. This is obtained by averaging the depth of excavation or the height of masonry on cross sections every 10, 15 or 20 feet. This average line will form polygons which should be given monthly numbers in rotation to correspond with the numbers of the contractor's estimates. Earth excavation may be shown by red lines; rock excavation by blue lines and masonry by black lines, the figures to be of the same color.

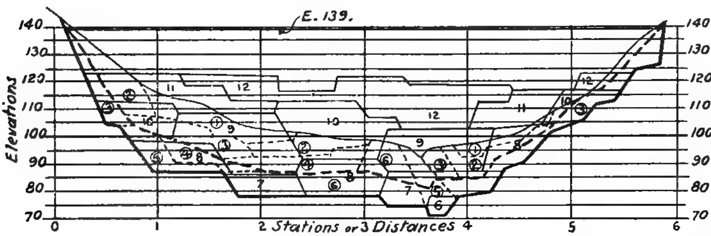


Fig. 263

This profile shows that the work of excavation lasted six months and that at the end of the first year the masonry had attained generally the elevation 120.

Progress of Expenses (Fig. 264).—Lay off, on the horizontal lines of a sheet of cross section paper, the number of years required to complete the contract, each year divided into 12 equal parts (months). Mark the starting point zero. On the vertical line corresponding to the last estimated contract (time limit), lay off to any scale the total amount of the contract as calculated from the estimated quantities and the contract prices. Connect the point thus obtained with the zero. This oblique line is the rate of expense which the added successive estimates should follow in order to complete the work in the time specified. Plot every month on that profile the total amount of this and the previous estimates connecting the points to the previous one. This second line will show whether there is rushing or retarding in the work as it will lie above or below the average line first determined.

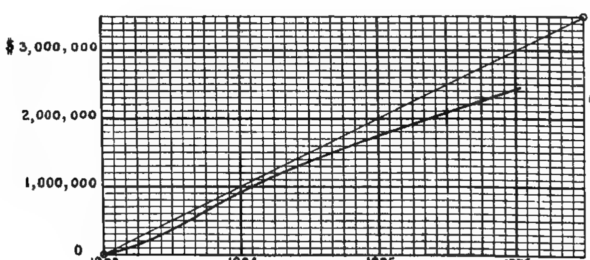


Fig. 264

Quantities Profiles.—Similar profiles may be prepared with regard to the several classifications of a contract such as: Earth excavation, rock excavation, rubble masonry, paving, rip rap, and so on, and be kept up every month, showing graphically the deviations of the actual work from the expected rate of progress.

Office Work.—Plots his field notes and draws the maps required. He also assists in making the monthly estimates.

He checks drawings and calculations.

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The Assistant Engineer

BY

Prof. JEAN P. GENTHON
Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK VI

THE INSPECTOR

WRITTEN FOR

THE CHIEF

Journal of the Civil Service

PUBLISHED BY
THE CHIEF PUBLISHING CO.
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1

THE INSPECTOR

Book VI

PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with the most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3°. The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examination in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON.

July 1, 1907. New York.

PRESS OF
THE CHIEF
NEW YORK

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1° Aptitude for mathematics.

2° Habit of observation.

3° Good memory.

4° System.

5° Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION.—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials or new appliances and processes are used for old or new purposes. Notice the several kinds of labor, and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory, which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results; principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books, always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figures and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations, in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around you will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public works have a Chief Engineer who prepares the work and directs its

construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

Chief Engineer.			Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist
			Inspectors.	
	Division Engineers.		Transitmen.	Chainmen. Axemen.
		Assistant Engineers.	Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen.

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.
Boston, Mass.
Buffalo, N. Y.
Charleston, S. C.
Chattanooga, Tenn.
Chicago, Ill.
Cincinnati, Ohio.
Cleveland, Ohio.
Detroit, Mich.
Galveston, Tex.
Grand Rapids, Mich.
Jacksonville, Fla.
Little Rock, Ark.
Louisville, Ky.
Memphis, Tenn.
Milwaukee, Wis.
Mobile, Ala.
Nashville, Tenn.
New London, Conn.
New Orleans, La.
Newport, R. I.
New York, N. Y.
Norfolk, Va.
Philadelphia, Pa.
Pittsburg, Pa.
Portland, Me.
Portland, Oreg.
Rock Island, Ill.
St. Louis, Mo.
St. Paul, Minn.
San Francisco, Cal.
Savannah, Ga.
Seattle, Wash.
Sioux City, Iowa.
Tampa, Fla.
Vicksburg, Miss.
Washington, D. C.
Wheeling, W. Va.
Wilmington, Del.
Wilmington, N. C.
Yellowstone Park, Wyo.

where examinations may be taken except that of Assistant Engineer, which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above-named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken at

Albany.
Amsterdam.
Auburn.
Binghampton.
Buffalo.
Elmira.
Hornelsville.
Ithaca.
Jamestown.
Kingston.
Lockport.
Malone.
Newburg.
New York.
Ogdensburg.
Olean.
Plattsburg.
Poughkeepsie.
Rochester.
Utica.
Syracuse.
Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewer construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman; topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps.

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service, appear regularly in "THE CHIEF."

BOOK VI

THE INSPECTOR

Inspector.—He who is assigned to a particular job to inspect the materials entering into it with regard to their quality, their manufacture or preparation, and the manner they are laid in the work.

Who his Superiors are.—He usually reports to the Division Engineer.

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service.

The manual is silent.

N. Y. STATE CIVIL SERVICE.

Inspector of Buildings.

Salary, $5.00 per day, when employed.

EXAMINATION.

	Subjects.	Relative Weights.
1.	Practical questions relating to various branches of inspection and the interpretation of architectural drawings	6
2.	Experience and education	4
	Total	10

Inspector of Public Works.

Salary, $5.00 per day when employed.

EXAMINATION.

	Subjects.	Relative Weights.
1.	Education and experience	4
2.	Practical questions on materials, construction and inspection.	6
	Total	10

Specimen Questions.

1. What differences should be observed between concrete con-

struction for watertight retaining walls and for ordinary dry foundations?

2. Name all the ways you can suggest of slighting cement masonry.

3. What are the common defects of stone, and how discovered?

4. In what cases would you advise the use of grout rather than a full bed of mortar in building stone-work?

5. Give the essential points in a first-class job of brick-work in cement, including materials and workmanship.

6. Describe some simple expedients to cheapen quick-sand excavation.

7. Describe the construction of the cheapest safe coffer dam, subject to a head of 12 feet of water and extending between impervious banks 100 feet apart.

8. Describe circumstances under which wheelbarrow excavation is the most economical method of handling earth. Same for casting with shovels; handling with carts; handling with cars.

9. Suppose in excavating for the foundations of a bridge abutment of a lock-wall, that the rock which was thought to be practically level at a certain depth, is found to fall away rapidly over a part of the foundation; what would you require of the contractor?

10. Does it require more or less earth than that in a given cutting to make an embankment of the same size? How is this provided for in making the embankment?

11. In building a canal, suppose the bottom to be but little below the natural surface; state under this condition every precaution that should be taken or may be necessary to prevent seepage at the junction of the bank with the ground and to insure the stability of the bank.

12. Describe the materials best fitted for puddling, and how they should be used in building a bank to insure imperviousness to the passage of water.

13. What are shakes? checks? What is sapwood, and how told? What is meant by brash timber, and how told?

14. What kinds of timber last best in wet places?

15. Where round drift bolts are to be used, as in crib-work, etc., state the requirements to be observed in boring for, driving them, etc., to get the strongest hold in the timber.

16. State all the defects you would look for in piles to be used on an important work.

17. (a) How would you determine that a pile has been driven to a bearing? (b) How would this be affected by piles brooming at the top, and what is the remedy? (c) Which is the most effective method of driving, by frequent blows with small fall of ram, or slow blows with a greater fall?

18. How should iron-work be prepared for repainting and what materials and method of application will give the most permanent coating?

New York City Municipal Civil Service.

Classification.—Schedule C, Group 1.

Salary, $1,200 to $1,800.

EXAMINATION QUESTIONS.

CEMENT TESTER.

Arithmetic.

1. Multiply.
2. Subtract.
3. Divide.
4. Convide 17-64 to a decimal.
5. The radius of a circle is 3 feet. Find the circumference and area.

Technical.

1. State what you consider to be the best proportion of water to use in mixing cement for testing.

2. Describe fully and clearly the process of mixing the cement and of placing it in the moulds.

3. State just where and how the briquettes should be removed from the moulds, and what subsequent care should be taken of them.

4. State what appearance of the briquettes at any time after mixing would lead you to be suspicious of the quality of the cement.

5. Describe the correct way of making the tests so as to get uniform breaking results.

6. What records of tests would you keep?

7. (a) How would you test for initial set of a cement? (b) What would you call quick and what slow setting?

8. State as far as you know them the differences between Portland and Rosendale cements.

9. How do you test for fineness of cements?

10. How would you test the weight of cements so as to get uniform results?

11. How would you be able to tell whether a cement is "over limed?"

12. How should cement be stored?

13-14 (a) Why are some cements darker than others? (b) Does this have anything to do with strength?

15. (a) Suppose a cement to set properly in the air and after placing in water to crack more or less, what would it indicate? (b) May such cement in some cases be improved in any way after delivery from the factory, and if so, how?

16. Describe the manipulation of the testing machine, placing briquettes in same etc., to produce uniformity in testing.

17. (a) What is the difference in behavior of cements hardening in air or in water as to expansion or contraction? (b) Does the admixture of sand lessen or increase this action?

18. State what advantages are obtained by grinding cement very fine.

INSPECTOR OF SEWER CONSTRUCTION.

Arithmetic.

1. Add 7365867 — 2345678 — 9876543 — 2154672 — 7658899 — 5342544 — 2534353.

2. Subtract 320045 — 290987.

3. Multiply 78096 by 4097.

4. Divide 2601024 by 4278.

5. and 6. How many cubic yards of earth are required to fill a trench 104 feet long, 8 feet high, 6 feet broad at the bottom and 12 feet broad at the top?

Technical.

1. What are the essential requirements of a well constructed sewer?

2. Suppose the top of the grade stake set at one end of a 25 ft. length of sewer was 13'-3" above grade, and at the other end 11'-17" above grade, how would you fix your grade line?

3. Where are headers used in a circular brick sewer and why are they so used?

4. (a) Is an arch center a full semi-circle? (b) Show by sketch the construction of a center, and how it is supported and lowered.

5. Describe the proper method of keying a brick arch.

6. It is necessary to rebuild 100 ft. of a 48" brick sewer with considerable flow of water through it; describe completely the operation.

7. How much clearance in a trench should be allowed each side of a sewer to obtain good work?

8. Describe (a) your inspection of bricks delivered on the work for a sewer; and (b) the only right way of laying the same to insure tight work.

9. Under what conditions would you think it desirable to leave the sheeting in a trench, and why?

10. What are the rules governing the insertion of spurs in brick sewers, as to location, direction, etc.?

11. Describe the rules that must now be observed in laying pipe sewers.

12. How soon can filling in be done about a pipe sewer? What governs this?

13. Describe a good job of tamping earth around a sewer, giving best arrangement of men, and other requirements.

14. What is the best way of controlling quicksand where it is met with in an excavation?

15. (a) What is the diameter of the largest vitrified sewer pipe in use in the city? (b) What is the diameter of the smallest brick sewers now built in the city?

Report.

Write a weekly report of at least two pages on the construction of a six-foot outlet sewer in bad ground. Give such items as you think should appear, including all difficulties met with during the week.

INSPECTOR OF SEWERS CONNECTIONS.

Arithmetic.

1. Add 1234562, 9876, 278349, 968, 7865, 2678394, 2738492.
2. Subtract 367842 from 478096.
3. Multiply 76954 by 4079.
4. Divide 1676044 by 2347.

Technical.

1. At what height must a sewer connection be inserted in a brick sewer? Why do you think this necessary?

2. How often are spur connections made with sewers?

3. In inserting a spur in a brick sewer is there any rule as to its direction, and the method of finishing the work, and if so, what is it?

4. What size is the regular spur for house draining?

5. How many houses are allowed to drain into one spur?

6. Are larger spurs allowed, and if so, under what rule?

7. Does the inspector of connections have any authority over the laying of the house sewer in the street? If so, what? Where does his authority end?

8. What report as an Inspector would you make; give the items.

9. How are the house connections located so that they can be found again?

10. Would you think it your duty to take any notice of the condition of a sewer, or of the pavement?

11. Give any other details as to your duties which are not called for by the previous questions.

INSPECTOR OF REGULATING, GRADING AND PAVING.

Arithmetic.

1. Add 482396 — 384576 — 640985 — 498764 — 589787 — 694539.

2. Subtract 289698 from 532074.

3. Multiply 789645 by 40876.

4. How many cub. ft. of masonry are there in a solid wall 28 ft. 8 in. long, 9 ft. 10 in. high and 5 ft. 4 in. broad?

5. Add $357 and 1-4; $4078.09; $.18; $769 and 3-4; $37 and 1-2; $86.88; $450 and $.095.

6. Take 3907482 from 8600401.

7. Multiply 9048 by 605.

8. Divide 548130 by 906.

9. Find the number of cubic yards in a ditch 4 feet 6 inches deep, 5 feet wide and 7 feet 8 inches long.

Technical.

1. (a) In grading a street through rock at what depth in reference to the finished surface of the street is the rock excavated to? (b) What is the reason for this?

2. In making a deep fill across a depression, which will make the most solid fill; to bring the bank up uniformly in layers, or to carry the grade across level by dumping from one end, and why?

3. (a) State the rule respecting the disposition of material in a fill. (b) What is the reason for this?

4. Where a fill has been made of ordinary earth, gravel, etc., is any allowance made for settlement, and if so, about what?

5. Suppose several grade-pegs on a line, to be lost, but one at either end to remain, how can you restore those lost with reasonable accuracy without the engineer's aid?

6. If inspecting a job of grading, what memoranda should you keep and include in your reports? State this in detail.

7. Where concrete is used in the foundation for a pavement, describe (a) the preparation of the ground before placing the concrete; (b) describe the best method of mixing the concrete; (c) describe the proper method of placing the concrete, joining new to old, and all precautions to be observed.

8. Describe a strictly first-class job of a Telford foundation, omitting all above the foundation.

9. Describe the finish of a Telford road, above the foundation, including the rolling, and how you tell when the rolling is completed.

10. (a) Describe a stone block such as is required for stone block pavement in the city. (b) State all the defects for which you would reject stone blocks.

11. State all the rules to be observed by a paver in selecting and placing stone blocks in position in a roadway.

12. State the requirements of proper ramming and "back-ramming."

13. Where tar is used in stone-block paving, state how you would determine whether it was of proper quality and in proper condition for use.

14. In making an asphalt roadway, at what temperature must the asphalt be to obtain good work?

15. In laying an asphalt roadway, name carefully all the points in which inattention to detail may be detrimental to the durability of the roadway, or cause its rapid destruction.

16. State what you understand by the term "regulating."

17. Where a high embankment is to be made in grading a street state: (a) whether it makes any difference how the material is distributed, and if so, what the rule is. (b) Give your reasons.

18. State fully and clearly everything to be done in preparing the ground surface for and laying the Telford road, all but the last four inches—that is, all but the finishing courses.

19. What do you understand to be the difference between materials classified as rock and those classified as earth in paying for grading jobs? How is the classification determined?

20. Describe in your own language a perfect paving block, considering (a) its form, finish and dimensions; (b) the materials of which it is composed and its physical condition.

21. Is it detrimental, in your opinion, to pave with stones of different depths in the same row in a pavement, and if so, why?

22. In what ways is poor work done by pavers in selecting and placing paving blocks?

23. What is meant by "back-ramming" and what is the object?

24. (a) Draw lines representing a street intersection and show the proper way of placing paving blocks at that point. (b) State why it is the best way.

25. (a) State how you can tell whether the paving pitch has been overheated or not. (b) What is the result of overheating on its wearing qualities?

26. (a) What is the proper temperature at which asphalt should be brought on the work? (b) What effects are produced by its being either too hot or too cold? State each.

27. Which would you consider the best finish to leave on the surface of a concrete bed for an asphalt pavement—that it should be smooth or left rough? State your reasons.

28. (a) In preparation of the surface of cement to receive binder or of binder to receive a subsequent coat of asphalt name two essential things to be guarded against. (b) State why.

29. How may water affect a pavement after it is laid, and where is this effect most likely to occur? (b) What is done to prevent this?

30. State as nearly as you can the causes for (a) the formation of long cracks across an asphalt street. (b) The shoving up into waves. (c) The breaking up or wear in spots.

Report.

You are sent to inspect where the pavement has been relaid after having been torn up by a railroad company. Assuming such facts as you please, write a report of your inspection of not less than a page, properly made out, and signed with your examination number, and not your name.

MASONRY.

Masonry.—The art of erecting structures with suitably arranged blocks of stone.

Divisions.—Foundation; erection.

Foundation.—Excavation; preparation of the bed.

Excavation.—For large and important hydraulic works, for high buildings, etc., ground should be removed to solid rock.

For ordinary works, it may not be necessary to excavate below a certain level where it is ascertained, by borings or test pits, that the underlying strata to the depth of 8 or 10 ft. are hard ground free from mud or quick sand.

Drill Holes.—For important work, either wash or diamond drill holes should be bored. These last will give a core showing the strata and character of the formation.

Under water, when it is necessary to reach solid rock, a caisson may be used.

Caisson—The Working Chamber.—A huge timber box with sharp lower sides, a floor about 7 or 8 ft. from the open bottom which forms the roof or ceiling of a space called the working chamber. This chamber connects with the top of the caisson and the outside by means of several vertical shafts (generally in pairs).

Shafts.—Shafts are used for the entrance and exit of workmen, for the removal of excavated materials, for water and compressed air pipes.

Compressed Air.—Compressed air keeps the chamber clear of water.

Air-Locks.—Shafts have air-locks.

How Caisson is lowered.—The caisson is first placed in position very carefully and allowed to sink to the natural bottom either by its own weight or by masonry being laid on the platform and around the shafts to overcome the buoyancy.

Work begins.—Compressed air is then admitted to the chamber and, when this is judged free of water, a gang of laborers is sent through the air-locks and shafts to the working chamber at the bottom, and they begin to excavate the chamber floor.

Preparing Rock Bed.—As the excavation progresses and more masonry is added, the caisson gradually sinks to bed rock which is then finally prepared for a bed after which the chamber space and the shafts are filled with masonry.

When the depth to be reached is not very great, a coffer-dam may be built.

Coffer-Dam.—A coffer-dam is a water-tight enclosure from the interior of which the water is pumped out, thus allowing the excavation to be made to a suitable bed and in the open air. A coffer-dam may be built with a double row of **sheet-piling** or **crib-work** leaving a space to be filled with **rammed clay.**

Built-up Crib.—In size and shape a little larger than the bottom course. It is built up in regular vertical compartments, the walls and partitions being square timbers notched at their intersections and bolted. It is built either ashore and towed to the spot; or in position between pontoons, gradually sinking it by partly filling some compartments with stones, masonry or concrete. Before reaching the bottom, small stones are dumped through the compartments left open which find their way under the crib thus preparing a level bed. Concrete may also be dumped through a large canvass pipe which opens automatically before it reaches the bottom.

Top Platform—Its Disadvantage.—The top of the crib is sometimes finished with a platform on which to lay the bottom courses of the masonry. But such a floor is unsafe in case of unequal settlement in the crib, as it facilitates the sliding of the structure; it is better to continue the masonry in the compartments.

Random Stone.—Large blocks of stone are dumped as quarried over an area considerably larger than the proposed structure. Small stones are also dumped to fill the interstices. The area of the dump

is gradually reduced until the heap attains a height of only a few feet below low-water when it may be easily leveled off for a bed.

Bearing Piles.—Should be free from bark and straight. Average diam. at top about 12"; sharp at lower end and sometimes iron shod. They are generally sunk with a steam pile-driver, beginning at about the center of the group and proceeding outwardly. Dist. between centers 2 to 4 ft.

In soft soil use white pine, spruce or hemlock; in harder ground use yellow pine, oak, elm, beech.

Random stone is dumped around and about the piles, and they may be further secured with timbers notched and bolted at their upper ends which are sawed off level.

Water Jet.—By means of water jets from a force pump, the bottom of a stream or lake may be easily scoured to a good level.

Compressed Air.—Compressed air may also be used for the same purpose.

Steel Cylinders.—Large riveted steel cylinders may be lowered to a sufficient depth in the manner of a coffer dam and filled with sand, concrete or masonry. Above the natural bottom of the water they may be also surrounded with random stone.

Preparation of the Bed—Iu Rock.—Take off the top or desintegrated portions, either to a rough level over the whole area which the bottom courses are to occupy, or step off the rock in benches.

If there exist local fissures, these may be enlarged to a square or rectangular shape and made deep enough for the building of piers which serve as anchors against any lateral force which might solicit the structure to slide on its base. If such piers are not deemed necessary, the fissures as well as all the irregularities in the top of the rock should be filled with broken stone embedded in mortar or concrete. Should some fissures be too large, they may be arched over.

In Earth.—The best earth bed for foundation is sand or gravel of sufficient thickness (8 or 10 ft.). If it is feared that the understrata may cause future sliding, wooden or metallic sheet-piling may be driven, around the foundation site, to a sufficient depth to insure stability of the enclosed bed.

ERECTION.—Stones, Binding Materials, Kinds of Masonry, Bonding, Finishing, Structures.

Stones.—They are natural or artificial.

Natural Stones.—They are quarried from the solid crust of the earth. The principal kinds are: Granite, sandstone and limestone.

Granite.—Its components are quartz, feldspar and mica.

Hard Granite.—Contains a preponderance of quartz. It is the best for construction.

Soft Granite.—Contains little quartz. Used only for filling.

Sandstone.—Found in irregular masses. Formed with small grains of quartz embedded in siliceous or argilaceous cement. When it contains iron it should not be used as a facing stone because of sure discoloration in time.

Limestone.—Found in beds. It contains lime, carbonic acid, sometimes alumina, silica, etc. It disintegrates when exposed to a great heat. The finest kinds, called marbles, are reserved for works of art.

Natural Stones.	Specific Gravity.	Aver. Weight lbs. p. cu. ft.	Compression lbs. p. sq. in.	Tension lbs. p. sq. in.	Modulus of Rupture.
Granite .	2.6	165	15,000	600	1,800
Sandstone .	2.5	150	10,000	150	1,200
Limestone	2.5	155	12,000	1,000	1,500

Characteristics of a good Stone.—Fine grain, homogeneous, compact texture, uniform density.

Stones must lay on their own Beds.—When using one kind of stone in a structure, the stones coming from the foot or root of the quarry should be used for the lower courses, those coming from the heart of the quarry for the middle of the work and those coming from the roof or sky of the quarry for the upper courses.

Each stone also should have in the structure the same bed it had in the quarry.

In a Structure, Stones should be arranged in geological Formation.—When using different kinds of stones, those of oldest formation should be in the lower portion of the structure, and those of more recent formation should have in the structure the same relative position as they have in geological formation.

Test unknown Stones.—When using an unknown kind of stone, it should be tested by long exposure to air, water, frost and fire, and its behavior noted.

A stone which increases 5% in weight after remaining 24 hours in water should be rejected.

Artificial Stones.—Solid mixtures of earthy materials. They are **bricks, terra cotta, concrete** and **beton.**

Bricks.—Blocks of burned clay, about 2¼"x4"x8¼"

How made.—Clay, or clay mixed with other ingredients and often with coal dust in a mixing machine after being sprinkled with water, is pressed (by hand or machine) into moulds and dried in the shade. The blocks are built up into a pile or furnace with intervals between them and long narrow spaces at the bottom where wood is piled up or gas is burned. The outside of the heap is calked with clay before lighting the fire which is moderate at first then gradually increased until in about 60 hours it has the greatest degree required, and this should be kept up to the end.

Characteristics of a good Brick.—It should be uniformly burned, have plane faces and right angles, fine grain of fracture, be free from cracks or flaws, stones or lumps and give a clear sound when struck.

Hollow Bricks.—Of like materials as the common brick but of a larger size and longitudinally perforated.

Their Advantages.—Used in arches and partitions—They deaden the sound and are lighter on the bearings.

Floating Bricks.—Made of **fossil farina** mixed with less than ⅓ in bulk of clay.

Their Advantages.—Used in ice-houses, kitchens, etc., aboard ships, arsenals, etc.—They are very light and bad conductors of heat.

Terra Cotta.—Finer quality of ingredients but otherwise much like brick. It is moulded into plain or ornamental blocks and pieces, and baked on shelves of specially prepared ovens.

Pieces of terra cotta may advantageously be filled with strong grout or concrete, thus increasing their bearing power.

Characteristics of good Terra Cotta.—All pieces should be inspected and tested before acceptance. The glazing should not be chipped off; the surface should resist the scratch of a knife; the fracture must be uniform in color and texture and it should emit a metallic sound when struck.

Concrete.—A mixture of cement, sand and gravel or crushed stone. Sometimes lime or mortar is used instead of cement. Asphaltum, coal tar, etc., may also be used in the mixture.

Proportions in Concrete.—The sea ends of **Mississippi Jetties,** which were built in 1879 by Captain Jas. B. Eads, are of concrete in the following proportions by volume:

Cement,	3	mixed with fresh water, about 10.5 % of the dry ingredients.
Gravel,	4.38	
Portland sand,	8.28	
Broken stone,	15	

When drying it reduced to about ⅝ of the original bulk.

In **Boston,** concrete in Piers and Retaining Walls of Bridges contains:

Cement,	1	Parts in volume.
Sand,	2	
Broken stone,	5	

The **New York Central R. R.** on some of their Bridges use the following:

Concrete.				
Class.	Where used	Cement	Sand	Stone
A	*Coping. Roadway; 1" thick.*	1	2	4
B	*Pier and Sidewalls*	1	3	6
C	*Bridge Rest – Pier Coping.*	1	1	2
D	*Foundation; 3" thick.*	1	4	7.5

At the **Little Falls, N. J., Filtering Plant,** the proportions used in the main walls are 1—3—7 and the cement was **Atlas.**

Concrete Blocks.—Concrete blocks are moulded in a variety of shapes. In the form of hollow bricks 24"x12"x8", they are now extensively used in basement and main walls of buildings.

Large concrete blocks are used for sea walls.

Stairways are made of concrete.

Water pipes are sometimes made of concrete.

How Concrete should be laid.—It should not be dumped from a distance. It must be laid in layers of 6" or 8" and carefully rammed, leaving the surface rough, the better to bond the next layer. When the upper layer has been laid, the top, after being rammed should be leveled off with board and trowel. The last course may be a thin layer of cement and sand topped with pure cement, sometimes colored with lamp black in order to imitate blue stone.

Beton.—Kind of concrete made with hydraulic cement and angular stones, used in foundations where it is dumped.

Proportions: { *Hydraulic mortar*, 1; *Angular stone*, 1.5

Artificial Stones.	Specific Gravity.	Aver. Weight lbs. p. cu. ft.	Compression lbs. p. sq. in.	Tension lbs. p. sq. in.	Modulus of Rupture.
Brick.	2.00	125	10,000	200	600
Terra Cotta.	1.92	120	5,000		
Concrete.	2.25	140	2,000	400	150

Binding Materials.—Mixtures of substances placed between the stones of a structure where they solidify, thus uniting all the stones into a monolith.

They are: Mortar, cement and plaster.

The principal ingredient in mortar is lime.

Lime.—It is obtained by burning limestone, marble, shells, etc., in kilns until the flame from the flue appears bright and smokeless. The product is **quicklime.**

Characteristics of good Quicklime.—It should be lumpy, with very little dust or none at all and no cinders. It slakes readily in water forming an impalpable and smooth paste free from sediment. If enough soft water be added it should dissolve.

Remarks.—(a) The best quality of lime is obtained from the hardest limestone. (b) **Alumina** and **magnesia** impair the quality of lime.

"Hydraulic" Lime.—Manganese and iron render lime hydraulic; that is to say, enable it to harden rapidly under water.

Mortar.—Mixture of sand and lime with water, 1½ to 2½ times the weight of lime, carefully slacked with hoe (preferable to shovel) to a uniform paste, in a box.

Hydraulic Mortar.—Made with hydraulic lime and the same weight of water.

Remarks.—(a) When the mortar is slacked, it should be covered with a tarpaulin or a layer of sand to retain the vapors.

(b) Water added afterward renders mortar lumpy and chills the lime.

(c) Too much water makes the mortar thin, slow and weak.

Cement Mortar.—A mixture of cement and sand; or cement, lime and sand.

Grout.—Thin mortar or cement mortar used under pressure for filling, by injection, cavities in masonry work.

Remarks.—(a) Cement and lime should be well mixed together before water is added.

(b) Cement mortar must be used before it sets.

(c) Darker sand, better mortar.

(d) Pit sand better than river sand.

(e) Fresh pit sand better than dried.

(f) Lime and cement, or old mortar, better than lime and sand.

(g) In common mortar, coarse sand is best, mixed sand next, fine sand last.

(h) In hydraulic mortar, mixture of coarse and fine sand is best, fine sand next, coarse sand last.

(i) Mortar diminishes in bulk when drying.

Use of Water.—As little water as possible to obtain a stiff paste.

Mixture of the Ingredients.—A mortar box is used (about 4'x8'x 12"). A bed of sand is laid uniformly over it. Lime is evenly distributed over the sand. Water is added and the slacking is done with an iron hoe preferably to a shovel until the components cannot be distinguished.

Proportions.

Ingredients.	Usual Mixture.	Better Grade.	Hydraulic.
Lime.	1	1	—
Sand.	3	2	1.5
Water.	2	2	1
Hydraulic Lime	—	—	1

Cement.—Baked clay pulverized. Often containing broken brick, tile, pottery.

Kinds.—Portland, and Rosendale or natural.

Portland Cement.—A mixture of clay and carbonate of lime finely pulverized, burned at a high heat in kilns and ground to a fine powder.

Dark in color; slow setting; attains great ultimate strength; weighs 90 to 100 lbs. per cu. ft.

Rosendale Cement.—Made from natural rock containing the proper proportions of clay and limestone.

Light in color; very quick setting; becomes ½ as strong as Portland; weighs 50 to 60 lbs. per cu. ft.

External Characteristics. (a) Color uniform; (b) finely ground; (c) sieve of 100 meshes per inch shall not retain more than 8% cement; (d) specific gravity not less than 3.10; (e) shall not contain more than 1.75% sulphuric acid.

Testing Cement.—Mix a little cement with water and make two cakes (briquettes) about alike with thin edges—½ in. thick—and leave them exposed to the air. After an hour exposure, put one in water for 24 hours, then examine the samples.

If { *Air dried* — Quite hard, considerable tensile strength when broken, clean, sharp fracture, no crumbling; *Water soaked* — Kept its shape, much harder than when immersed; } *cement is good for all building purposes.*

Water soaked — Bad checks or cracks; *unsafe under water*

If { *Air dried* — Very hard; *Water soaked* — Crumbles; } add ½ as much slaked lime; if it hardens, it may be used in wet places.

With Testing Machine. (Samples were water soaked).									
Kinds of Cement used.	*Aver. lbs. per sq. in. Tensile Strength*								
	Clear Cement.			*Cement Mortar.*					
				Cement, 1 Sand, 1			*Cement, 1 Sand, 3*		
	1 week	*4 wks.*	*1 year*	*1 week*	*4 wks.*	*1 yr.*	*1 wk*	*4 wks.*	*1 yr.*
Portland.	*400*	*525*	*625*				*112*	*150*	*275*
Rosendale.	*80*	*125*	*350*	*40*	*65*	*250*			

Sampling Cement.—If in bags, take from the center. If in barrels bore a hole at one end and insert a brass tube which push to the bottom; withdraw the tube and shake the contents to use in tests.

Preservation of Cements.—Cements should be stored in a dry place protected from the weather and on boards several inches above the floor.

Plaster.—Or **gypsum** is a sulphate of lime.

How used.—Slaked on a hand-board, just before using, with just enough water to make a paste that will not run.—Must be used immediately as it hardens rapidly.

Where used.—In interior work only, because it exfoliates and loses its hardness when exposed to air.

Used to give a nice finish to interior surfaces as walls, ceilings.

Brick partitions are sometimes built with plaster as a binder.

Ornamental pieces are made of plaster.

Characteristics.—It adheres strongly to brick and stone, poorly to wood; hence the necessity of securing good clinches behind the lathing of partitions.

Cautions.—Plaster expands when drying, so that plaster work should be isolated from brick walls by studs and laths.

Remarks.—Lime-water gives a stronger plaster than ordinary water. Strongly glued water gives a plaster stronger yet which may be polished to a fine stucco.

KINDS OF MASONRY.—Stone, concrete, brick.

Stone Masonry.—(a) Ashlar or cut stone.

(b) Rubble or rough stone.

(c) Hammered stone.

Ashlar or cut Stone Masonry.—Made of the best grades of stone, uniform in color and texture; all faces dressed—visible faces polished.

This is an expensive kind of masonry.

Rubble Masonry.—Same faces only are roughly dressed.

Kinds of Rubble.—Block stone, common rubble, dry rubble, riprap.

Block Stone Masonry.—Kind of rubble in which the stones are very large and the faces of which are more or less dressed.

Common Rubble.—Stones of ordinary size with smaller stones for filling.

Dry Rubble.—The spaces between the stones are not filled with mortar.

Hammered Stone Masonry.—Stones are used as they come from the quarry, a selection being made on the work for facing stones and corner blocks.

CONCRETE MASONRY.—Monolithic, cyclopean, re-enforced, block.

Monolithic Masonry.—Structure made of a single block of concrete laid in successive layers not over 8" thick, each layer applied before the underlying one has finally set.

Wooden Moulds and frames are used to confine the concrete within the architectural lines of the structure.

Cyclopean Masonry.—Structure made of concrete into which blocks of stone as large as can be quarried, transported and handled are dumped, together with smaller stones for filling.

When cyclopean masonry is used in the construction of dams, it is important that the large blocks do not touch each other, so as to avoid the direct transmission of strains.

Re-enforced Masonry.—Concrete masonry into which are buried iron or steel rods, generally twisted, to strengthen and bind it.

Block Masonry.—Moulded concrete blocks.

BRICK MASONRY.—The building material is brick or terra-cotta blocks.

Wall.—A stone structure called

In a Building: (a) **Front Wall.**—The one fronting the street.

(b) **Rear Wall.**—Opposite the front wall.

(c) **Side Wall.**—Separating adjacent houses.

(d) **Party Wall.**—Side wall built over the property line.

(e) **Partition Wall.**—Separating rooms.

In the Field: (a) **Stone Fence.**—Enclosing or separating properties or field.

(b) **Retaining Wall.**—Sustaining earth on one side.

(c) **Dam.**—Retaining water on one side.

(d) **Jetty.**—Retaining water on two sides; or water on one side and earth on the other.

(e) **Abutment.**—Retaining wall sustaining the end of a bridge.

(f) **Bulkhead.**—Water end of a pier.

Parts of a Wall.—Footings and foundations, body, coping, face, back, filling, batter.

Footings and Foundations.—The first courses laid on the prepared bed. They are made wider than the wall in order to distribute the load over a larger area of the bed. These courses are stepped up until the thickness of the wall is attained.

Body.—Portion between the footings and the coping.

Coping.—The top course of a retaining wall. It is of a better class of material and projects slightly on the outside. The joints are normal to the slopes of the coping.

Face.—Outside portion; that exposed to view.

Back.—Inside portion—opposite the face; that not exposed to view in retaining walls.

Filling.—Interior portion; between the face and back.

Batter.—Slope of the face. Ratio of horizontal to vertical, as 1-12, which means that the face of the wall deflects one horizontal in twelve vertical.

Bond.—Manner of arranging and uniting the stones.

Shape of Building Stones.—The general shape is that of a regular parallelopipedon, the sides of the base being the length and breadth of the stone, and the height its thickness.

Size of cut Stones.—Thickness between 1 and 2 ft.; breadth not less than the thickness; length about 3 times the thickness.

Joints.—Spaces between the stones ordinarily filled with mortar.

Beds.—Horizontal joints.

Thickness of Joints.—From $\frac{3}{16}$ to $\frac{5}{8}$ in., according to the character of masonry. In fine brickwork joints may be as small as $\frac{1}{8}$ in.

A wall with more stone and less mortar is stronger.

Direction of Joints.—Should be normal to the direction of pressure; this prevents sliding. In vertical walls one set of joints is horizontal, the other vertical.

Broken Joints.—When the batter is greater than $\frac{1}{2}$ or 1 horiz. to 2 vert., the bed joints are often broken off at right angle to the face (Fig. 265) forming an oblique joint of from 4 to 6 in.

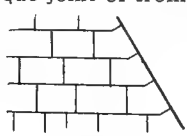

Fig. 265

Disadvantages.—This arrangement has the double disadvantage of being costly and of permitting water to enter the joints where it may freeze in winter thus forcing the stones apart.

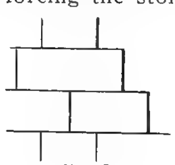

Fig. 266

Stepping.—A cheaper way is to step off the courses (Fig. 266). In abutments of arches, often the bed joints are not horizontal but normal to the thrust.

Course.—Horizontal layer of stones.

Regular Coursing.—That in which all the stones of a layer have equal thickness.

Names of Stones in a Wall (Fig. 267).—They are either stretchers, headers or fillers.

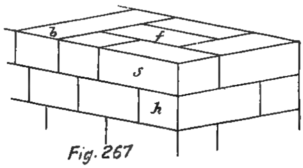

Fig. 267

Stretcher.—A stone with its length showing in the face of the wall (**s**).

Header.—A stone with its breadth showing in the face of the wall (**h**).

Binder.—A header taking the full thickness of the wall (**b**).

Filler.—A stone entirely in the body of the wall with no visible part (**f**).

Bond by Stretchers (Fig. 268).—Only stretchers are seen in the face of the wall; then ordinarily the breadth forms the thickness of the wall.

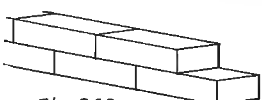
Fig. 268

Bond by Headers and Stretchers (Fig. 269).—1°. Headers and stretchers alternate in a course and alternate in the courses, the center of a header being over the center of a stretcher.

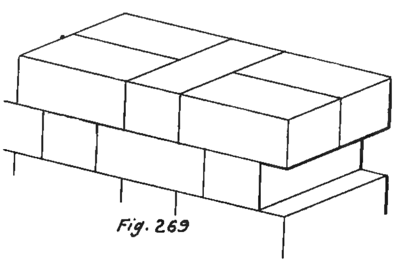
Fig. 269

2°. Two stretchers and a header in succession (Fig. 270); the center of a header in a course is over the **V** joint of the two stretchers in the course below.

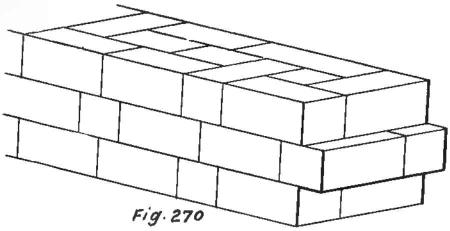

Fig. 270

Cramped Bond (Fig. 271).—When the face and back of a wall leave a space to be filled with rubble, broken stone or concrete. Stones on the face and back should be tied by iron cramps at intervals. It is better, in that case, to use long headers of a length equal to the thickness of the wall.

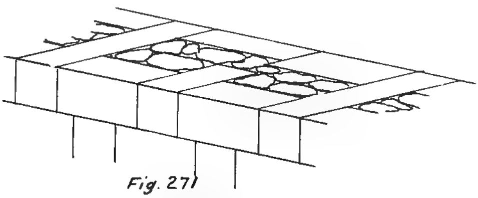

Fig. 271

Reticulated Bond or **Network** (Fig. 272).—Used only in panels between horizontal courses and strong vertical pillars. The stones are square and they are arranged in layers forming an angle of 45° with the horizontal.

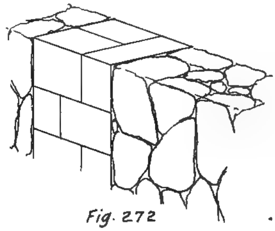

Fig. 272

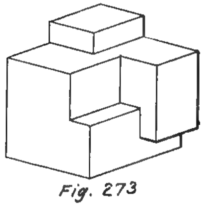

Fig. 273

Broken Bed Bond (Fig. 273).—In walls destined to bear violent shocks as sea-walls, light-houses, etc., the beds of the stones are often cut in some portions of their surface, to fit with corresponding projections in the stones of the next course. The courses are often united by iron rods or straps besides.

Rubble Work (Fig. 274).—When executed with care is very strong.

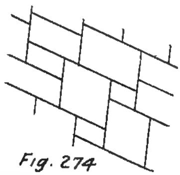

Fig. 274

The stones are prepared by giving them a rectangular general shape, simply cutting off sharp angles.

Stones should be washed before being imbedded in mortar into a thick bed of which they are pounded.

Stones of different thicknesses and different sizes are used side by side.

Care should be taken:

1°. To have the beds horizontal;

2°. To use frequent headers;

3°. To avoid having more than two vertical joints on a line;

4° To fill up the spaces in the vertical joints with spawls and mortar well rammed;

5°. Best stone with best cut should be reserved for angles.

Remark.—Such angles are sometimes built with well burnt brick.

Brick Masonry stronger and lighter.—Stronger and lighter than stone masonry because mortar adheres to brick more strongly than to stone and brick weighs less per cu. ft. than stone.

English Bond (Fig. 275).—Courses of stretchers alternate with courses of headers.

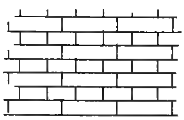

Fig. 275

Flemish Bond (Fig. 276).—Courses formed with alternating headers and stretchers. In the successive courses a header is laid over the center of a stretcher.

Finishing.—Adorning the face of the masonry and completing it.

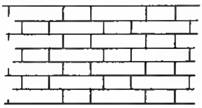

Fig. 276

Finishing cut Stone.—Chisel Draft—All angles and generally all joints should be made apparent by a chisel draft from 1 to 1½ in. wide with true horizontal and vertical edges. By this means slight inaccuracies in the joint are corrected.

Rock Face.—The space between the draft lines is left as it comes from the quarry.

Crandalled.—Oblique ridges worked at about 45° with a pointed chisel.

Double-crandalled.—Two sets of ridges crossing one another practically at right angles.

Broached.—Grooves dug in the surface obliquely and generally parallel.

Pointed.—Oblique broken grooves.

Rough Pointed.—Intervals of about 1 inch.

Fine Pointed.—Intervals of about ½ inch.

Tooled.—Vertical lines made with chisel 3 to 4 in. wide.

Droved.—Vertical lines made with smaller chisel.

Vermiculated.—Rough irregular grooves somewhat deep leaving irregular winding ridges. Effective in base courses when the rest of the wall is of a smooth finish.

Bush-hammered.—Leaves the surface granulated. Should not be used on soft stone, but on medium hard qualities.

Patent-hammered.—For granite and hard stones. The degrees of finish are 6-cut, 8-cut, 10-cut from the number of blades per inch in the hammer.

Smooth-finish—Polished.—Obtained by rubbing, now generally done by machinery before the stone is brought to the work. This finish is reserved for the hardest and most valuable kinds of stones, such as granite and marble.

Finishing the Joints.—On cut stone and concrete block work, the joints are raked out to a depth of 1½ or 2 inches when the stones are set. After the work is complete, these spaces are neatly filled with cement, this is called **pointing.** On the vertical joints the face of the pointing is often made concave; in the horizontal joints it is

finished at an angle of about 60° with the horizontal, thus forming a recess on the upper portion; this prevents the infiltration of water in the joint.

On brick work the same kind of a batter is given to the mortar in the horizontal joints when the bricks are laid.

Finishing Concrete Work.—The upper portion of a finished concrete work is generally made with **granolithic** concrete which is a mixture of cement and granite or hard sandstone ground to a uniform grade of sand. If it presents a large surface like that of a walk or a piazza, it is generally divided into regular figures like diamonds, squares, hexagons or their combinations, some of which figures may be made of different color by the addition of pigments in mixing the top layer. Lamp black gives the concrete the appearance of blue stone.

Structures.—Principal structures are walls and arches.

Walls may be divided into building walls and retaining walls.

Building Walls may be divided into foundation walls, outside walls, bearing walls and partitions.

Foundation Walls.—Walls below grade or curb.

Thickness of Walls (N. Y. Building Laws).

Dwellings, Schools, Hotels.

Height above Curb. ft.	Foundation. Stone. in.	Foundation. Brick. in.	Outside Bearing Walls.
115			
	36	32	For 25 ft., 28"; next 25 ft., 24"; next 40 ft., 20"; above, 10".
100			
	32	28	For 35 ft., 24; next 35 ft., 20"; above, 16".
85			
	28	24	For 20 ft., 20"; next 40 ft., 16"; above, 12".
75			
	24	20	For 25 ft., 16"; above, 12".
60			
	24	20	First Story, 16"; above, 12".
50			
	20	16	12"
35			
	20	16	Basement, 12"; above, 8"
0			

Factories, Warehouses.

Height above Curb. ft.	Foundation. Stone. in.	Foundation. Brick. in.	Outside Bearing Walls.
100			
	36	32	For 25 ft., 28"; next 25 ft., 24"; next 25 ft., 20"; above, 16".
85			
	32	28	For 25 ft., 24"; next 35 ft., 20"; above, 16".
75			
	28	24	For 25 ft., 20"; above, 16".
60			
	24	20	For 40 ft., 16"; above, 12".
40			
	20	16	12"
0			

Above those heights, the lower part must be 4" thicker for each 25 ft. additional or fraction; the upper 115 ft. or 100 ft. remaining as hereinbefore given.

These figures don't apply to steel construction where the weight of the walls is carried by the framework.

Foundation Walls.—Below 12 ft. depth, foundation walls should be 4" thicker for every 10 ft. additional.

Footings of Foundation Walls should be stepped up to an angle of 60°.

In Factories, etc., over 25 ft. clear span, walls must be 4" thicker for each additional 12.5 ft. of span.

Partitions.—Non-bearing partitions 8" if less less than 50 ft. high.

Retaining Wall (Fig. 277).—One sustaining loose solid material or backing on one side called its **back.** The other or exposed side is its **face.** The bottom of the face is the **toe.**

The batter on the face is never more than 1/8; it is generally 1-24 or 1-12.

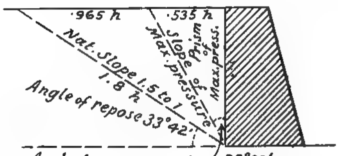

Fig. 277

Angle of Repose or natural Slope.—Any loose material when not confined gradually settles to a certain slope called its **natural slope.** This slope forms with the horizontal an angle called the **angle of repose.** It is generally admitted that a slope of 1.5 horizontal to 1 vertical is safe in almost all cases; this slope corresponds to an angle of 33° 42'. If the backing is filled horizontal with the top of the wall and the back of the wall is supposed vertical, the mass to be retained is a right angled triangle of which the line of natural slope is the hypothenuse = 1.8 **h,** the other sides are the height = **h** of the wall and the horizontal = 1.5 **h** at the top of the wall.

Angle of maximum Pressure, Slope of maxim. Pressure, Prism of max. Press.—If the angle of repose be 33° 42', the angle at the bottom of the inverted triangle retained is 90°—33° 42'=56° 18'. If this angle is bisected ½ (56° 18')=28° 09', the line thus drawn will divide the horizontal into two parts proportional to **h** and the hypothenuse 1.8 **h,** or equal respectively to 0.535 **h** and 0.965 **h.** The angle 28° 09° is the **angle of max. pressure;** the bisectrix is the **slope of max. pressure,** and the prism of material included within the triangle whose sides are the bisectrix, **h** and the segment 0.535 **h** of the horizontal is the **prism of max. pressure,** so-called because that particular prism presses more against the back of the wall than any other. The value of that pressure of the prism acting horizontally and applied to the back at

a distance $\frac{h}{3}$ from the bottom is: $p = \text{Weight of prism of max. pressure} \times \frac{\text{horizontal}}{\text{vertical}}$

In general $p = \frac{(.535h + s)w}{2} \times \frac{.535h}{h}$; or $p = \frac{.535w(.535h + s)}{2}$ (1)

For a V^{al} wall and per running foot, $p = .535h \times \frac{h}{2} \times w \times \frac{.535h}{h}$; $p = \frac{\overline{.535}^2 h^2 w}{2}$,

in which **w** is the weight of a cubic foot of backing.

In a Wall battered (or stepped) on the Back, the angle to be bisected is that formed by the sloping back and the line of natural slope and is greater than 28° 09', then the segments of the horizontal side of the triangle must be calculated. Also, the direction of pressure **p**, always normal to the back will not be horizontal.

Angle of Wall Friction.—Angle at which a wall of the character of masonry under consideration should be inclined to the horizontal to let the backing (dry sand, earth, etc.) just slide over it. This angle is practically the angle of repose given above as 33° 42' for a slope of 1.5 to 1.

Amount of Friction (Fig. 278)—Draw to a scale the line of pressure **p** due to the prism of max. pressure; at its point of application draw a line making with **p** an angle of 33° 42' (angle of wall friction) and produce it till intersected by a parallel to the back drawn through the free end of **p**; complete the parallelogram. The side along the back of the wall will be the friction due to the backing acting downwards; this force combined with **p** will have as a resultant the diagonal whose value is the final pressure upon the back of the wall.

$$P = \text{Weight of prism of max. pressure} \times \frac{\text{horizontal}}{\text{vertical}} \times \frac{1}{\cos. 33°42'} = p \times \frac{1}{\cos. 33°42'}$$

$$\text{or} \quad P = p \times 1.202 \qquad (2)$$

In formulas (1) and (2) the prism of max. pressure is formed by the back, the horizontal side and the slope of max. pressure;

and *horizontal* in the fraction $\frac{\text{horizontal}}{\text{vertical}}$ is in all cases the projection, on the horizontal side, of the slope of max. pressure.

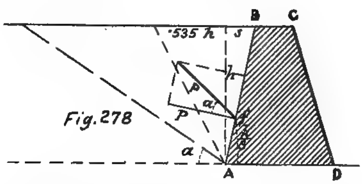

Fig. 278

Surcharged Walls.—The backing slopes up from the back, (sometimes even from the nose) of the coping. Then the point of application of **P** will be higher up due to that portion of the prism of max. pressure above the horizontal at the top of the wall.

Transforming a Profile.—When the section of a vertical retaining wall has been ascertained, as being able to support the backing, a slightly wider base, say 1.1 times or 1-10 wider may be taken as the base of a battered back wall the batter of which may be made ¼, or may be stepped off on that line with offsets of 8 or 9 inches and 32 or 36 inches high.

Precaution against Displacement by Frost.—To prevent displacement by frost, the back is suddenly sloped up from the back at the

line of frost, say 6 ft. below the top, to the coping, and that batter is heavily coated with cement, mortar or coal tar.

Thickness of retaining Walls.—Some engineers use the following thicknesses, **h** being the height of wall:

Dry Rubble (well built)	$0.5h$;
Mortar Rubble or *Brick*	$0.4h$;
Cut Stone or *First class Rubble* (ranged)	$0.35h$.

Arch.—Structure supported at the ends only. The following are the principal kinds:

Lintel.—A flat arch. Large stone covering an opening and supporting weight.

Circular Arch.—One drawn with a semi-circle. The arc=180°.

Segmental Arch.—One formed with only a portion of a semicircle.

The $Arc < 180°$

Elliptical Arch.—One affecting the form of an ellipse. It is oftener drawn with 3, 5 or 7 arcs and is called an arch with 3, 5 or 7 centers. The radii are equal 2 and 2 at equal distances from the supports.

Rampant Arch.—One of the supports is higher than the other. The radii are unequal.

Lintel.—The safe load on lintels is given by the formula.

$$W = c\frac{bd^2}{l}$$

in which W = *safe load* (uniformly distributed) *in tons of 2000 lbs.*

b = *Breadth of lintel in inches*;

c = *Coefficient of safety*

sandstone	0.08
limestone	0.1
granite	0.12
bluestone	0.18
slate	0.36

d = *Depth of lintel in inches*;

l = *Length of span in inches*.

For concentrated Load at the Center, safe load will be ½ of that given above.

Circular Arch.—Term reserved for an arch formed by a semicircle. This arch is always formed with an odd number of stones.

Voussoir,—Any of the stones forming an arch.

Key Stone.—The central stone, or middle voussoir of an arch.

Springers.—The voussoirs resting directly on the supports.

Haunches.—The voussoirs between the springers and the key.

Spring Line.—The horizontal diameter of the arch, or the horizontal chord on the top of the supports in a segmental arch.

Joints.—They are normal to the arc forming the arch, and generally of equal length.

Intrados.—The inner curve of the arch, in the face of the wall.

Extrados.—The outer curve passing through the further end of the joints.

Soffit.—Under (concave) face of the arch.

Spandrels.—Portion of the wall built over the voussoirs, between a vertical drawn at the end of the spring line and a horizontal drawn through the center of the extrados.

Abutments.—The supports of the arch.

Span.—Width between the abutments, or diameter of the arch if semi-circular. The span is represented by **2a.**

Rise.—Vertical distance from the center of the span to the crown of the intrados. The rise is represented by **b.**

Radius.—Radius of the arc forming the intrados of the circular or segmental arch. The radius is represented by **R.**

Arch Ring.—Portion of the face of the wall between the intrados and extrados.

Radius of a segmental Arch, given the span 2a and rise b.

$$R = \frac{a^2 + b^2}{2b}$$

Depth of Keystone.—Formula given by **Trautwine:**

$$d = c\left(\frac{\sqrt{R+a}}{4} + 0.2\right)$$

in which a = *half-span in ft.*

c = *Constant* – $\begin{cases} 1 & \text{for cut stone ,} \\ 1.125 & \text{for second class work ;} \\ 1.25 & \text{for fair rubble or brick :} \end{cases}$

d *depth in ft.*

R = *Radius in ft.*

Area of Arch-Ring.—

$$A = \frac{\pi}{2}\left(R^2 - r^2\right).$$

When intrados and extrados are concentric, **d** being the depth of the voussoirs, the area of arch-ring becomes

$$A = \frac{\pi}{2}(2r+d)d$$

Determination of the Stability of an Arch (Fig. 279).—Suppose it to be a segmental arch and to be loaded with a masonry wall of the same weight per cub. ft. as the arch stones.

1° Draw ½ the arch to a large scale with the extrados line and the top of the wall supported by the arch.

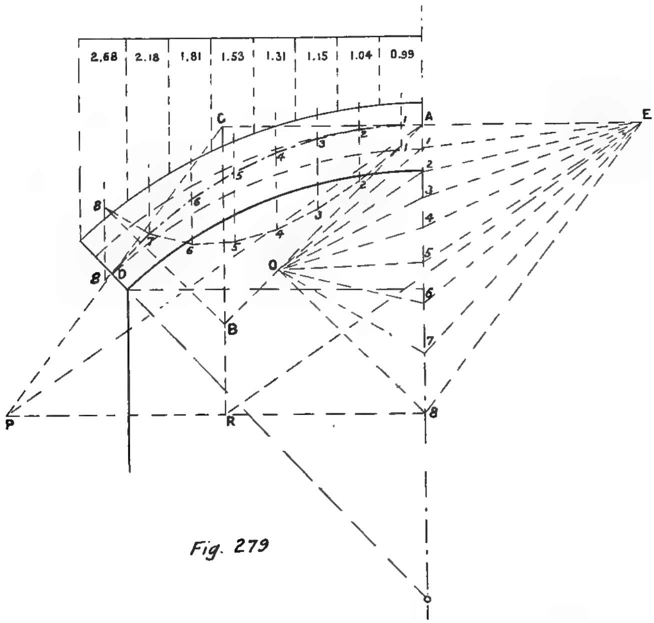

Fig. 279

2° Draw also two curves dividing the arch-ring into three portions of equal depth. The middle portion is called the **middle third.**

3° Draw 2 verticals at the skew-back line; one at the intrados the other at the extrados. Draw also a vertical through the center of the arch.

4° Divide the space between the center vertical and the intrados vertical into a number of parts or sections of equal width by means of verticals. In our figure it is divided into 7 sections.

5° Scale the verticals from top of wall to intrados and calculate the areas of the several sections. These areas alone are given in the figure.

6° Draw a horizontal through **A** the upper point of the middle third at the crown, and a set of verticals 1-2-3....8 through the center of gravity of each section.

7° Lay off from **A** downwards, on the center vertical, and to any convenient scale, distances.

A_1, A_2, A_3 ..... A_8 respectively equal to areas of sections 1, 1+2, 1+2+3 ..... 1+2+3+···+8

8° Through the last point obtained 8 and through **A**, draw 45° lines converging to **O** on the left, and join **01, 02, 03, 04**....**08** on the vertical of loads.

9° From point where **AO** intersects vertical **1**, draw **1-2** parallel to **01**, and in succession draw **2-3** parallel to **02**; **3-4** parallel to **03**.... and **7-8** parallel to **07**. From the last point obtained draw line **8B** a 45° oblique to an intersection with **AO.**

10° Draw vertical **BC** to an intersection with horizontal **EAC.** This vertical passes through the center of gravity of the arch and its load. Draw line **CD** through the point on the skewback intersected by the lower curve of the middle third. **CD** is the direction of the thrust of the arch on the skewback.

11° Through point **8** on the vertical of loads, draw horizontal **8RP**; the portion **RP** comprised between vertical **B** and thrust line **CD** is the thrust at the crown in direction (horizontal) and magnitude, and **CP** is the direction and amount of the thrust at the skewback. Lay off the horizontal thrust **RP** from **A** to **E** and join **E1, E2, E3,**.... **E8.**

12° From point **1** on horizontal **A** draw **1-2** parallel to **E1,** then in succession **2-3** parallel to **E2; 3-4** parallel to **E3**...., **7-8** parallel to **E7.** As a verification or check, this last line **7-8** must pass through point **D.** The curve which would connect the angle points of the last polygonal line drawn is the **line** or **curve of resistance.**

If the curve of resistance is entirely within the middle third, the arch is stable.

It may be safe enough if the line of resistance, although falling for a short distance outside of the middle third is still close to it; otherwise and principally if it intersects the intrados line, it is unsafe and the depth of the arch ring must be increased.

SEWERAGE.

Sewerage.—System of pipes and conduits provided for the flow of foul and rain water

Separate System.—When a system of pipes and conduits leads away foul water or sewage (it is called the **Sewage System**), and another system takes care of rain water (it is called the **Drainage System.)**

Combined System.—When one system only carries away both sewage and rain water.

What System is cheaper.—The separate system, because in a number of streets, at the head of the watersheds, collectors for surface water may be omitted in that system.

When the separate System should be used.—In localities where it is necessary to purify the sewage before finally disposing of it This purification could not be effected on the great amount of liquid issuing from outlets in the combined system.

When may Sewage be set free without Purification.—When it can be discharged in a large body of water not used for water supply, such as a large river, a lake or the sea.

Best place for Sewage Outlet.—At a point in a lake or the sea where a current may carry the sewage away from shore. If no such current exists, the outlet should be located as far from shore as possible and away from inlets and bays.

Elevation of Outlet.—If on a tidal body of water, it is preferable that the invert of the outlet be at mean high water elevation, so that the flow will not back up at flood tide. If grade of invert is established at mean low water elevation, provision should be made for storing up sewage in tidal chambers closed by tidal gates which may act automatically.

Acreage taken care of by Circular Sewers.
(from *John Roe's* Tables)

Slope or Pitch of Sewer	Clear diam. of Sewer in ft.						
	2	3	4	5	6	8	10
level	39	120	277	570	1 020	2 850	5 825
1 in 480	43	135	308	630	1 117	3 025	6 250
1 in 240	50	155	355	735	1 318	3 500	7 175
1 in 160	63	203	460	950	1 692	4 500	9 250
1 in 120	78	257	590	1 200	2 180	5 825	11 050
1 in 80	90	295	670	1 385	2 486	6 625	
1 in 60	115	318	730	1 500	2 675	7 125	

Remark.—House drains should not be less than 4 inches.

Acreage taken care of by Small Circular Drains.
with corresponding slopes.

Clear diam. of Pipe in inches. Pitch = 1 in					Acres.
4	6	8	12	18	
120					0 250
40					0 . 4375
30					0 . 5
20					0 . 6
	60				1
	40				1 . 2
	20	120			1 . 5
		80			1 8
		60			2 . 1
			120		4 5
			80		5 . 3
			60		5 . 8
				240	10 .
				120	17 .

Usual Grades for Drains					
Clear Diam. in.		*Pitch or Fall.*	*Clear Diam. in.*		*Pitch or Fall*
Drain.	*Fresh air Inlet.*		*Drain.*	*Fresh air Inlet.*	
2		· in 20	8	6	1 in 80
3		30	9	7	90
4	4	40	10	8	100
5	4	50	12		120
6	5	60	18		180
7	6	70			

These grades give a velocity of flow of not less than 275 ft. per minute which is considered to be a desirable minimum.

A System is not exclusive.—When the site of a town contains several watersheds, one or more of these watersheds or sections may be provided with one system while another system may be used for the rest of the town. This selection depends on local conditions, as the proximity to large bodies of water, to farming low lands, to large sand or gravel beds, the existence of water drainage previously established, the state of the public treasury, etc.

When a Sewerage System becomes a Necessity.—As soon as a public water system is introduced in a town, because then the amount of soiled water is so greatly increased that it becomes a public nuisance.

Steps to be taken to lay out a Sewerage System. 1° **Areas of Watersheds.**—From a contour map of the locality showing elevations of streets and changes of grade, divide said locality into watersheds or sections and calculate the areas of these sections.

2° **Location of Sewers.**—Determine the location of the trunk sewers, the collectors and street drains in each section.

3° **Grades of Sewers.**—Determine the grades of the sewers so as to give a mean velocity of not less than 275 ft. and establish them below frost line.

4° **Sizes of Conduits.**—Determine the sizes of trunks, collectors and street drains, considering the area which each has to take care of.

5° **Ground Water Drains.**—Provide drains for ground water (springs. infiltration, etc.), preferably under the sewer; this will render basements and cellars dry and prevent said water from forcing its way through the joints of the pipes or sewers. Let these drains run out into the nearest stream or body of water, if the latter is not used for water supply.

6° **Manholes.**—Provide manholes at all junctions and at all changes of grade. They are generally cylindrical, built of brick or concrete, often provided with an iron ladder. Their purpose is twofold (a) inspecting and repairing; (b) flushing the sewer. They generally have a cast-iron perforated cover for ventilation. If intended

for flushing they should have a gate on the lower side to allow the water to accumulate in the manhole up to a certain level when the gate is opened thus releasing the water. This may be done automatically.

Flushing.—Flushing may be done with public water.

When Flushing is necessary.—Flushing is necessary where it is not possible to secure grades with enough fall, as in low bottoms.

7° **Catch-Basins..**—Provide catch-basins at each corner of the streets (often a single pipe will answer when the drainage of a side of a street is very small), just under the sidewalk corner. It is generally square or rectangular, built of brick or concrete and deeper than the pipe or pipes leading to the sewer in order to allow for the collection of silt or heavy matter and the occasional removal of the same. The opening in the curb should be grated. A trap should be provided to prevent the escape of sewer gas. A good trap is one or two sheet iron shutters suspended with hinges and resting on slightly battered frames.

8° **Overflows.**—Provide overflows in the collectors and trunk sewers so as to allow storm water to escape into nearby streams or into other pipes when it reaches a certain elevation.

9° **House Connections.**—Provide house connections every 25 ft. in closely built localities, or more in residential sections. If the sewer is deep, the connection may be carried vertically to below frost and there quarter bended. If the sewer is at a high elevation, the connection is made with its grade a little higher than that of the sewer; this facilitates the flow.

10° **Ventilation.**—Provide air vents at intervals of about 400 ft. Some ventilation is effected through manholes.

Minimum Grade for Sewer—Baumeister's formula.

$$\text{Min. Grade in per cent} = \frac{100}{5d+50}$$

in which d = Clear Diam. in inches.

What Water is to be carried in Sewers.—In the separate system, the sewers should be designed to carry.

1° The daily water consumption which varies with each town, but which may be assumed at 100 gals. per day per inhabitant. It is also safe to assume that 75 gals. per capita is to find its escape during a period of about 12 hours.

2° In many cases it will have to accommodate roof water. In the combined system the sewers will have to carry in addition all surface water and water from springs and rain. Storm water should be reckoned at the rate of one inch per hour.

How Connections should be made.—Intersections of pipes, collectors, trunk sewers and also house connections should be made at an acute angle in the direction of the flow; generally an ⅛ connection (45° **Y**).

Size of Pipes.—In the separate system 8" to 30" vitrified clay, salt-glazed surface. House connections not less than 4" depending on size of house.

Shape of Sewers.—Inverted egg shape. This form presents the greatest depth and flow for the least amount of sewage., thus preventing deposits. Where a steep grade prevails, a circular section may be found preferable as it contains less material for maximum area of opening. The elliptical form may be adopted where the flow is large and more uniform and the grade medium.

Materials of Sewers.—Generally brick. Concrete is preferred on uncertain soil, and stone on steep grades where the water carries much sand. A combination of these materials may be better in some cases.

A previous knowledge of the ground formation should have been obtained from existing excavations, wells, test pits, etc.

Care in laying Pipes.—Have a good bed of uniform grade, (concrete if possible), providing hollows for the bells so as to avoid unequal settling and loosening of joints when refilling.

Pumping.—It may be necessary in certain sections of a town to use pumps in order to raise the sewage to a higher elevation and there collect it or dispose of it. Reservoirs (covered if in a populated district) should be built in order to collect sewage during the night, and this sewage should be screened before reaching the pumps; the reservoirs are scraped occasionally.

Sewage Disposal.—Where it is not possible to empty sewers in tide water or in large streams (not used for water supply), it may be disposed of 1° as manure 2° by filtration.

Sewage in Farming.—This method of disposal is akin to irrigation, but can only be used on porous, light soil and for certain crops. The author knows large cities where the sewage is collected nightly by farmers, in long barrels similar to street sprinklers, and used by them as manure in truck farming.

Filtration.—The sewage is run out from the trunk sewer through small diverging pipes emptying alternately over various sand and gravel beds of a depth of 4 or 5 feet. A system of drains under the beds will carry the liquid away. When clogged with sludge the top of the beds is raked off and the matter carted away.

Straining.—By metallic screens or by filters. This process does not purify sewage.

Sedimentation.—Sewage is run out into tanks where it is allowed to settle. Expensive and not applicable to large towns.

Chemical Treatment.—Settling basins are used, but the chemicals should be added to the sewage and well mixed with it before it reaches the basins. Lime is mostly used; sometimes sulphate of alumina is added to it The basins may be used either continuously or at intervals.

Sludge Disposal.—Pressed into cakes and used as fertilizer or burned.

PAVEMENTS.

Pavement.—A hard, solid covering for the surface of a street or road.

Parts of a Pavement. Foundation, body, joints.

Foundations.—Bed prepared to receive and bear the pavement proper, and to distribute over a larger area the local stresses received. The style and size of the foundations depend 1° on the importance of the traffic; 2° on the kind of pavement to be laid; 3° on financial conditions.

Materials used in Pavements.—Stone, wood, asphalt.

Stone Pavements.—The stone used may be 1° natural; 2° artificial.

Natural Stones used.—Limestone, sandstone, trap, granite.

Pavements with natural Stones.—1° Cobble; 2° Cut blocks or Belgian; 3° Broken stone.

Cobble Stone Pavement.—The cobbles are small boulders of approximately a uniform size. They are generally of a quartzy nature and very lasting. They present a good footing for the horses, but they are very noisy and offer a great resistance to traction.

Foundations.—The ground is excavated to a depth of about 8 inches, rammed and covered with a bed of sand 1½ to 2 inches thick.

Laying the Cobbles.—The cobbles are laid in rows, either across the roadway or diagonally to it, and each one is hammered to a solid bearing.

Joints.—The joints are filled with sand as the work progresses. When a certain area has been paved, sand is thrown on the top of the cobbles.

Ramming.—The cobbles are then rammed down hard and evenly.

Belgian Pavement.—A pavement made of square or rectangular blocks of cut stone. The best material is granite.

Foundations.—The excavation is carried to at least 12 inches below grade, or deeper where bad ground is found, such as mud.

Preparation of the Bed.—Holes and deep excavations are filled with good material, as gravel, to the bottom of the general excavation which should be approximately level after being rammed.

Laying the Bed.—This is generally a concrete floor, from 3 to 4 inches thick, allowed to harden before being covered with a layer of sand about one inch thick.

Cobble Stone Foundations.—When a cobble stone pavement is to be replaced by one of a better kind, a few rows of cobbles are removed and the ground dug to a greater depth than usual, say 16 or

18 inches. In the bottom are laid the next rows of cobbles and the excavation is continued under the emplacement of these; the next rows of cobbles are again laid in the bottom of the trench and the work is carried on in that way to the end of the street. The cobbles may then be rammed and will form a good bed for the layer of concrete.

Shape of Belgian Blocks.—The bearing surface is a rectangle, the dimensions of which vary greatly. They may be made 4 or 5 inches wide, 8 or 10 inches long and about 6 inches deep.

Even Width more important than even Length.—As the blocks are to be laid with their length across the street and in regular rows, the width of the block forming the length of a row should be as uniform as possible, consistent with the roughness of the cutting.

Blocks to be as much as possible of even Size.—The evenness of length and width produces neatness of appearance. The evenness of depth contributes to uniform bearing on the bed and more equal distribution of loads.

Rejection of Blocks.—Ill-shaped blocks, blocks which chip under the sharp blow of a light hammer should be rejected.

Selection of the Blocks.—1° Blocks of the same width should be used in the same row.

2° Blocks from the same quarry should be used together, therefore.

3° Blocks from different quarries should not be used promiscuously, but they should be assorted and laid together in order to insure uniformity in the wearing of the surface.

Laying the Blocks.—If a particular gutter of cut stone is specified, it is laid in advance of the blocks and between the gutters or in their default between the curb-stones the blocks are laid

1° in rows generally perpendicular to the length of the street:

2° with their length in the direction of the row and their width across the row.

3° When a block has been set, it is hammered to a good bearing and to a uniform surface with the blocks already laid.

4° The rows should be straight—and break joint not less than 2 inches.

Ramming.—The rammer should not be less than 3 inches; it should not weigh less than 50 pounds.

Each stone should be rammed down to a solid bearing, and if by doing so the surface is distorted, the blocks should be removed and replaced by deeper or thinner ones as the case may be.

Joints.—Joints may be simply the filling of the spaces with sand and gravel before ramming. They are now more generally filled with gravel and paving-pitch after the ramming is done and in the following manner:

Gravel is heated to 250° F. and thrown in the joints to a depth of 2 inches on top of which hot pitch is poured as long as it filters through until it covers the gravel about 1 inch when more hot gravel is thrown and more hot pitch, then again gravel to within ½ inch of the surface which space is filled with pitch. Sand is sprinkled on top.

Why the Gravel should be heated.—Cold gravel would chill and thicken the pitch and prevent its free flow through the interstices. The top of the pitch at the surface would be crusty and this would break and crumble under traffic or in freezing weather.

Advantage of Pitch filled Joints.—Gravel and pitch filling when well made constitute clastic joints in which brittleness under shock is minimized.

Paving Pitch used in New York City:

Refined Trinidad Asphaltum	*20*	*parts*
No. 4 Coal-tar distillate	*100*	*"*
Petroleum residue	*3*	*"*

Broken Stone Pavements.—They are the Telford and the Macadam.

Telford Pavement.—It is of varying thickness and the excavation is made according to specifications to a uniform grade. Then

1° The bottom of the excavation is rammed or rolled.

2° Large irregular quarried stones about 8" thick are laid with their largest face against the bed.

3° The intervals are filled with smaller stones or chips.

4° Too great projecting vertical angles are broken off.

5° The whole is rolled.

6° A course of broken stones is then laid.

7° Binding (as clay or earth) is spread over it.

8° This is rolled in turn.

9° Other courses of broken stone and binding may or not be added and rolled.

10° Screenings are spread and repeatedly rolled with a steam-roller.

Use of Water.—Before any rolling is done water is sprinkled (not poured) evenly over the work. Too much water should be avoided as it endangers the foundation, although it facilitates the rolling.

Amount of Binding.—The binding to be used should about equal the volume of voids in the broken stone. Two much binding helps rolling but lessens longevity.

Size of the Broken Stone.—The size of the broken stone should be specified as to pass through a ring of a certain diameter. Such

a ring should be provided for the inspector who should make a frequent use of it.

What Stones to reject.—All tailing or long flat stones should be rejected as not compacting with adjoining stones under the roller.

Clean Stones only to be used.—All stones should be free from loam and clay.

Amount of Rolling.—This depends on many conditions such as the grade, the amount of water and binding used, the class of the work, the size of the roller. A general rule is to keep on rolling as long as the bed sinks under and in front of the roller, and to leave the surface smooth and hard.

Compaction by Rolling.—Rolling compacts the materials as laid about ¼ in volume.

How to Roll.—1° Roll in the direction of the street, the sides first and the center last. The operation may have to be repeated several times.

2° Roll slowly and uniformly, overlapping but little the portions previously rolled.

Determination of the Voids.—Take a vessel of known capacity and fill it with the loose material previously wetted thoroughly. Pour water in the vessel until it overflows. The volume of the water so required, divided by the volume of the vessel will give the proportion of the voids.

Determination of the Weight of a cu. yd. of broken Stone.—Determine the volume (by means of the vessel as above stated) of a piece of the stone and its weight.

From the result thus obtained, determine by calculation the weight of a cubic yard of that stone. Multiply that weight by the proportion of voids and the result will be the weight of a cubic yard of the broken stone.

Area covered by a cu. yd. of broken Stone.—We have **Ah=1** in which **A**=area in sq. yds., and **h** the depth of the layer in yds. If **h** is given in inches, then **Ah"**=3x12 and **A**=36÷**h"**.

Rule: Divide 36 by the thickness in inches.

After rolling, the volume of the material is reduced about ¼ and the formula to be used is then A=27÷**h** in which **h** = in inches the final thickness.

Macadam Pavement.—A Macadam pavement differs from the Tilford only in the omission of the bottom course or large stones. It is otherwise treated as previously explained for a Telford pavement.

Artificial Stone Pavements.—Brick pavements—Concrete pavements.

Brick Pavements.—Made with cross-rows of paving bricks much like a Belgian pavement.

Foundations.—Either concrete, broken stone, sand or sand and gravel well rolled and covered with a sand layer of from 1½ to 3 inches in thickness.

Qualities of good Paving Bricks.—1° Acid proof.

2° Give clear ringing sound when struck together;

3° Rough to the touch and not apt to receive a polish;

4° Fracture free from pebbles, air holes and laminations; should be close-grained, uniform and compact;

5° Should not scale-off or chip under a quick blow of a hammer on the edges;

6° Should absorb very little water; 1-600 of its weight in 48 hours is a maximum.

Brick Tests.—1° **Resistance to crushing** is ascertained by a testing machine.

2° **Resistance to breaking** or **modulus of rupture** is given by formula:

$$R = \frac{3Wl}{2bd^2}, \quad \text{in which} \begin{cases} R = \text{Modulus of rupture}; \\ W = \text{Breaking weight}; \\ l = \text{Distance between supports}; \\ b = \text{Breadth}; \\ d = \text{Depth}. \end{cases}$$

The breaking weight **W** is ascertained by placing a brick on edge on two rounded knives 7 inches apart and loading the sample at the center until fracture.

A common test consists in letting a brick fall flat from a height of about 4 feet upon another brick standing edgewise. It should not break.

3° **Resistance to Abrasion** is ascertained by placing 4 or 5 bricks, previously weighed, in a **rattler** (instrument used to clean castings) together with scraps of cast iron—about 100 lbs. of half pound pieces. Revolve the rattler about 20 times in half an hour, remove the bricks and weigh them. Replace them in the rattler and repeat the operation for another half hour; weigh again and calculate loss.

4° **Absorption.**—Dry the bricks to be tested and weigh them. Immerse them in water for 24 hours, or 48 hours and weigh again. The difference in weight will give the amount of absorption. Bricks absorbing one or more per cent. in 24 hours should not be accepted as paving stones as they are apt to crack in frosty weather.

Laying the Bricks.—They are laid on edge, in straight courses perpendicular to the curb, with their length in the course, their thickness forming the width of the course. The joints must break not less than 3 inches. Bricks must be well hammered in the course and against the bricks of the adjacent course previously laid.

Where Portions of Bricks may be used.—Only at the starting and closing of a course and never in the course. These portions of bricks act as keys.

Ramming.—Ramming is to be begun when about 30 ft. of pavement has been laid. The rammer should not weigh less than 50 lbs. Remove all bricks that sink below the surface level and throw more sand or use deeper bricks.

Joints.—Materials used for joints are either sand, grout or pitch.

Characteristics of Paving Bricks.—Weight: 5½ to 7½ lbs., according to size.

Specific gravity: 1.9 to 2.7;

Resistance to crushing: 7000 to 18000 lbs.;

Resistance to breaking: 1400 to 2000 lbs.;

Absorption: 0.15 to 3% in 24 hours.

Bricks.	Dimensions.	Weight.	Number.
Standard.	2½ x 4 x 8 in.	7 lbs.	requires 58 to a sq. yd.
Repressed.	2½ x 4 x 8½	7½	" 56 " " "
Metropolitan.	3 x 4 x 9	9½	" 45 " " "

Concrete Pavements.—Mostly used for alleys or streets with a light traffic and for sidewalks. The concrete used is mostly composed of sand, gravel, crushed stone, and hydraulic cement, to which is added a mineral substance to increase its resisting power, and in this case the process of manufacture is generally patented.

Varieties of Concrete Pavements.—The principals are called monolithic, granolithic, metalithic, ferrolithic, kosmocrete, etc., and their names are derived from the process of manufacture or from the indurating material used.

Foundations.—Dug to a depth of about 8 inches; ground well rammed and covered with a bed of cinders, gravel and sand or broken stone, also well rammed to a uniform surface. The concrete is then placed on this foundation, well rammed and floated. Sometimes a concrete containing a greater proportion of broken stone is first laid, and on top of this a layer of concrete with smaller sized broken stone is used.

Precautions against Frost.—This is done by providing elastic joints at regular intervals. To that end, the concrete is laid in regular figures as squares, lozenges, hexagons, etc., confining it by wooden strips or separators about ½ inch thick which are removed after the setting, and the space which they occupied is filled with an elastic material as sand and pitch.

Wood Pavements.—The blocks used are yellow pine, cedar, juniper or cypress. They are used either in the natural state or me-

chanically impregnated with a composition for preserving them by reducing their absorptive power; it may be coal-tar, creosote, asphaltum, etc.

Foundations.—The blocks are sometimes laid simply on the rammed foundation, or on a bed of sand, sand and gravel. broken stone, concrete, etc.

Nicolson Pavement.—The blocks rest on a foundation of boards and the joints are filled with gravel and coal-tar.

Laying the Blocks.—The same as brick, hammered to a close joint.

Ramming.—The rammer should be lighter if the bed is concrete.

Joints.—Filled with sand, paving-pitch or Portland cement grout.

Example.—A newly laid wood pavement in New York City has a concrete bed of about 4 inches. This was allowed to set and was covered with a layer of sand about 1 inch thick on which the blocks were set close-jointed. The blocks were cut to shape around manholes, vent, gas and water covers. When a portion was completed, it was covered with a dry mixture of sand and cement which was watered by means of a hose and worked with a hoe into the joints in the form of a grout. The whole was left to dry several days before traffic was allowed. There is no gutter, the pavement proper taking the full width of the street from curb to curb. The blocks are impregnated with coal-tar and creosote.

Note: This pavement is very slippery.

Asphalt Pavements.—The material employed is mostly bitumen or asphalt used in a mixture with a more induring material.

Bitumen and asphalt are mixtures of hydrocarbons.

Bitumen.—The term is generally applied to those mixtures of hydrocarbons which are found in the earth under a more solid form, as those from Sicily, France, Switzerland, etc., in Europe, and the bituminous sandstones of Texas, Kentucky, California, etc., in this country.

Asphalt.—The term is reserved for those less compact and more semi-fluid masses of hydrocarbon substances found on the surface of some lakes as the Asphaltite lake (Dead Sea) of Judea and the pitch-lake of Trinidad.

Maltha.—A thick mineral pitch supposed to be formed by evaporation and drying of petroleum.

Aggregate.—The term is used to denote those materials which are used in asphalt paving to render the mixture more enduring, more resistant to the traffic.

Matrix.—The term is used to denote the asphalt cement or asphalt mastic which are mixed with the aggregate to render it cohesive.

Refined Asphalt.—The crude product is placed in open vessels and indirect heat is applied (the material itself being inflammable) under which treatment the light oils and water evaporate; the vegetable matter rises to the surface where it is skimmed off, and the mineral matter settles at the bottom. The refined asphalt is decanted in barrels.

Bitumen Blocks.—Bituminous limestone is pulverized and mixed with 8 to 10 per cent. of refined asphalt; it is melted and thoroughly stirred. The mixture is then moulded into cakes or blocks of different shapes while still hot. The hexagonal shape is preferred. Blocks weigh from 55 to 60 lbs.

Asphalt Cement.—Is prepared from refined asphalt by melting it at 300° F. when about 15% of hot residuum oil (from petroleum) is added and the mixture stirred during 2 or 3 hours. Proper consistency or tempering is obtained by the admixture of maltha (at a lower temperature). At 90° F. the cement should draw into threads, and at 75° a finger-nail should leave its mark.

Paving Mixture.—It is obtained by adding the materials composing the aggregate to those composing the matrix.

Bitumen Mastic.—Bitumen blocks are broken into small fragments, and in that form they are thrown into a caldron containing about 5 per cent. of refined asphalt already melted. A wood fire is lighted under the cylinder and the whole is stirred constantly when melting.—Coal fire is injurious and coke is even more so and should never be used as fuel.—Gravel not larger than a pigeon's egg is added with sharp sand and stirring is continued for 2 hours or so at 300° F. An excess of asphalt should be avoided in warm climates. It is then ready for use.

Asphalt Mastic.—Mastic is also made from refined asphalt and from asphalt cement.

Uses of Mastic.—Mastic is used principally for floors and sidewalks.

Paving Composition.—Made with a mixture of asphalt cement, sand and stone dust in the following proportions:

Asphalt Cement	*15 %*	Proportions may vary according to
Sand	*75 "*	(a) consistency of cement;
Stone Dust	*5 "*	(b) hardness desired.

How the Ingredients are mixed.—The sand and stone dust are first mixed then heated before being thrown in the hot cement; this is done in order not to chill the asphalt. The mixture is thoroughly stirred until the components cannot be distinguished and appear to form a homogeneous mass. The mass should give off puffs of smoke and the asphalt should drop freely from the stirrer.

Foundations.—The foundations are generally a concrete bed 4 to 6 inches thick which should be set and thoroughly dry before the composition is applied in order to prevent the formation of steam holes and seams and subsequent cracks in the bearing surface.

Laying the Composition.—It is sometimes laid in two coures. The lower one, called **binder,** is generally ½ inch thick and contains about 5% more cement in order the better to bind the foundation and to form an elastic cushion for the shocks of the traffic. The upper course or bearing surface is about 2 inches thick (after rolling), contains a greater proportion of aggregate and is therefore more wearing. The mixture is hauled hot to the spot, dumped and spread with rakes, sometimes finished with trowels, in bands about 3 feet wide. Before laying the next band the joint should be heated.

Surfacing.—The top of the asphalt should be sprinkled with fine, sand, or chalk powder before rolling in order to prevent adhesion.

Rolling.—Rolling is resorted to in order to compress the mixture. Two rollers are often used. The first one, or compressor, has a narrow rim and weighs about 5 tons; the second one, or finisher, has a broad rim and weights about double the first.

Rolling is done longitudinally and from curb to center.

Tamping-irons heated in a movable basket are used where the rollers cannot reach, as around man-hole or hand-hole covers, along the gutter, etc.

Duration of Rolling.—The rolling should occupy about one hour for every thousand square yards of surface.

Shrinkage by Rolling.—The compression after the rolling is done is about 40 per cent. of the original thickness of the layer. If therefore a thickness of 2½ inches of finished paving is desired, the layer, as first placed, should be:

$$x = \frac{2.5 \times 100}{60} = 4\tfrac{1}{6} \text{ in. thick}$$

Area covered by a cu. yd. of Composition.—

If 1.5 in. thick, 27 sq. yds.

If 2 in. thick, 18 sq. yds.

If 2.5 in. thick, 12 sq. yds.

Weight of a cu. yd. of Composition.—About 4500 lbs.

Amount of refined Asphalt required for a cu. yd. of Composition.—About 590 lbs.

Where Asphalt Pavement should not be used.—An asphalt pavement should not be laid where the surface is likely to be saturated with greasy substances.

Thickness of Asphalt Floors or Coverings.—

Over arches ½ inch.

Sidewalks ¾ to 1 in.

Warehouses 1¼ in.

HEAT.

Heat.—Rapid and irregular vibratory motion of the molecules of a body.

By adding heat to a body the motion of the molecules is increased.

Liquefaction.—When that motion becomes intense enough that the molecules cannot hold together the body becomes liquified.

Liquifying Point.—Moment when a body passes from solid to liquid state.

Vaporization.—By applying still more heat to a body in a liquid state, we increase the more the motion of the molecules. This motion may reach a point when the molecules move away from each other and escape into the atmosphere; the body then becomes a vapor,—has been vaporized.

Evaporation.—When the phenomenon of vaporization takes place slowly and only at the surface of liquids.

Temperature.—Degree of heat of a body, compared to the heat which other bodies exhibit under known conditions.

Thermometer.—Instrument designed to measure temperature.

Centigrade, Reaumur and Fahrenheit Therm. have been described in **Pneumatics.** Their principle is expansion and they are used to measure ordinary temperatures.

For very low temperatures, the principle of change of electrical resistance is applied.

For very high temperatures, thermo-electric effects are utilized.

Expansion of Solids.—Increase in the dimensions of a body subjected to rising temperature.

Co-efficient of Length Expansion.—Elongation per degree Centrigrade of a solid of 1 unit of length at temperature O°**C.**

Consult tables for different kinds of solids.

Co-efficient of Area Expansion.—Twice the co-efficient of linear expansion.

Co-efficient of Volume Expansion.—Three times the co-efficient of linear expansion.

Length of a Rod of Length L at n°C,, 1 being its Length at O°C.

$L = l(1 + en)$, e being the lin. coefficient of the material of the rod,

or $$l = \frac{L}{1+en}.$$

Expansion of Liquids.—It is volume expansion only. For most liquids, expansion is variable and increases rapidly with temperature.

Exception.—Water contracts, as temperature rises from 0°**C** to 4°**C.** at which latter point it obtains its maximum density. Below 0° and above 4°**C** it follows the general rule.

Expansion of Gases.—It is practically the same for all gases:

$$\frac{V}{273} \text{ (} V \text{ being their volume) per degree C, or } \frac{V}{491} \text{ per degree F.}$$

Volume or Pressure V of a gas at n° C whose temperature is v at 0°C.

$$V = v\left(1 + \frac{n}{273}\right),$$

or, if in degrees F $\quad V = v\left(1 + \frac{n-32}{491}\right)$ in which v is the temp. at 32 F.

Contraction of Gases.—It is the converse of the expansion. The bulk (or pressure) decreases

$$\frac{V}{273} \text{ per degree C, or } \frac{V}{491} \text{ per degree F}$$

Absolute Zero.—From this it seems that by lowering the temperature of a gas to — 273° **C** or — 459° **F** there would be no more pressure but equilibrium between the particles. This point is called the **absolute zero** of temperature.

In liquefaction of gases a temperature of —260°**C** has been attained.

Notes on Liquefaction.—Laws of Fusion—1° A substance always melts at the same temperature, under the same pressure.

2° Temperature remains constant from beginning to end of melting.

3° A liquid solidifies at the temperature of melting.

Laws of Vaporization.—Similar to laws of fusion.

All substances have not the same boiling point (consult tables).

Distillation.—Process of the removal of a liquid from another or from a solid.

Instrument of Distillation.—A **still,** which is a closed vessel called **retort,** laid over a fire and containing the substance to be distilled. A pipe on the top of the vessel allows the vapors to escape to a coil of pipe or **worm** called **condenser,** surrounded by cold water which condenses it, the liquid falling into a receiving basin.

Heat Units.—1° **Thermal Unit:** Amount of heat required to raise a pound of water from 59° to 60° **F.** This is the British unit and is used for steam and fuel.

2° **Calorie:** Amount of heat required to raise a gram of water from 15° to 16° **C.** This is the French unit and is used for scientific work.

Thermodynamics.—Treats of the relations between heat and mechanical power.

First Law.—When heat is applied to work, the quantity of heat is equal to the quantity of mechanical energy.

French Unit.—427.3 kilogrammeters of work are required to raise from 15° to 16° **C** 1 kilogram of water at sea-level and latitude 45°.

English Unit.—778.8 foot-pounds of work are required to raise from 59° to 60° **F** 1 pound of water at sea-level and latitude 45°.

Second Law.—Heat cannot pass from a cold to a hot body.

Third Law (Boyle).—The product of volume **V** of a perfect gas by pressure **P** is constant for the same temperature.

$$V\,P \quad V'P' \; : \qquad P' = \frac{VP}{V'}$$

Fourth Law (Boyle and Charles).—The product VP is proportional to the absolute temperature **T.**

VP=CT: **C** being a constant which for air is 53.2.

Isothermal Expansion.—State of a gas expanding and doing work when supplied, from outside, with heat sufficient to keep it at a constant temperature.

Adiabatic Expansion.—State of a gas expanding and doing work after the supply of heat has been cut-off. Its temperature falls rapidly.

The relation between pressure and volume is then

$$PV^n = C \;,$$

C being a constant and **n** varying with the several kinds of gas.

For air **n**=1.405.

Work in Expansion done by a single Stroke of a Cylinder Piston.—

$$w' = pAl$$

In a full stoke $\quad w = 2pAl$

In a minute $\quad W = 2pAln.$

in which w', w and W = Work done in *ft. lbs.*

p = Pressure, *lbs.* per *sq. in.*

A = Area of Cylinder, *sq. in.*

l = Length of Piston stroke, *in.*

n Number of full strokes in a min.

Application of the expansion of gases and their resultant work is made in the steam engine, the hot-air engine, the gas engine; in refrigerating machines, etc.

BOILERS.

Steam Boiler.—A metallic closed vessel called **shell,** made of steel plate ¼" to ¾" thick, destined to be partly filled with water to be heated from an external source and to generate steam.

Principal Attachments.—**Supply pipe, pump** or **injector** with cocks or valves to feed the boiler with water.

Glass Water Gauge.—With **gauge cocks,** to indicate the level of water in boiler.

Indicator or **Pressure Gauge** to indicate in lbs. per sq. in. the pressure of steam in the boiler. (To this should be added about 15 lbs. of atmospheric pressure to have absolute pressure.)

Safety Valve.—A lever destined to allow the escape of steam when it exceeds a certain limit. It sometimes actuates dampers to regulate the draught.

Blow-off.—With valve, to reduce amount of water in boiler or empty it completely.

Steam Pipe.—With valves, to carry steam where needed.

Manholes, Handholes.—With covers and fastenings for same, to inspect or clean parts.

Sometimes also **Heater** to warm water before injection into boiler.

Alarms for high or low water, to give warning of the same.

Fusible Plugs.—Which melt and allow water to escape thus quenching fire.

Classes of Boilers.—Stationary Boilers, Locomotive Boilers and **Marine Boilers,** according to the use they are put to.

Kinds of Boilers.—Cylindrical Boiler—Set in masonry, 2-3 filled with water, surmounted by a dome to which the steam pipe and safety valve are attached.

It is heated externally, the hot gases passing under and along the boiler to the flue in the rear.

Flue Boiler.—The hot gases are carried in one or two large flues internally fired and passing through the water, with as many fires as there are flues.

Cornish Type has one flue and one furnace.

Lancashire Type has two flues and two furnaces.

Galloway Type has one or two flues with corrugated sides and tubes placed perpendicularly to the direction of the flue, such tubes opening at both ends in the boiler of which they form a portion, thus 1° adding to the heating surface, 2° increasing the effectiveness of the gases by reducing their speed through the flue and 3° acting as stays in the flue whose longevity is thus increased.

Multitubular Boiler.—Is externally fired. It contains numerous flues, or rather fire tubes, 3" or 4" diam. within the lower ¾ portion of the boiler. These fire tubes increase the heating surface.

Water Tube Boiler.—The water is in the tubes and the hot gases surround them thus heating the water.

Horizontal Boilers are set horizontally with a slight inclination towards the blow-off pipe generally placed in the rear.

Vertical or upright Boilers.—They are set vertically to economize floor space.

Sectional Boilers.—The shell and corresponding tubes are made in short detacheable sections, so that the boiler may be made larger or smaller.

Horsepower of a Boiler.—

$$HP = \sqrt{SA}$$

in which HP Nominal horse power of boiler.
S = Heating surface in sq. yds
A Grate area in sq. ft.

For Cylindrical double flue: $HP = \frac{2rl}{6}$

in which r = Radius of boiler;
l = Length of boiler.

Approximate Quantities per Horsepower.—Capacity of boiler = 1 cu. yd.

Heating surf. = 1 sq. yd.

Water consumption = 1 cu. ft. per hour.

Grate area = 1 sq. ft.

Flue area = 28 sq. in.; over bridge 18"

Grate.—Its length is from 6 to 7 ft. in large boilers. This is about the dist. a man can throw coal.

In many cases the length is limited.

Grate Area.—A=W÷w r in which **A**=grate area in sq. ft.

W=Total water evaporated in lbs. per hour.

w=Water evaporated with 1 lb. of coal.

r=Rate of combustion in lbs. per sq. ft. grate area per hour.

Rate of Combustion r varies with the draft and kind of boiler.

With *Ordinary Draft* (Chimney):		
Cornish boiler,		4 to 15 lbs. coal p. sq. ft. grate area p. hour;
Ordinary factory "		12 18
Marine "		15 25
Coal-Quick rate	Anthracite,	15 20
	Bituminous.	20 - 30
With *Force Draft*:		
Marine boiler,		60 - 130 lbs. coal
Locomotive "		40 - 120

Steam Space.—Generally ⅓ the boiler. That space should be equal (experience) to steam consumed in 20 seconds.

$$s = \frac{20p \times 4.86}{3600} = \frac{2.43p}{90},$$

in which s Steam space in *cu. ft.*
$20 =$ 20 seconds;
p Steam pressure, *lbs. p. sq. in.*
$4.86 =$ Specific vol of steam at 75 *lbs. pressure (1 HP)*, which is $75 + 15 = 90$ *lbs.* absolute pressure,
$3600 = 60^{min.} \times 60^{sec.}$

Tube Space:

$t = \dfrac{2\pi r l n}{1728}$; in which t Tube space in *sq. ft.*
r External radius of a tube in *in.*
$l =$ Length " " " " "
$n =$ Number of tubes

Water Space:

$w = \frac{2}{3} s = \dfrac{2.43p}{135}$; in which $w =$ Water space in *cu. ft.*

Size of Boiler:

$$b = s + t + w = \frac{2.43p}{90} + \frac{2\pi r l n}{1728} + \frac{2.43p}{135} = \frac{2.43p}{54} + \frac{2\pi r l n}{1728}.$$

l being known or assumed, the radius of the ends will be calculated by the formula: $R = \sqrt{\frac{b}{\pi l}}$.

Kind of Boiler	*Ratio of*					
	Heating Surface to	Grate Surface	Grate Surface to	Horse Power	Heating Surface to	Horse Power
Cornish	27	32				
Lancashire	26	33	.1	.165	2.75	4.25
Horizontal - Internally fired	40	50				
Water tube	35	65			10	12
Locomotive			.02	.06	1	2
" - forced draft	30	34				
Marine	28	32				
" - return tube	25	38			3.25	4
Multitubular			.4	.6	14	18
Plain cylinder			.5	.7	5	10
Vertical			.6	.7	15	20

Chimney.—

$$\text{Area } A = \frac{15\,c\,HP}{\sqrt{h}};$$

in which A Area of chimney at top in *sq. in.*
c Coal consumed per *HP* per hour in *lbs.*
HP Horsepower of boiler;
h Height of chimney (above grate);

from which $h = \dfrac{225\,(c\,HP)^2}{A^2}$ if A is assumed.

Draft.—

$V = 36.5\sqrt{h(T-t)}$; in which V = Velocity of draft in ft.
h = Height of chimney in ft.
T = Temperature of air at bottom of chimney.
t = Temperature " " " top of chimney.

Materials Used in Boilers. Cast Iron.—Used only for fittings, parts of water-tubes and ends of low-pressure cylinders.

Cast iron resists corrosion and is cheap.

Cast iron is unreliable in texture and is brittle. The parts must be made heavy

Wrought Iron.—Used for fastenings and stays.

Wrought iron is not brittle and has a high tensile strength.

Steel.—Is now mostly used in boiler construction.

Steel has a great tensile strength, is tough, uniform in texture and generally free from internal defects.

How the Pieces are United. Lap Joint.—When the ends of the plates are placed one on top of the other. They are united by rivets passing through both plates.

Such a joint is generally used laterally or crosswise.

Butt Joint.—When the ends of the plates are brought close together and a strap or **butt** is placed on top of them. They are united by rivets passing through the butt and one plate.

Such a joint is generally used lengthwise.

When the ends of the plates are confined between two butts, we have a double butt joint.

Edges of Plates to be Planed.—After drilling holes and bending plates to shape, the edges are planed and beveled to facilitate calking.

Rivet Holes.—Holes are either punched or drilled.

Punching is sometimes used in wrought iron plates.

Drilling is most generlly used.

Rivets.—Made from mild steel or iron bar. One head is made by means of a cup or pan die. The straight portion or **shank** is slightly beveled.

Rivet Heads.—They are either button, cone or countersunk.

Riveting. 1° Hand Riveting.—Used only when machine riveting is not possible.

Two riveters insert the red-hot rivet and form the second head, either by hammering or by using a die called **snap** of the required shape.

2° Machine Riveting.—Preferable because it is more uniform, more accurate and faster.

Heating Rivets.—The ends only of iron rivets are heated to a white heat. Steel rivets are heated uniformly to a bright cherry red in a fire rather thick with a moderate draught.

Why Rivets should be put in red hot.—Because (a) they are worked more easily; (b) they contract in cooling and clinch the plates making a tight joint.

Countersunk Head.—Necessary only when straps or mountings are to be attached to boiler. Otherwise should be avoided because they weaken the plates.

When and where used, a countersink is previously drilled in one plate.

Pitch.—Distance between rivets.

Single riveted Joint.—When only one row of rivets is used.

Double riveted Joint.—When two lines of rivets are used.

Staggered riveting.—In a double riveted joint, when a row of rivets is opposite the spaces of the second row. It is also called **zig-zag** riveting.

Chain riveting.—When the rivets in a row are opposite the rivets in the other row.

Testing rivets. (a) A cold rivet should bend around another rivet (of same diam.) in the form of a horseshoe and show no crack or flaw.

(b) A hot rivet should bend around another rivet in the form of a ring and show no crack or flaw.

(c) A hot rivet head should flatten to 2½ diameters and show no crack or flaw.

Strength of riveted Joints. (All holes drilled).

Single Row—Lap or Butt.—62% of plate strength.

Double Row—Lap or Butt.—75% of plate strength.

Single Row—Double Butt.—67% of plate strength.

Double Row—Double Butt.—79% of plate strength.

STEAM ENGINE.

Parts.—Cylinder—A fixed hollow metallic cylinder into which steam is admitted from the boiler. It is sometimes surrounded by an outer cylinder and steam is admitted between the two, to prevent the too rapid condensation of steam in the cylinder. It is called a steam jacket.

Piston.—A solid short cylinder of same external diameter as the cylinder proper into which it fits and is free to move to and fro under the expansive force of steam acting upon one of its ends or faces.

How the Piston acts.—Steam is admitted into the cylinder for a portion only of its stroke and is then **cut-off** automatically. By the expansion of this amount of steam the stroke is completed.

Piston Rod.—A cylindrical steel rod securely fixed to one end of the piston at its center, and moving in a line with its axis which is that of the cylinder.

Cross Head.—A connecting piece attached to the free end of the piston rod and whose object is:

1° to guide the piston in its motion along the axis of the cylinder, by sliding between **crosshead guides;**

2° to transmit its motion to the **connecting rod.**

Connecting Rod.—A steel rod (forged) jointed at one end with the cross head and encircling the crank pin at the other.

Crank Pin.—A cylindrical steel pin uniting the connecting rod with the crank.

Crank.—One or two arms of a shaft, or a disc keyed to the end of it, and either of which is of a length equal to ½ the piston stroke. The crank transforms the rectilinear motion of the piston into the circular motion of the shaft.

Shaft.—Transmits the rotary motion of the crank to the fly wheel.

Fly Wheel.—A heavy wheel of large radius put in motion by the shaft which is attached to its center. Its object is two-fold.

1° Its momentum acts in regulating the motion of the crank.

2° By means of a belt on the plane face of its periphery it gives motion to the several parts of the machinery.

Eccentric.—A disk attached to the shaft at another point than its center. It is encircled by an eccentric strap terminating in a rod and stem actuating a slide valve in the steam chest.

Slide Valve.—It moves to and fro in the steam chest, directing the steam alternately to the front and back of the piston in the cylinder. A space within it receives the exhaust steam.

Steam Chest.—A box placed on one side of the cylinder, into which steam is received directly from the boiler. The steam is admitted into the cylinder through apertures or **ports** opening at each end of the cylinder; these apertures are shut and opened alternately by the steam valve so that the steam may act either on the back or the front of the piston.

Clearance.—The cylinder is made a little longer than the stroke of the piston so as to leave a little space called **clearance** on the front and at the back of the piston at the beginning and at the end of the stroke, for the admission of steam, and into the clearance the steam ports of the chest open.

Governor.—A contrivance put in motion by the shaft and designed for either of the two following purposes:

Cut-off Governor.—1° to regulate the amount of steam admitted into the cylinder; it is then a **cut-off governor;** or

Throttling Governor.—2° to regulate the pressure of steam admitted into the chest by throttling it in the main steam pipe; it is then a **throttling governor.**

The governor is generally a spindle put in motion by the shaft. At the top of the spindle two levers are hinged, each carrying a heavy iron ball or circular weight at the other end. Rods are hinged to these levers near the ball, sometimes at the center of the weights, and their other ends are hinged to a strap or ring encircling the spindle and free to move up and down. The ring or strap rests on the end of a lever (or is even hinged to it) which controls the throttling or cut-off valve. When the engine is apt to **race** the governor revolves at a greater speed, the balls move further apart on account of centrifugal force; the strap or ring is forced up the spindle, and the lever which it controls throttles or cuts off steam.

Safety Valve.—(theoretical)—

$p = \frac{WL}{\pi r^2 l}$; in which p Pressure in *lbs.* p. sq. in. on safety valve,
W = Weight in *lbs.*
L = Length of lever arm for W, in *ft.*
l " " valve arm, in *ft.*
r Radius of valve in *in.*

Taking into account the weight **w** of the lever and that **w'** of the valve, the condition of equilibrium is

$p = \frac{1}{\pi r^2}\left[\frac{1}{l}(WL + wl') + w'\right]$; in which l' = Distance from C of grav. of lever to fulcrum.

Generally $r = .0437\sqrt{A}$; in which A Area of fire-grate,
then $p = \frac{1}{.006A}\left[\frac{1}{l}(WL + wl') + w'\right]$

Theoretical Horsepower of Engine.—

$T.HP = \frac{144\ vp}{33\ 000}$; in which v = Vol. of steam discharged in *cu. ft.* per min.
p = Pressure, *lbs.* per sq. in.

In this formula, allowance should be made for friction, etc., in order to obtain actual **H. P.**

Indicated Horsepower.—

$I.HP = \frac{PLAN}{33\ 000}$ in which P = Mean press. on piston, *lbs.* p sq in. (during one stroke);
L = Length of stroke in *ft.*
A = Area of piston, *sq. in.*
N = No. of strokes p. min.

KINDS OF ENGINES.

Simple Engine.—Has only one cylinder and at the end of the stroke the steam is exhausted into the atmosphere or into a condenser (as in marine engines where the supply of fresh water is limited).

Compound Engine.—Has two cylinders placed alongside of each other or in tandem. The second or low pressure cylinder is of a larger diameter than the first or high pressure cylinder and receives its exhaust. Each has its piston rod, cross-head and connecting rod. Their cranks are generally placed at right angles to each other thus neutralizing the effects of their respective dead centers (beginning and end of stroke).

Formulas: $\frac{V}{V'} = \frac{1}{\sqrt{e}}$; in which V = Vol. of high pressure cylinder ;
V' = " " low " "
e = Ratio of expansion.

or $\frac{r}{r'} = \frac{1}{\sqrt[4]{e}}$, in which r = Rad. of high press. cyl. } The length of the cylinders or
r' = " " low " } stoke being the same.

Triple Expansion Engine.—Has three cylinders of increasing diameters: High pressure, intermediate and low pressure.

The Volumes are generally $1 : \begin{cases} 2.25 \\ 2.75 \end{cases} : \begin{cases} 5 \\ 8 \end{cases}$

Quadruple Expansion Engine.—Has four cylinders.

Low Speed Engine.—Makes from 60 to 90 revolutions per minute.

High Speed Engine.—Makes from 200 to 1000 revolutions per minute.

Horizontal Engine.—The piston moves in a horizontal direction.

Vertical Engine.—The piston moves in a vertical direction.

Vertical engines are used where space is to be spared, as in marine engines.

Steam Turbine.—The power is applied directly to the shaft. There are two principal systems.

De Laval Turbine.—On the shaft is mounted a wheel on the face of which, near the periphery, are a number of buckets. Steam is brought to strike these buckets (thus moving the wheel and shaft), by means of several tuyeres or nozzles equally spaced and placed at the proper angle. The bore of the nozzles increases from the thread outward so that the steam may act on the wheel at the pressure of condensation.

A governor controls a throttle-valve which regulates the admission of steam.

Parsons Turbine.—In the turbine of the Parsons Type, the shaft is enlarged to a long cylinder called **Rotor,** the surface of which is provided with parallel rows of crescent shaped blades. This cylinder is surrounded by a jacket called **Stator,** fixed to the bearings, and also provided with like blades of contrary directions to those of the cylinder and situated between the rows of the latter. The steam is admitted intermittently into a chamber at one end of the cylinder where it strikes the blades of the jacket and those of the cylinder alternately; causing this (and the shaft which is solidary) to rotate. There are generally two or more cylinders of increasing diameters following one

another in tandem on the shaft; they are also provided with blades and there the expansion takes place. They act as the several cylinders of an expánsion engine.

METALLURGY.

Metallurgy.—The art of extracting metals from ores.

Metals used in Construction.—The principal metals used in construction are iron, copper, lead, tin, zinc, aluminum. The most important is iron.

Kinds of Iron.—Cast iron, wrought iron and steel.

Iron Ores.—Black ore or **Magnetite,** contains 40 to 60% iron.

Red ore or **Hematite,** contains 65 to 70% iron.

Brown ore or **Limonite,** contains about 60% iron.

These ores contain foreign substances: Alumina, silica, sulphur, phosphorus, etc.

Blast Furnace (Fig. 280).—Vertical vessel in which iron ore is reduced; it is from 80 to 100 ft. high, the outside is of sheet iron and is lined on the inside with fire brick.

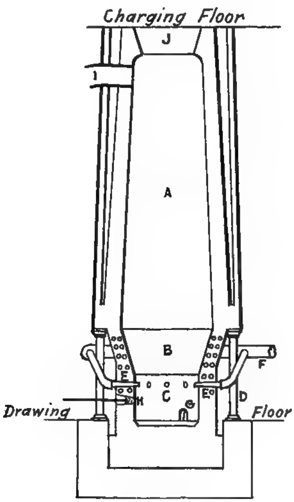

Fig. 280

Parts of a Furnace.—1° The shaft **A** is the top portion, a very elongated frustrum of a cone about 15 ft. diameter at the top or throat gradually increasing to about 20 ft. diameter at the bottom, in a length of about 60 ft.

2° The boshes **B** or middle portion, an inverted frustrum of a cone about 20 ft. diameter at the top or bosh line, which coincides with the bottom of the shaft, gradually decreasing to a diameter of about 14 ft. at the bottom, in a length of about 10 ft.

3° The **hearth** or bottom portion **C**, a cylindrical vessel of about 14 ft. diameter and about 10 ft. high.

These three portions form a single vessel resting on a solid foundation and supported at the boshes by columns or piers **D**.

Tuyeres.—A number of fire-proof pipes called **tuyeres E** enter the hearth at about 8 ft. from the bottom; they are connected on the outside with a large pipe **F** called **bustle pipe**, itself connected by pipes to the **Blowing engine.** This system furnishes a blast of air to the furnace. The air is generally heated by stoves on its way from the engine to the bustle pipe.

Crucible.—That portion of the hearth below the tuyeres.

Cold Water Circulation.—Cold water circulate around the boshes and the crucible where the heat is more intense.

Crucible Openings.—1° The **Tap Hole G** on the side and at the lower portion of the crucible; it is plugged with clay which is drilled just before the molten metal is drawn out.

2° The **Slag Eye H,** situated just below the tuyeres; it is closed with a metallic plug attached to a long handle. This hole is opened at intervals during the smelting process for the purpose of removing the slag or floating impurities.

Double Lining.—The shaft has generally a double lining of fire brick, leaving a space often filled with broken slab. This arrangement provides for the expansion of the inner lining.

Escape of Gases.—A large pipe **I** opening on the side and near the top of the shaft leads the gases and products of combustion to the stoves and boilers.

Charging Arrangement.—The throat of the shaft **J** is closed by a charging apparatus composed of a **hopper** into which the materials are dumped, and which is closed by a **bell** actuated by levers and counter-weights.

Fuel.—Coke, charcoal, anthracite. The best is a mixture of coke and anthracite.

Flux.—Substance used to combine with the impurities of the ore; they use limestone, dolamite, oyster shells.

Charging the Furnace.—The furnace is first dried. Cord wood is then piled up to about the middle of the boshes and a charge of coke is dumped on top, then successive layers of fuel, flux and ore, gradually increasing this last until the normal charge is attained. These materials are weighed in barrows called **buggies.**

Firing the Furnace.—After charging, the wood is lighted and the blast is turned on.

Blast.—At first little pressure is used; it is increased gradually.

Slag.—It is drawn off at intervals, sometimes even continuously.

Tapping.—Metal begins to flow 18 or 20 hours after firing. The molten metal is drawn about every 5 hours (or oftener) by drilling the tap hole.

Sow and Pigs.—They are channels dug in sand and into which the metal is allowed to flow. The **sow** is the main channel; the **pigs** are branch channels.

Furnace Production.—From 200 to 600 tons a day, according to size.

Run of a Furnace.—3 to 5 years.

Grades of Pig Iron.—According to fracture.

Gray	1 - Dark, rough fracture	- Foundry work exclusively;
	2 - Mixed grain, less rough	do. do.
	3 - Close grain	do. & rolling mill;
Mottled	- Dotted with graphite	- Rolling mill;
White	- No grain, smooth fracture	- do do. & Bessemer steel

Pig Iron to be remelted.—Pig iron contains silica, carbon, sulphur, phosphorus, manganese, etc., and must be remelted; this is done in the **cupola.**

Cupola.—A smaller kind of cylindrical blast furnace, where the pig iron is refined and made into cast iron; or in a

Reverberatory Furnace.—Where it is puddled, gathered into balls, hammered or rolled and made into wrought iron.

Effect of Carbon in Cast Iron.—Carbon in combination with iron produces hard and brittle iron white in color. When crystallized in the form of graphite and evenly distributed in the mass, the iron is gray in color, soft, expands more than white iron in solidifying and contracts less in cooling.

Effect of Silicon.—An increase of silicon in cast iron is an increase of crystallized carbon; it makes the iron softer. A large amount makes the iron stiff and weak.

Effect of Phosphorous.—It weakens the iron when there is more than 1½ per cent. It increases fluidity and decreases shrinkage.

Effect of Sulphur.—Sulphur in castings should not exceed half of one per cent. as it impairs good flowing and renders them unsound.

Effect of Manganese.—Manganese increases hardness and shrinkage of cast iron; it also helps iron to hold more carbon.

Steel.—Is made from wrought iron which is encased in charcoal, heated to red heat and kept so for about 8 days when it is taken out as **blister steel,** on account of being covered with scales. For purer steel, the same operation is performed on blister steel.

Crucible Steel.—Blister steel cut in fragments, melted in a graphite or black-lead pot or **crucible** and cast in an ingot afterward hammered into bars.

Bessemer Steel.—Made from pig iron in a **converter.** The converter is a large cylinder mounted on trunnions and terminating in an open frustrum of a cone from which mouth it is filled and emptied; its bottom is clamped, and through it tuyeres pass which blow a cold blast through the molten metal. The converter is lined with silicon lime or magnesia, and the action of this lining on the metal frees it of carbon, silicon, sulphur, phosphorus, to a greater or less degree.

Open Hearth Process.—Several chambers generally in pairs are provided below the **hearth** which is in the form of a hammock. These chambers are partly filled with fire bricks arranged in checker work, leaving spaces between them. The fuel used is gas, which burns on the hearths. The products of combustion are carried to a set of chambers below where the bricks become very hot. After a time the gas and air are turned into these chambers where they are highly heated before being led to the hearth where they burn, the products of combustion going to heat two more chambers. The current is so reversed every 40 or 50 minutes.

Martin Process.—In it pig and scrap are used.

Siemens Process.—In it pig and ore are used.

Combination Process.—In it pig, ore and scrap are used.

Effects of Carbon in Steel.—Up to 1.5% it increases both Tensile and Compressive Strength, raises Elasticity but reduces Welding power and Malleability.

Effect of Silicon.—Should not contain more than half of 1%, it would become brittle. Silicon increases hardness and Tensile Strength; it also prevents blow-holes.

Effect of Phosphorus.—It makes steel hard and apt to break; it reduces Elongation and increases Elastic Limit. Causes **cold shortness.**

Effect of Sulphur.—When hot, the steel is brittle under the hammer or roller; this is called **hot shortness.**

Effect of Manganese.—Counteracts in part the effect of sulphur, preventing hot shortness. Increases Toughness, Elongation and Tensile Strength. If in excess, it may cause **cold shortness.**

Special Steels.—Nickel Steel—Used in armor plate, tubing for bicycles, etc.; it is obtained by adding metallic nickel or nickel ore to the liquid steel in the open-hearth.

Aluminum Steel.—Used in castings and light structural steel.

Manganese Steel.—Is forged into car-wheels, dies, etc.

Case-Hardening.—When iron is red hot, it is sprinkled with potassic ferrocyanide in powder and suddenly cooled, when the surface becomes very hard steel.

Tempering.—A shaped steel piece is first hardened more than necessary then heated slowly to soften it until the right hardness is obtained.

TABLE OF IRON AND STEEL.

Name.	Aver. Specific Gravity.	Melting Point. F.	Tensile Strength. lbs. p. sq. in.	Weight, lbs. p. cu in.	Weight, lbs. p. cu. ft.
Cast iron	7.35	2100°	13 000 to 30 000	.26	445
Wrought "	7.6	3.000°	40 000 60 000	.278	485
Steel; 0.1% carbon	7.6	2500°	50 000 65 000	.283	490
" 0.2 "	7.67	2500°	60 000 80 000	.283	490
" 0.4 "	7.73	2500°	70 000 100 000	.283	490
" 0.6 "	7.8	2500°	90 000 - 120 000	.283	490

Alloy.—Several metals melted together.

Composition of some Alloys.

Name	Copper	Tin	Zinc	Lead	others	Notes
Aluminum Bronze	90				Aluminum, 10	High Tensile Strength
Babbitt's Metal	3	89			Antimony, 8	Bearings
Bearing Metal	77	8		15	Phosphorus, trace	Heavy Bearings
Bell Metal	75	25				Bells - Brittle
Brass	$66\frac{2}{3}$		$33\frac{1}{3}$			Pipes, Sheets, Tubes, Wire
Fusible Plug		10		86	Bismuth, 4	Safety Plugs for St. Boilers.
Gun Metal		91	9			Bearings, Castings, Ordnance
Manganese Bronze		89	10		Manganese, 1	Pumps, Propellers
Solder.		50		50		Common Solder

Colors in Iron Caused by Heat.

C°	F°	Color
210	410	Pale yellow
221	430	Dull "
256	493	*Crimson
261 370	503 698	Violet, purple and dull blue, bet 261°C & 370°C, it passes to bright blue, to sea green, and then disappears.
500	932	Begins to have light coat of oxide; looses much harness; becomes more impressive to the hammer; can be easily twisted
525	977	Becomes nascent red
700	1 292	Sombre red
800	1 472	Nascent cherry.
900	1 652	Cherry
1 000	1 832	Bright cherry.
1 100	2 012	Dull orange
1 200	2 192	Bright orange.
1 300	2 372	White.
1 400	2 552	Brilliant white *Welding heat.*
1 500 1 600	2 732 2 912	Dazzling white

HIS DUTIES.

What he has to inspect.—He may have to inspect: 1° **The material**; 2° The **work.**

Must know the Specifications.—He must be conversant with the specifications and have a copy of those sections which relate to his branch of the service.

Shall give his whole Attention.—He should give his whole attention and full time to the faithful performance of his duties, remembering that upon his vigilance depend the good execution of the work and the safeguarding of the financial interests of the City, State or Government.

The Division Engineer and the Chief Engineer both rely on the honesty and accuracy of his reports for their approval of the estimates on which payments to the contractors depend.

THE INSPECTOR.

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THE ASSISTANT ENGINEER

BY

PROF. JEAN P. GENTHON
Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK VII

THE DRAUGHTSMAN

WRITTEN FOR

THE CHIEF

Journal of the Civil Service

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THE DRAUGHTSMAN

Book VII

PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with the most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3°. The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examination in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON.

July 1, 1907. New York.

PRESS OF
THE CHIEF
NEW YORK

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1°. Aptitude for mathematics.

2°. Habit of observation.

3°. Good memory.

4°. System.

5°. Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION.—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials or new appliances and processes are used for old or new purposes. Notice the several kinds of labor, and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory, which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results; principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books, always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figures and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations, in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around you will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public works have a Chief Engineer who prepares the work and directs its

construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

			Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist
Chief Engineer.			Inspectors.	
	Division Engineers.		Transitmen.	Chainmen. Axemen.
		Assistant Engineers.	Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen.

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.
Boston, Mass.
Buffalo, N. Y.
Charleston, S. C.
Chattanooga, Tenn.
Chicago, Ill.
Cincinnati, Ohio.
Cleveland, Ohio.
Detroit, Mich.
Galveston, Tex.
Grand Rapids, Mich.
Jacksonville, Fla.
Little Rock, Ark.
Louisville, Ky.
Memphis, Tenn.
Milwaukee, Wis.
Mobile, Ala.
Nashville, Tenn.
New London, Conn.
New Orleans, La.
Newport, R. I.
New York, N. Y.
Norfolk, Va.
Philadelphia, Pa.
Pittsburg, Pa.
Portland, Me.
Portland, Oreg.
Rock Island, Ill.
St. Louis, Mo.
St. Paul, Minn.
San Francisco, Cal.
Savannah, Ga.
Seattle, Wash.
Sioux City, Iowa.
Tampa, Fla.
Vicksburg, Miss.
Washington, D. C.
Wheeling, W. Va.
Wilmington, Del.
Wilmington, N. C.
Yellowstone Park, Wyo

where examinations may be taken except that of Assistant Engineer, which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above-named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken at

Albany.
Amsterdam.
Auburn.
Binghampton.
Buffalo.
Elmira.
Hornelsville.
Ithaca.
Jamestown.
Kingston.
Lockport.
Malone.
Newburg.
New York.
Ogdensburg.
Olean.
Plattsburg.
Poughkeepsie.
Rochester.
Utica.
Syracuse.
Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewer construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman; topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps.

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service, appear regularly in "THE CHIEF."

BOOK VII

THE DRAUGHTSMAN

Draughtsman.—He who prepares plans, sections, elevations, maps, profiles, diagrams, from data furnished him by the engineers.

Who His Superiors Are.—The draughtsman is under the orders of the Division Engineer or the Chief Engineer.

In large offices doing important work, an assistant engineer or a Chief Draughtsman is in charge of the Draughting Bureau.

Classes of Draughtsmen.—Tracer or helper; Architectural Draughtsman; Mechanical Draughtsman; Topographical Draughtsman.

Tracer or Helper.—Assists the draughtsman in storing supplies, receiving and distributing them, preparing color washes, cleaning drawings and filing them, making blue prints, making tracings, etc.

Architectural Draughtsman.—Draws plans, elevations, etc., for buildings.

Mechanical Draughtsman.—Draws plans for iron structures, as bridges, machines, structural iron buildings, etc.

Topographical Draughtsman.—Draws maps of the ground from field notes and, together with the engineer, determines the best location for structures. He makes preparatory and comparative estimates. He draws property maps, calculates the areas and writes descriptions; he calculates and adjusts traverses, etc.

All Office Work.—The work of the Draughtsman is essentially an office work.

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service—Tracer or Helper.

Title.—Copyist topographic.

Age Limit.—20 years or over.

Examination.

Subjects.	Relative Weights.
1. Drawing, small specimens of topographic drawing to reproduce, preferably in india ink	50
2. Lettering, short words in different styles, also numbers	50
Total	100

New York State and County Civil Service.

Title.—Tracer.

Age Limit.—Minimum age 18 years.

Salary.—Usually \$40 to \$50 a month.

Specimen Questions.

1. Ink in the drawing attached.

2. Letter, in some suitable style, the following title to the drawing: "Design for a Railroad Arch Culvert."

3. Combine figures 1, 2 and 3 into one drawing to the scale $40' = 1''$

4. Trace figure 4 and ink in your tracing.

New York City Municipal Civil Service.

No mention in manual for tracer.

Junior Draughtsman.—Schedule B—Groups 1 and 2—Grade 1.

Salary.—\$750 to \$1,050.

QUESTIONS PROPOSED AT EXAMINATIONS.

DRAUGHTSMAN'S HELPER.

Mathematics.

1. Add 29 ft. 7½ in., 11 ft. 0¾ in., 7 ft. ⅞ in., 6 ft. 9⅝ in., and 17 ft. 11 and 5/16 in.

2. Reduce 2463¾ in. to feet and decimals of a foot.

3. Subtract from the answer of (2) 122.6875 ft. and reduce the result to inches.

4. How many cubic yards are there in a wall 207 ft. 6 in. long, 3 ft. thick, 12 ft. high at one end and 6 ft. high at the other?

5. What is the area of metal in a cast iron column 8 in. outside diameter and 1 in. thick?

6. How many pieces of tracing cloth 24 in. by 36 in. can be cut from a roll 60 in. wide and 24 ft. long? How many if a roll is 48 in. wide and 24 ft. long?

7. How many brick, allowing 21 to a cubic foot, are in a brick wall 20 in. thick, 21 ft. high, and 46 ft. long?

8. Multiply 231 by 672 and take 23½ per cent. of the product.

9. Add 42 ft. 6 in., 16 ft. 2 in. and 15 ft. 3 in.

10. Add: 20", 15¾", 6⅛" and 9 3/16".

11. Reduce 44⅜ in. to feet and decimals of a foot.

12. What is the difference between 3 ft. 7⅝ in. and 3.6354 ft.?

13. Find the area and circumference of a circle which has a diameter of 6 ft. 7 in.

Technical.

(Sketches or drawings may be done in ink or pencil.)

1. Define and illustrate by sketches of a table (a) plan, (b) elevation, (c) middle vertical section.

2. Make a neat tracing of these sketches.

3. Draw neatly a box 10 ft. long, 4 ft. high and 6 ft. wide in perspective, with one corner toward you.

4. What is the method of stretching drawing paper preparatory to using colors? How should colors be applied?

5. (a) How is tracing cloth treated to prevent drawing ink from running? (b) Which side of tracing cloth should be used?

6. How are blue prints made?

7. Show three different methods of indicating the scale of a drawing and state which is best, with reasons.

8. How long a piece of drawing paper will be required for a map of a small park, 430 feet by 250 feet to a scale of 1 equals 50?

9. (a) What is the difference between an inch and a tenth? (b) Between 3 sq. ft. and 3 ft. sq.

10 and 11. Indicate by quickly made but neat drawings: (a) a wooden beam, (b) concrete, (c) rock-faced masonry, (d) random coursed masonry, (e) a Corinthian column cap, (f) a Doric column cap, (g) a frieze.

12. What colors are used to indicate (a) wood, (b) concrete, (c) common brick, (d) iron work, (e) earth?

13. Describe two ways of laying off an angle, one of which is the most accurate way. Give reasons why it is so.

14 and 15. Draw neatly the following in your best style: ABC,

abc, 123 (in any quick, plain style); EGM, egm, 456 (in substantial block or Roman style); OQS, oqs, 789 (in round writing or Gothic style).

16. Define and illustrate by scale drawing the following geometrical figures: (a) square; (b) equilateral triangle; (c) isosceles triangle; (d) hexagon; (e) parallelogram.

17. Make a tracing of the above named figures; show names and scale.

18. How are maps enlarged?

19. What is the difference between a scale in tenths and one in twelfths? For what class of drawing is each used?

20. In plotting an angle what method would you employ? and why?

21. Define the terms "Plan," "Elevation," "Section."

22. (a) Draw a circle with a diameter of 2 inches and (b) inscribe an equilateral triangle; (c) draw a tangent to the circle at any point.

23. In using tracing cloth what precautions should be observed?

24. State how you would indicate on a drawing of a structure the different materials of which it is composed.

25. In applying colors to a drawing what precautions should be observed in order to produce neat work?

26. Describe the process of making blue prints.

27. Define "Perspective" and give an example.

28. How should drawing pencils be prepared for use?

29-30. Make neatly in your best style the following words:

Plan—New York.
Map—Brooklyn.
Profile—Queens.

31. What do you understand to be the duties of Junior Assistant Draughtsman?

32. How are original drawings reproduced so that various copies may be sent to contractors or used on construction?

33. What kinds of materials would the following colors indicate on drawings: (a) Blue gray, (b) red, (c) purple, (d) blue, (e) slate, (f) light blue, (g) yellow?

34. How many general drawings and what are they?

35. What scale is used for (a) general drawings; (b) detail drawings for school buildings?

36. How do the sides of tracing cloth differ? What is the advantage of using each side as compared to the other?

37. After having to erase ink on tracing cloth, how can you prevent the new ink from running?

38. How is the blue print made from a tracing?

39. What is a "T square" and how is it used?

40. Describe the two "triangles" most used and how lines perpendicular to the drawing board are ruled with them.

41. Why should ink on the outside of a right line ruling pen be wiped away? What is the best way of getting ink on a right ruling pen?

42 and 43. Draw neatly a column with ornamental cap and base.

44 and 45. Reproduce in your best style lettering: Front Elevation, Grammar School No. 35, Scale ¼ in. to 1 ft.

ARCHITECTURAL DRAUGHTSMAN.

Federal Civil Service—Supervising Architect's Office, Treasury Department.

Title.—Architectural Draughtsman.

Age Limit.—20 years or over.

Examination.

Subjects.	Relative Weights.
1. Building materials and construction; details of construction; specification forms	25
2. Drawing and designs; plans; elevations to scale from specifications	30
3. Free-hand drawing; ornamental and projection; free-hand perspective; decoration; shadows	25
4. Training and experience	20
Total	100

New York State Civil Service.

Title.—Architectural Draughtsman.

Salary.—$3 to $5 per day.

Open to both sexes.

Examination.

Subjects.	Relative Weights.
1. Practical and theoretical questions and drawings	6
2. Experience and education	4
Total	10

Specimen Questions.

1. Under what circumstances are the following scales used in making drawings: 1/8" scale, 1/4" scale, 1" scale and full size?

2. Describe the usual methods of indicating in the drawings the various materials to be used.

3. Give the average dimensions of the following parts of a staircase: Height of rise, width of tread, height of handrail; and how is this dimension measured?

4. A brick wall of a building exerts a pressure of 5 tons per sq. foot at its base; how would you prepare the footings for such a wall (a) when quicksand is found; (b) when good, hard clay is found? Make a drawing of the above, giving the thickness of wall, width, thickness and materials of footings.

5. Calculate the dimensions and the distance on centers of spruce beams which are to be used in the floor of a warehouse, the span being 20 ft.

6. What weight will a wall built of good, sound brick sustain per square foot if (a) laid in Portland cement mortar; (b) laid in Portland cement and lime mortar; (c) laid in lime mortar?

7. A building is to be twenty stories high. What method of construction would you employ, and why? Describe this method in detail.

8. Make a diagram of a scissor truss, a hammer-beam truss, a king-post truss, a queen-post truss, and a belly truss.

9. Calculate the size of a bluestone template which it would be necessary to place under each end of a girder carrying a distributed load of 235 tons, the girder composed of two 15" steel I-beams. How much bearing should the girder have?

10. State the safe deflection of a steel beam.

11. When is it necessary to place stone bond plates in a brick pier?

12. Describe the various methods of carrying the weight over openings in a wall.

13. Name the five orders of architecture, and state the height of the columns in each order, expressed in diameters. Make a free-hand sketch of the capital of each order.

14. Make a drawing to scale, as carefully as you can, of the entire Doric order.

15. Make a drawing of a classic pediment, showing the method of determining the slope.

16. Draw to a scale of 1/4" to the foot the plan of a bathroom 6'0" x 8'00", having a door at one end and a window at the side. Indicate in the conventional way on this plan a bathtub, a water closet, a wash basin, and figure the approximate dimensions of each fixture.

17. Make a drawing to scale of the plan, elevation and section of a brick fireplace, showing the hearth, the flues, etc., and the method of constructing the throat. Also draw in dotted lines the framing of the floor, showing header and trimmers.

18. Draw in free-hand perspective the cornice and entablature of the Corinthian order.

19. Draw to a scale of 1″ to the foot an ornamental wrought-iron railing and newel post.

20. Make a complete sectional drawing, showing the proper thickness of the wall for each story, footings, etc., of the front bearing wall of a five-story building with cellar, the floor load to be taken at 125 lbs. per sq. foot, span 20 feet. The wall to be faced with 4″ stone ashlar in the first story and pressed brick for the upper stories. The cellar wall of rubblestone. The bottom, sand. The section to be taken through the windows. Make this drawing complete in every detail and figure same.

New York City Municipal Civil Service.

Architectural Draughtsman.—Group 2, Grade 2.

Age Limit.—Over 18 years old.

Salary.—$1,200 to $1,800.

QUESTIONS PROPOSED AT EXAMINATIONS.

Arithmetic.

1. A building is 80 ft. long; the roof has a pitch of 6 ft. What is the length of the roof?

2. A cone-shaped tank has a depth of 9 ft., the bottom diameter is 11 ft. and its top diameter is 7¼ ft. How many gallons of 231 cu. in. will it contain?

3. A cellar has on one street a length of 137 ft. and on a parallel street a length of 197 ft. 8 in. The perpendicular distance between the streets is 98 ft. 6 in. How many cubic yards of excavation will there be if the depth be 22 ft. with vertical sides?

4. A brick segmental arch subtends an angle of 68°. The radius of the intrados is 30 ft., the radial depth is 3 bricks laid rowlock; length is 5 ft. How many bricks required?

Technical.

1. Make a pencil sketch of a dormer window, showing a portion of a sloping roof.

2. Make sketches of the capital and base of each of the principal architectural orders.

3. Make a sketch of one-half of the pediment of one of the orders and state which it is.

4. Make sketches of the sections of built iron columns, as follows: Of L bars and plates; of channels and plates.

5. Show by sketch the method of a Z-bar column at a point where the sections above are reduced.

6. Make sketches of the most used fireproof floors, showing two panels of each.

7. State what loads are used per foot for office floors, school floors and floors for warehouses for heavy goods.

8. Is it sufficient in designing a floor to consider only the supporting power of the beams, or is there a limit to the loading for some other cause, and if so what?

9. What point would you be particular about in making a drawing for an iron casting requiring great strength, so as to obtain homogeneity, freedom from internal strains, etc.?

10. Make a sketch showing the timbering around a chimney-breast, and the proper method of supporting the hearth to an open fireplace.

11. Suppose there are knots in one edge of a wooden floor beam, does it make any difference whether that edge is placed up or down, and why?

12. Make a sketch of the front of a fire engine building.

MECHANICAL DRAUGHTSMAN.

Federal Civil Service—Ordnance Department-at-Large.

Title.—Mechanical Draughtsman.

Age Limit.—20 years or over at date of examination.

Salary.—Entrance salary, $1,000 per annum.

Examination.

Subjects.	Weights.
1. Mathematics (comprising arithmetic, algebra to quadratics, geometry, mensuration, logarithms, use of tables, elementary problems in mechanics, and use of slide rule)	15
2. Materials (comprising all the materials used in machine construction)	15
3. Practical calculations (involving the interpretation of formulas and the correct working out of results for special cases)	20
4. Drafting (involving competent knowledge of machine construction and the ability to draw neatly to scale)	25
5. Training and experience	25
Total	100

DRAUGHTSMAN-ENGINEER.

Supervising Architect's Office, Treasury Department.

Examination.

Subjects.	Weights.
1. Mathematics (pure mathematics up to and including calculus, theoretical and applied mechanics)	25
2. Materials and design (comprising knowledge of steel, iron, fireproofing, etc.; designs of columns, girders, trusses, etc.)	40
3. Drawing (involving ability to draw designs neatly to scale)	15
4. Training and experience	20
Total	100

New York State Civil Service.

Title.—Bridge Draughtsman.

Experience Required.—Three years' practical experience in draughting on structural steel and bridge work.

Examination.

Subjects.	Relative Weights.
1. Questions on riveted joints, standard bridge details, roller bearings, conventional signs and drawings	6
2. Experience, education and personal qualifications	4
Total	10

Specimen Questions.

1. Calculate the stresses in the middle panel of a through riveted Pratt truss of 133 ft. span, 20 ft. depth and 7 panels, due to a dead load of 1,000 pounds per lin. ft. and a live load of 2½ tons concentrated at one end of the middle panel, the loads to be distributed equally between two trusses 20 ft. apart.

2. Design the lower chord and one of the vertical members for the middle panel of the Pratt truss described in question (1), showing the connections at the panel-point.

3. A Warren girder, 80 ft. long, is formed of equilateral triangles with sides 16 ft. long. Weights of 2, 3, 4 and 5 tons are concentrated, respectively, at the first, second, third and fourth apex along the upper chord. Determine the stresses in the diagonals due to these loads.

4. A frame in the form of an inverted queen-truss is composed of a horizontal top-beam 40 ft. long, two vertical struts 3 ft. long, and three tie-rods, of which the middle one is horizontal and 15 ft. long.

(a) Find the stresses produced in the several members when a single load of 6,000 pounds is concentrated at the head of each strut. (b) If a wheel loaded with 12,000 pounds travel over the top-beam, what members must be introduced to prevent distortion? What are the maximum stresses to which these members will be subjected?

5. The platform of a bridge for a clear span of 60 ft. is carried by two queen-trusses 15 ft. deep; the upper horizontal member of each truss is 20 ft. long; the load upon the bridge is 50 pounds per square foot of platform, which is 12 ft. wide. Find the stresses in the several members.

6. Find graphically the stresses in a pony parabolic bow-string truss of 6 panels, 80 ft. span and 10 ft. rise, (a) for a dead load of 1,000 pounds per linear foot; (b) the maximum and minimum stresses in the diagonals in the third panel from one end due to a concentrated load of 3 tons crossing the bridge.

7. Choose and sketch section of the endpost of a bridge to withstand a compression of 120 tons; length 28 ft., taking the unit stress from the following formula, $P = 10500 - 60 \frac{l}{r}$, where l is the length of member in inches and r is the radius of gyration in inches.

8. Letter in some suitable style the title: "Design for a Steel Through Pratt Truss Highway Bridge. Span 110 ft."

New York City Municipal Civil Service.

Mechanical Draughtsman—Heating and Ventilation—Sanitary. Schedule B—Group 3. Grade 2.

Salary.—$1,200 to $1,800.

Structural Steel Draughtsman. Schedule B—Group 2—Grade 2.

Salary.—$1,200 to $1,800.

QUESTIONS PROPOSED AT EXAMINATIONS.

MECHANICAL DRAUGHTSMAN.

Mathematics.

Candidates for first grade answer questions 1 to 4, inclusive. Other candidates answer all questions. Question: A room 28 by 42 ft. in size, with 14 ft. ceiling, has a rectangular space 6 by 10 ft., and the full height of room cut out in one corner by the shape of the building. It also has eleven windows, 2′ 6″ by 7′ 6″

1. What is its contents?

2. What is the area of its window space? What of floor?

3. What is the area in square feet of walls, exclusive of ceiling, floor and windows?

4. Extract the square root of 6278.1 to three places of decimals.

5. Disregarding friction, how may 1¼ in. pipes in groups of 6 be supplied by one 4½ in. and one 6 in. main?

6. The main hot air duct of a system is 3 ft. by 4′ 6″ in size and air is passing through it at the rate of 40 ft. per second. How much air per minute is being passed? The velocity of exit into rooms is 1½ ft. per second, and amount of air required per person is 3,000 cu. ft. per hour. What would be the total area of outlets, and how many persons would be supplied?

Technical.

Candidates for first grade answer questions 1 to 8, inclusive.

Candidates for second grade answer questions 1 to 11; and

Candidates for third grade answer all except 8.

1. In a mechanical drawing, what is meant by "projection" of an object, and how many projections or views are necessary to completely describe an object?

2. What is a "sketch," a finished drawing," a "tracing," a "blue print"?

3. Describe the T square, 45 and 60 degrees triangles, drawing pen and compass, stating the use of each and the care and attention necessary to keep in best condition and adjustment for use.

4. Describe the operation of the "direct," "indirect" and "direct-indirect" systems of heating, and state for what uses each is best adapted.

5. Describe the operation of a "natural system" of ventilation for a building, of the "Plunum System," the "Exhaust System."

6. In a direct steam-heating system, what becomes of the water from the condensed steam that collects in the radiators while in operation? What is "water hammer," and how can it be prevented?

7. In what part of the room should the opening for the admission of pure and heated air and for the exhaust of impure air be located, and what is the reason for this?

8. Draw in your best manner the following title: "Heating and Ventilating System, Public School No. 200, Aug. 26, 1903." Pay particular attention to the proportions of the letters and fill in at least one-half of them.

9. In case a double system of piping is employed in a steam-heating system, how should the return compare with the supply in size, and what is the reason? If a single system of piping is employed, what care should be taken in choosing size of pipe and placing same, and for what reason?

10. In laying out a system of air-supply ducts, state fully what care should be exercised, designating the sizes of mains and branches,

changes of direction and location of outlets. Also what velocity of exit at outlet should be considered in the design?

11. Would the construction of the building, location of a room and amount of window space, etc., have any influence on the proportioning of the heating apparatus or air ducts for that particular room? If so, what care would you take in planning for same?

12. Make pencil sketches to 1/8 in. scale of cellar, and first and second floor plans of a two-story school building of considerable size, showing the location and arrangement of all heating and ventilating apparatus and sizes of all ducts and openings.

13 and 14. Reading plans.

Note.—The sketches required will be rated from the standpoint of correctness of design, general grasp and knowledge of the subject and neatness rather than excellence of finish and detail.

STRUCTURAL DRAUGHTSMAN.

Mathematics.

1. What is the area of metal of a round cast-iron column 1 in. thick and 11 in. outside diameter (three places of decimals)?

2. Extract the square root of 911 to four places of decimals.

3. A building 40 ft. by 108 ft. has a peaked roof with a rise of 15 ft. How many square yards of roofing felt is required to cover it (neglecting lap)?

4. Reduce the following measurements to feet and four places of decimals of a foot: 17 ft. 11 in.; 3.915 ft.; 77.25 in.; 2 ft. 6 3/8 in., and 3 ft. 9 15/16 in.

5. Reduce the same measurements as in question 4 to inches and three places of decimals of an inch. Add them and reduce the result to feet, inches and fractions of an inch.

6. The span of a roof is 80 ft. The rise at the center is 30 ft. What will be the length of the rafters?

7. Taking the weight of steel at 490 lbs. per cubic foot, what will be the weight of a pin 8 in. in diameter and 17 in. long?

8. What must be the area of a cast-iron column base to support a load of 200 tons? (The resistance of cast iron to compression being taken at 16,000 lbs. per square inch.)

9. What shearing stress will a wrought-iron bolt 1 1/2 in. in diameter and 18 in. long sustain? (The shearing strength of wrought iron being taken at 5.500 lbs.)

10. What tensile stress will a steel tie rod 2 1/8 in. and 16 ft. long sustain? (The tensile strength of steel being taken at 16,000 lbs.)

Technical.

1. (a) What are the largest sections of I beam, channel and angle rolled? (b) What the lightest weight 20 in. and 18 in. I beams?

2. Show by sketches the standard connection angle of a 15 in. and a 10 in. I beam.

3. Show the conventional signs for rivets and field holes.

4. How close can ¾ in. rivets be driven (a) to each other, (b) to the edge of piece?

5. What points must be considered in designing floor construction and connections?

6. Same for columns.

7. (a) What is the least proportion of depth a plate girder should have to the span; (b) in general, how should stiffeners be spaced?

8. How would you secure transverse or wind bracing on a building where cast-iron columns are used?

9. Show by sketches a cast-iron column base for a 12 in. round column.

10. Show by sketches the usual connection of a cast-iron column for a 15 in. I beam.

11. (a) What is the least bearing a beam should have on masonry? (b) How are such beams arranged?

12. What is the object of the rods? Describe or show by sketch how they should be arranged for several beams of 20 in. span.

13. Describe and illustrate two methods of fireproofing of beams and columns.

14. Design a stone footing of courses 2 ft. thick for a load from a column of 250 tons for ground which will take 4 tons per square foot safely.

15. In designing a roof what are the loads to be considered and how are these determined?

16. How to compute strength of rivets, what constants used?

17. How to compute the number of rivets in any section of the flange of a plate girder.

18. How to compute spanning and size of stiffeners near the ends of a plate girder.

19. How to compute economic lengths of cover plates.

20. Is the full cross-section of a rod with screw threads on available to compute the stress? When is it available?

21. How is the stress in the horizontal diagonals connecting the upper chord transmitted to the masonry?

22. Proper proportions for an eye bar.

23. Draw column made up of **Z** bars, of plates and angles.

24. Best method of attaching wind bracing to upper chords. Give sketch.

25. What is camber? How computed?

26. Would you use the same working strains for field rivets as for shop rivets? If not, why?

27. What precautions should draughtsmen use to insure sound castings?

28. Single beam supported at two ends with two, three or more loads; find shear at any section and maximum bending moment.

29. Constants (ultimate stresses for):

1. Bridge with heavy traffic.
2. Ordinary highway bridge.
3. Roof truss.

30. Maximum and minimum distance between rivets. Distance of rivets from edge of plates.

The following questions were given at an examination for this position held April 10, 1906:

31. Name the various kinds of stresses and state the effect they produce.

32. Under what conditions would the deflection rather than the absolute strength govern the size of a beam to be used?

33. State briefly the general rules for the spacing of rivets.

34. State briefly the points to be observed in designing riveted joints.

35. State briefly the points to be observed in designing pin connections.

36. State how the weakening of a bolt or rod by cutting a screw thread on it may be obviated.

37. State what is meant by a "Strain-sheet." Describe its use and illustrate by a sketch.

38. Show by sketch the conventional signs used to indicate the different styles of riveting, both in the shop and the field.

39. (a) What is the use of lattice or lacing bars in built columns? (b) Show each form by sketch and state at what angle and interval they should be attached. (c) Has the intersecting lattice type any advantage over the single-lacing type?

40. What is meant by an erection sheet? What information should it contain? Illustrate by sketch.

41. Illustrate by sketch the arrangement of plates and shapes to form columns as follows: (a) 2 Tees. (b) 4 Tees. (c) 2 Channels. (d) 3 **I** beams. (e) 8 **Z** bars.

42. What limits the length of span on which rolled beams may be used?

43. Show by sketch a standard connection angle for a 24 in. **I** beam.

44. Design and draw to a suitable scale for shop use a turnbuckle having given the following dimensions:

Diameter of rod, 2¼ in.

Diameter of screw, 3 in.

Length of buckle over all, 15 in.

45. From the following dimensions draw to a scale of 1½ in. to 1 ft. a cross-section of a box girder. (Supply all the details and information necessary to construct it):

Web plates, 30 in. x ½ in.

Flange plates, 16 in. x ⅜ in.

Angles, 3½ x 3½ x ½.

TOPOGRAPHICAL DRAUGHTSMAN.

Federal Civil Service.

Title.—Topographic Draughtsman.

Age Limit.—20 years or over.

Examination.

Subjects.	Relative Weights.
1. Drawing (small specimens of topographic drawing to reproduce)	35
2. Lettering (short words in different styles, with numbers)	35
3. Mathematics (comprising arithmetic, algebra to and including quadratics, plane and solid geometry, plane trigonometry, logarithms, mensuration and projections)	30
Total	100

New York State Civil Service.

Title.—Engineering Draughtsman.

Salary.—$4 to $5 a day.

Examination.

Subjects.	Relative Weights.
1. Practical and theoretical questions, including mensuration and the solution of plane triangles, use of logarithms, free-hand lettering, reduction of field notes and plotting, mapping....	7
2. Experience and education....................................	3
Total	10

Specimen Questions.

Directions: The work of computation must be shown in full. Logarithm tables will be furnished by the examiners.

1. Two straight roads meet at an angle of 58¼° Find the distance between a tree on one of the roads a half mile from their point of meeting, and a flagpole on the other road 3,640 ft. from the same point.

2-4. From the following bearings and distances, balance the survey, plot to suitable scale and compute the area in acres by the method of co-ordinates:

Station A..................	N. 51° 50′ E.	Distance, 1,063 feet
Station B..................	S. 29° 45′ E.	Distance, 410 feet
Station C..................	S. 31° 44′ W.	Distance, 769 feet
Station D..................	N. 61° 0′ W.	Distance, 713 feet

Letter the bearings and distances on your map in stump writing.

5. Indicate the following by means of neat free-hand pen drawings, each to occupy a space 1″ x 1½″; (a) quarry-faced masonry; (b) brickwork in Flemish bond; (c) broken range masonry; (d) forest with both evergreen and deciduous trees; (e) marsh with stream through it.

6. Draught the following title in letters of three different sizes, using stump or round writing or some other style of free-hand lettering that you can do neatly and rapidly: "Plans for Improving the Highway Between Buffalo and Albany, a Distance of 310 Miles, Passing Through Medina, Rochester, Clyde, Syracuse, Oneida, Whitesboro, Utica, Little Falls and Schenectady. Scale, 1″ = 100′ "

7-8. Draw plan, elevation and cross-sections for a semi-circular arch-culvert, of 15 ft. span, under a highway 22 ft. wide, allowing 2 ft. between the top of the arch and surface of road, with parapets 2 ft. 6 in. wide, 3 ft. high. Design suitable wing-walls for a flood 3 ft. deep. Supply all other needed data.

9-10. Draw 5 ft. contour lines from the elevation points given on this sheet; mark the elevations for each contour and indicate the probable location of streams.

From the contour sheet as you have prepared it, draw on separate sheet two sections of the ground, one through the line N.S., the other through E.W. Indicate the vertical scale used.

New York City Municipal Civil Service.

Topographical Draughtsman.—Schedule B—Group 1—Grade 2.

Salary.—$1,200 to $2.100.

QUESTIONS PROPOSED AT EXAMINATIONS.

TOPOGRAPHICAL DRAUGHTSMAN.

Mathematics.

1. What is a logarithm?

2. Extract square and cube root by logs.

3. What is the mathematical value of sin., cos., tang. and cotg.?

4. Solve an oblique triangle having 3 sides given.

5. Solve a right-angled triangle, given the hypothenuse and one angle.

6. A street curves from due N to due E with a given R. What is the arc between street lines (width of street being given)?

7. Given a street rising—**a** foot in the mile and falling **b** foot in another mile—give rate of grades and figure out.

8. Transpose a given number of chains and links into feet and fractions to three decimal places.

9. Prove:

$$tg.\ 2b = \frac{2\,tg.b}{1 - tg^2 b}.$$

10. Compute the area of a hexagon inscribed in a circle whose radius is 27 ft.

11. Divide 18076 by .0804 and extract the square root of the result to two places of decimals.

12. Find the sum of 8 2/3, 5 3/8, 12¾, .097 and 38/94, and give the result in decimals.

13. A field is trapezoidal in shape with one side at right angles to the two parallel sides, which are, respectively, 800 and 1,200 ft. long. The inclined one has a length of 1,000 ft. Find the area of the field.

14. Divide 7739.875 by 2.5839. Multiply the result by .06134 (use logs.).

15. Extract the square root of 28004.0463 to four places of decimals. Use no logs.

16. In a circle whose radius = 18 yds. 2′, find area of a segment bounded by the arc of a quadrant and the corresponding chord.

17. A field with six sides has distances and courses as follows:

a to b	N.	150′
b to c	N. 50° E.	450′
c to d	S. 80° E.	260′
d to e	S. 33° W.	300′
e to f	S. 25° E.	350′

Find course and distance from f to a.

18. Given the parallel sides of a right-angled trapezoid and the slant side, find the area.

19. Extract square root of 27863.2732.

20. How many square feet in 1 acre? How many square chains?

21. 5⅜ plus 6 15/23 plus 7 13/21 plus 8 165/732; find sum and express in the form of a decimal.

22. The radius of a circle is 11′; find the area of the inscribed hexagon.

23. Demonstrate that the side of an equilateral triangle inscribed in a circle is to the radius as the square root of 3 is to unity.

24. Extract the square root of $\frac{4483}{6887}$; give the answer in a decimal, correct to three places.

25. A walk 9 ft. wide surrounds a square garden, the area of the walk being ¼ of an acre. What is the length of a side of the inclosed square?

26. A city lot in the form of a trapezoid cost $2,250 at 40c. a square foot. If its parallel sides were 39 ft. and 51 ft., what was its length?

27. Compute the area of a hexagon of which the radius of the circumscribed circle is 11 ft.

28. Extract the square root of the following number: 70913078.

29. Find the sum of 3 2/5 plus 4⅛ plus 1 11/40 plus 3 18/265, and express the fractional portion in decimals.

30. A field is trapezoidal in shape, with one end at right angles to the two parallel sides. The parallel sides are, respectively, 713 ft. and 1619 ft. long and the inclined end has a length of 1278 ft. What is the area of the field?

31. Extract the square root of 4930.6271.

32. What is the area of a triangle of which the sides are 50, 60 and 70 ft?

33. Add 17′ and 11½″; 4′ 7″; 13.375′; 35½″; 16′ 9″ and ½′ Express the result in feet and inches and feet and decimals of a foot.

34. A rectangular field is 5⅛ times as long as it is wide and contains 2¼ acres. What are the dimensions of the field?

35. What is the area of a roadway 60 ft. wide of which the center line is a quarter circle, with a radius of 500 ft.?

The following questions were for Topographical Draughtsman, tenth grade:

Experience.—Ten years as assistant engineer.

36. Show by most direct method the way to find the side of a regular hexagon of same area as an equilateral triangle 150 ft. a side.

37. Given the sides and angles of a triangle, give formula for the radius of the inscribed circle.

38. $a - b\sqrt{a^2 - b^2 + c - d - c^2 + 5}$; substitute values $a = 1$; $b = 2$; $c = 3$; $d = 4$ and calculate.

39. Find the area of a sector in a circle 25 ft. radius, the versed-sine being 18 ft.

40. The difference of two numbers is 15; the cube of the greater is 89, and the cube of the lesser is 45. Find the numbers.

Technical.

1. What information will a well-prepared topographical map furnish?

2. Describe briefly the three kinds of survey done by a topographical party to furnish data for topographical map.

3. Draw carefully the following, using conventional signs in spaces 1½″ sq.:

Sandbar and Mudflat; Railroad and Fenced Railway, with Station.

4. Buildings and Highway; Ploughed Field and Fruit Orchard; Corn Field and Grassland.

5. House and Outbuildings, with Roadway, Paths, Shade Trees and Garden, Stream Running Through Marsh.

6. Describe plane table and method of using it.

7. What is a pantograph, and for what used?

8. Show by contour lines a stream running in a ravine between two hills and make profiles at right angle to stream at different points corresponding to contours.

9. State and describe fully the three methods of determining the area of a piece of land which has been plotted.

10. State accurately and fully all the items that should be obtainable from a complete profile of any street in the city.

11. Do profiles of a street ever show any levels other than on the center line of the street, and if so, what? Also, to what datum are city levels referred?

12. Describe a contour line and state how far apart they are in city maps.

13. What are "hachures"? What direction should be given them with relation to adjacent contour lines?

14. Suppose a set of contours surround a depression and another set surround an elevation, how can you tell the depression from the elevation?

15. In what case might contours run into each other? Can they ever cross each other as full lines?

16-17. Make a diagram of the sine, cosine and all the other trigonometrical functions of an arc.

18. In making computations, do the sine, cosine, etc., always have a positive sign? If so, how must they be taken?

19. In making a plot of a piece of ground with a number of sides in the survey of which there is an error, how do you make the plot "close"?

20. What are the principal causes of error in angle surveying? What limit allowable in city work?

21. What is the principal cause of error in line measurements, and how much allowable in city work?

22. In making an accurate plot of a survey, how would you plot it, and give your reason for thinking it most accurate?

23. How many feet in a Gunter's chain? How many inches in a link?

24. How many square feet and how many square chains in an acre?

25. Explain clearly what you understand by the logarithm of a number.

26. What operations are facilitated by using logarithms, and how?

27. What is the trigonometrical relation of and between sines and angles of any plane triangle?

28. Do the terms + and — remain the same for an angle over 90° and for one below 90° for sine, cosine, tang., cotg.? If not, what are they for each?

29. Draw correctly the following lettering, using your best style: Map of Property in 23d Ward of the City of New York. John Jones. Scale 25′ = 1″.

30. Describe the object of a topographical map.

31. Describe the method of making a topographical survey.

32. Sketch page of field book with such notes.

33. What is a profile, and what does it show?

34. Is it generally drawn to a uniform scale? If not, why?

35. Name colors that are used to represent different objects of a topographical survey.

36. What are the most accurate ways of laying off angles on a plan?

37. What is a traverse table, and what is its use?

38. Explain the traverse table in detail.

39. Give table of land and square measures.

40. Describe method of making corrections if a plot does not close in plotting.

41. Make specimen "Italic" "Script," "Roman" and "Block" lettering for maps.

42. Draw profile of a line of levels at 100′ sta., showing eight stations and a stream and road crossing the line. Draw grade line, also all figures you would in actual case.

43. An angle may be plotted by a chord, scale, protractor and co-ordinates. Explain cases.

44. What is the shortest distance between two parallel lines? Show how this enables you to draw two parallel lines.

45. What is meant by latitude and departure?

46. How many square feet in 3 acres 1 rod — 17 rods?

47. Draw isometric projections of a cube.

48. Having all but one of the distances in a closed survey, how do you get the remaining one?

49. How do you draw maps in reference to the North point?

50. The parallel sides of a trapezoid are 50 ft. and 60 ft.; their distance is 100 ft; divide it into two equal parts by a parallel to the bases.

51. Give the notes of a boundary survey of a piece of property; describe how you would plot them and show graphically how you would distribute an error in closure.

52. (a) Why is it not best to use a protractor to plot such a survey? (b) How would you look for a considerable error in angles in such notes?

53. What form of notes would you require to enable you to plot an irregular shore line of a lake or river?

54. Describe the method of plotting a course of a survey by chords.

55. (a) Show by sketches two methods of showing the scale of a drawing 100 ft. to 1 inch; (b) show by sketch the method of indicating the points of the compass on a map.

56. Draw a small topographical map 2 in. square, of two small hills, with a brook with steep banks between, running out through a nearly level meadow; show contours.

57. Same by use of hachures.

58. Same of a river with marshy shore on each side and sandy shore, steep bank, wooded with several varieties of trees on the other.

59. Same part of a farm with plotted land, cultivated land, orchard and buildings.

60. Show your skill in lettering by arranging the following title and finishing at least four different letters to each line: Map of Blackwell's Island, Borough of Manhattan, Greater New York City. From Survey made in 1872. Scale 100 ft. to 1 in.

61. How would you prepare paper before making a drawing on same, and what precautions are necessary?

62. What do you consider a reasonable degree of accuracy of a traverse survey, and how would you correct same?

63. What survey notes are necessary to make a topographical survey?

64-65. Make sketches in 1" squares showing "Ploughed Field and Woodland;" "Farm Buildings, Fences and Cultivated Lands;" a "Bay with Marshy and Wooded Shores;" a "Stream Entering a Bay with Sandy and Marshy Shores."

66. Make a sketch, a street starting at a low hill, crossing a stream and cutting into a steep hill, and give all necessary data for building such a street.

67. On a five-sided figure assume four courses and distances and show how to compute other course and area of the parcel.

68. What field notes are especially required to make a map of part of the Borough of the Bronx?

69. Show in hachures a cone inclined at an angle of 45° on a plane.

70. What objection is there to plotting angles?

71. Construct this title: Topographical Map of Part of the Borough of the Bronx. City of New York. Work out enough of the letters to show your skill in lettering; also give scale.

72. What general method for finding the general accuracy of a closed survey?

73. How would you balance a survey? Show also graphical method.

74. Show how you would find missing side in a survey.

75. How should a map be placed on the paper?

76. Show (in your best method) two nearly parallel ravines running down a hill, being almost nothing at the top and getting deeper before it strikes the level country below. Show by hachures.

77. Show broad stream flowing into a reservoir.

78. What maps must a topographical draughtsman make?

79. Show in 2″ squares evergreen trees, oak, etc.; orchard, meadow, houses, etc.

80. (a) Contour Lines—State under what conditions a contour line will have a "sharp re-entrant" in it. (b) State what contour lines run into each other.

81. Hachure Lines—Directions for same. How is relative steepness indicated by them?

82. Suppose a street to have a reverse curve in it, which lines pertaining to the curves would lie in the same straight line?

83. Is topography ever shown by any other method than by lines? If so, describe it.

84. What is meant by degree of a curve in a street or railway?

85. Having plotted the survey of a plot of ground, how would you make an approximate estimate of its area without use of tables?

86. Draw a complete profile of a set of levels on an ungraded street 1000 ft. long, with considerable variations of surface, showing everything required for letting the job of grading the street. Do this in the best manner you can.

87. Make a topographical drawing (in your best style 2″ square, showing farm buildings, houses, etc.; stone fences, orchard, meadow, corn field, road and hardwood trees (shade) along the same.

88. Make a topographical drawing (in your best style) about the same length of a wide stream with precipitous banks.

89. Do same for a small pond with marshy shores, changing gradually to upland.

90. In your best style letter the following: A Topographical Map of a Portion of Borough of Queens. From Surveys Made in 1897. Put scale as you think it should be.

91. (a) What information should be furnished by a well-prepared topographical map? (b) For what purposes are topographical maps used in this city?

92. Describe how the area of a piece of land is determined, by (a) the use of a planimeter; (b) by "double meridian distances;" (c) what other method is used?

93. How should the scale to which a map is drawn be shown?

94. In computing the parts of a triangle, how many different cases may arise? To which one may two correct answers be given, and under what conditions?

95. A piece of property 500 ft. square shows elevations taken every 100 feet as follows:

100	115	115	115	120	120
105	115	120	120	130	120
115	118	130	140	140	130
115	120	125	130	130	120
112	115	115	125	120	115
108	112	110	118	110	110

Plot this to approximate scale and show contours at 5 ft. intervals, beginning at elevation 100.

96. Assuming that you are to plot a course running 1060 ft. N., 16 deg. 40 min. W., describe two ways of doing it, one of which must be accurate, and state why it is so.

97-98. Show 10,000 ft. of a profile of the center line of a sewer, with grade 2.5 per cent., running under rough ground. Make vertical scale 1 in. equal 10 ft., horizontal scale 1 to 2000. Show all necessary information.

99. Make topographical drawings 2 in. square, in your best style, with pen or pencil, of a country road on a hillside, heavily wooded; show a bridge and small stream. Scale about 1 in.—30 ft.

100. Same for mill pond, showing dam and mill, and using contours. No special scale necessary.

101. Same for precipitous bluff and ravine by hachures.

102. Same for a small park, showing evergreen and deciduous trees.

103. Draw neatly, in your best style (with pen or pencil), the following caption; lay out the work carefully as to spacing and arrangement, and complete at least two letters of each line: Map of Stapleton and Vicinity, Borough of Richmond, City of New York. From Surveys Made in 1903. Scale 1 in. to 500 ft.

104. (a) What drawings does a topographical draughtsman have to make? (b) Suppose there were notes of a closed survey of a field, how would he examine them as to their reasonable correctness?

105. If errors are found in latitudes and departures, (a) how would tabular corrections be applied? (b) How can the amount and direction to be applied at the end of each course be determined graphically?

106. (a) If the course and length of one side be missing, how

can it be supplied? (b) Is any assumption made by doing this, and if so, what is it?

107. (a) How many square chains (of 66 ft. per chain) are there in an acre? (b) How many square feet are there in an acre?

108. How should a drawing of a survey be located on the paper?

109. (a) When a transit survey of a street or other continuous survey of considerable length and with a number of angles in it is to be plotted, what method should be pursued to avoid, as far as possible, errors in plotting? (b) What is the most accurate method of laying off an angle on paper?

110. Illustrate all the ways of indicating, either by description or otherwise, the scale to which a drawing is made.

111. In a reverse curve are there any trigonometrical lines common to the two branches of the curve, and if so, what are they?

112. Make a careful topographical drawing, about 2 in. square, in your best style, of the position of a hill containing two ravines, or depressions approximately parallel, starting with nothing a little below the top and reaching a considerable depth before debouching upon the plain below; a swelling to occupy the space between the ravines; all slopes to be represented by hachures.

113. Make a careful topographical drawing, 2 in. square, of a reservoir and dam, with wide stream above and below.

114. Draw the same of a farm, with one-quarter in evergreen trees; one-quarter in hardwood trees; the rest showing buildings, orchard, meadows, etc.

115. Show your skill in lettering and arrangement by drawing a caption to a drawing as follows: Map of a Part of the Borough of the Bronx. March, 1902. Scale 100 ft. per inch.

116. How would you proceed to enlarge a map so as to be able to fill in additional details?

117. How would you plot the survey of a line of considerable length with a number of angles, so as to avoid error in plotting the angles?

118. What notes should you receive in order to plot a curve, such as a railroad, and how would you do it?

119. Describe the planimeter and its use.

120. (a) What notes should you receive to enable you to locate an irregular shore line? (b) How would you compute the area of a piece of property bounded by an irregular shore line?

121. Show a form of stadia notes and state what reductions have to be made in the office.

122. Make a neat drawing in ink or pencil, of a city block 200′ x 400′ to scale 50′ equals 1″. Show one end with rocky cliff and sandy beach and shore line; the other end with city lots and several houses;

the main portion of an old residence, with grounds, outbuildings, driveway, pine and oak trees, a fruit orchard, garden and corn field.

123. Arrange the following caption and fill in the letters in your best style, using at least three kinds of lettering: Topographical Map of Bronx Park, Borough of the Bronx, City of New York. From Surveys Made in 1903. Scale 100 ft. to 1 in.

124. Describe briefly the making of a topographical survey of a watershed for purposes of water supply.

125. (a) Make a table from the following bearings and distances and describe the method of balancing the survey and computing the area in acres. Use the method of co-ordinates: (b) Explain why certain quantities are subtractive and others additive:

Sta. A	N. 51° 50′ E..	Distance 1063 ft.
Sta. B	S. 29° 45′ E.,	Distance 410 ft.
Sta. C	S. 31° 44′ W.,	Distance 769 ft.
Sta. D	N. 61° 0′ W.,	Distance 713 ft.

126. How would you determine the area of a piece of property one side of which was bounded by an irregular water course?

127. Show the form of notes of a stadia survey and explain them.

128. What are "contours" and "hachures"? Illustrate by sketch of two hills with a ravine between.

129. Plot to approximate scale contours at even 5′ from the following elevations, taken 50 ft. apart:

100	105	108	112	115	120	125
95	100	105	110	115	120	120
95	100	103	105	110	115	115
100	100	102	103	105	110	112
105	105	105	105	103	110	110
110	110	110	110	115	115	115
113	115	115	115	120	120	120

130. Show a form of level notes for the center line of a street for at least 1000 ft.

131. State the mathematical relations between the sine, cosine, tangent, cotangent and secant.

132. In a space 6″ x 4″ divided into six nearly equal spaces, show by neatly drawn standard topographical signs (a) a pond with marshy shore on one side; (b) with meadow land adjacent; (c) then ploughed land and a corn field; (d) a farmhouse, with outbuildings, fences, etc., on a road; (e) the other shore, sandy, with a steep bluff, on which grow (f) mixed oak and pine trees.

133. Correct, punctuate, arrange, lay out completely and finish enough letters to show your skill in lettering the following title:

Topographical map of the Borough of queens greater city of New York from surveys made in 1904 Scale ...... (Put the scale as you think it should be.)

The following questions were for Topographical Draughtsman, tenth grade:

Experience necessary: Ten years as assistant engineer.

134. Make up office force for large topographical office.

135. Tools, etc., necessary.

136. How to adjust a system of triangles.

137. What error would you allow on primary triangulation of a borough?

138. What is an optical square, and how used?

139. What is a planimeter, and how used?

140. Give all the methods you can think of for plotting contours.

141. What method is used instead of plane table? Describe plane table.

142. When is stadia work accurate enough?

143. What methods would you use for plotting a large geodetic survey?

144. What is **personal equation,** and how eliminated?

145. Write a report on topographical work.

146. What methods used for reading angles accurately?

147. How should the stations in a triangulation system be located?

SCIENTIFIC REQUIREMENTS.

That which precedes (see Book I., The Axeman; Book II., The Chainman; Book III., The Rodman; Book IV., The Leveler; Book V., The Transitman; Book VI., The Inspector); also Framed Structures, Drawing.

FRAMED STRUCTURES.

Parallelogram of Forces (Fig. 281).—Let **A** and **B** **(a)** representing, in magnitude and direction, two forces applied at **O.** Complete the parallelogram **AOBC**; the diagonal **OC** will be, in magnitude and direction, the resultant of the forces **A** and **B.** The arrows show the way in which the forces act.

Triangle of Forces.—Remark that in **(a)**, triangle **AOC** = triangle **COB** and **AC** = **OB** and is parallel to it. We may draw that triangle separately **(b)**, in which **a**, **b** and **c**, respectively, represent forces **A, B** and **C** of **(a)**. It is a **triangle of forces** and is also called the **force diagram** or the **reciprocal figure** of the forces **A, B** and **C** in **(a)**.

Direction of Resultant in the Triangle of Forces.—Note the direction of the forces **a** and **b (2)**; it is from left to right, going round the triangle. The direction of the resultant **c** is opposite; it goes round the triangle from right to left.

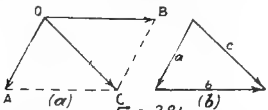


Fig 281

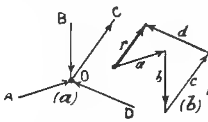


Fig. 282

Polygon of Forces (Fig. 282).—Let **A, B, C** and **D** represent, in magnitude (length to scale) and direction (arrow and angle), forces acting at a common point **O**, Fig **(a)**. Draw Fig. **(b)** as follows: Draw a line **a** parallel to **A** and equal to it; draw a second line, say, **b**, from the end of **a** parallel and equal to a second force **B**, and in the direction shown by the arrow on **B**; from the end of **b** draw **c** parallel and equal to a third force **C**, and of same direction; from the end of **c** draw **d** parallel and equal to the last force **D** and of the same direction. Finally, connect the end of **d** with the beginning of **a**; that line will be, in magnitude and direction, equal and parallel to the resultant of forces **A, B, C** and **D**, and that direction is contrary to the general direction, **a, b, c, d,** of the other forces, which is from left to right when going around the polygon, it acts from right to left. The figure thus formed is a **polygon of forces** or a **force diagram** or the **reciprocal figure** of the forces **A, B, C. D.**

Frame.—Rigid structure, composed of straight members, called struts and ties.

Stress.—Tendency of the molecules to displacement under the action of forces.

Compression.—A stress produced by a push.

Tension.—A stress produced by a pull.

Tie.—A member subject to tension.

Neutral Member.—A member subject neither to compression nor to tension.

Truss.—A frame used to support weight.

Condition of Rigidity of a Frame.—It should be designed so that a stress in one member cannot produce a stress in another member. This is effected by designing it in the form of adjacent triangles, in which an elongation or compression of a member will simply change the form of the triangles.

Redundant Members (Fig 283).—A member not necessary to establish the rigidity of the frame, as **AC** in **ABCD**. The five mem-

bers, **AB, AD, BD, BC and CD,** determine a rigid structure; **AC** is a redundant member and should not be introduced. Any elongation in **BD** will cause a compression in **AC** and an elongation in **BC, CD, BA** and **AD;** any compression in **BD** will cause an elongation in **AC** and a compression in **BC, CD, BA** and **A.D.** If, therefore, **AC** and **BD** are not constructed of the right lengths, the other four members will be permanently subjected to unknown stresses, which it will not be possible to calculate.

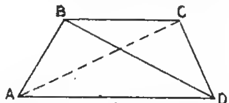


Fig 283

Forces Acting on Frames.—1° Weights they have to carry; 2° Pressures exerted, as by wind. 3° Weight of their own materials. 4° Reactions of supports.

Conditions of Equilibrium of a Frame.—The total forces of weights and pressures must equal the reactions of the supports.

Dead Loads.—Weight of materials supported.

1° In roofs, they are purlins, boards, shingles, slates, tiles, zinc, glass, etc., or any other material used in the construction.

2° In bridges, they are filling, roadbed, ties, rails, etc.

Joints.—Points of connection of the members of a frame.

Loads Referred to the Joints.—The loads and pressures are supposed to be carried and supported at the joints, as explained in the triangular frame.

Strains Induced in the Members of a Triangular Frame (Fig. 284).—Let **ABC** be a triangular frame, formed with two equal rafters, **BA, BC,** and a tie, **AC,** resting on supports at **A** and **C,** and supporting a load, **2P,** uniformly distributed. Each rafter carries **P,** half, of which **½P** is supported at **A** and **C,** and the other, **½P + ½P = P,** is supported at the joint **B.** Force **P** pushes the molecules of members **BA** and **BC** towards **A** and **C.** The abutments resist that thrust and react in opposite direction, pushing the molecules towards **B.** The total amount of this reaction is the total load, **2P,** because the system is in equilibrium; that system is symmetrical and the reaction at each point of support is **½ 2P=P.**

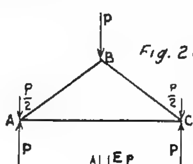


Fig. 284

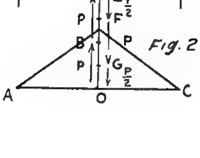


Fig. 285

Members **BA** and **BC** are in compression. Under the action of that push at **A** and **C**, the molecules of tie **AC**, are pulled apart or away from **A** and **C**, and that member. **AC**, is in tension, hence its name of tie. The arrows about a joint indicate the directions of the forces acting on that joint, either towards or away from the joint.

How Compression and Tension Are Represented.—Compression by heavy lines, tension by fine lines. When numerical values are given it is customary to consider tension (a pull) as positive + and compression (a push) as negative —.

Reciprocal of the External Forces—(Loads and Reactions) (**Fig. 285.**)—Force. **P**, may be supposed to be applied at **O**; the reactions and forces ½**P** acting at **A** and **C** may be supposed to move uniformly towards the center, while keeping at equal distances from it, until they finally act at **O** with an intensity = **P** + **P** = **2P** for the reactions and ½**P** + ½**P** = **P** for the loads. These loads added to **P** will give a force **2P** = **EO**, acting downward. These forces, acting at the same point. **O**, in opposite directions and with equal intensities, = 2**P**, are in equilibrium as in the given system and their sum is zero. Furthermore, reaction **DO** may be supposed carried in its own direction from **O** to **E**. The figure **EF** + **FG** + **GO** — **OB** — **BE** is the reciprocal of the external forces acting on the simple triangular truss, **ABC**.

Notation in a Truss and its Reciprocal (Fig. 286).—Let it be the same triangular truss. Write capital letters in all spaces of the frame diagram (**a**) separated by members or by forces, **M, N, P, Q, R, S.** A force will, in that diagram, be represented by the two letters on each side; for instance, the load carried at the apex will be called **MN**; the stress in the right-hand rafter will be called **NP**; a joint will be represented by three letters, such as **MRQ** for the left-hand joint.

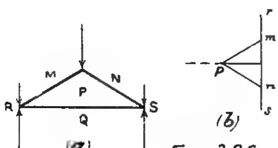


Fig. 286

In the reciprocal, the same small letters are used and they represent lines. In (**b**), for instance, load **RM** is line **rm**, load **MN** is **mn**, load **NS** is **ns**; reaction **SQ** is **sq** (carried upward), and reaction **QR** is **qr**.

Reciprocal of the Stresses or Internal Forces.—After drawing the force diagram, **rmnsqr**, draw **mp** parallel to **MP**, **qp** parallel to **QP** and **np** parallel **to NP**. It is clear that **mp** and **np** will intersect at the same point as **mp** and **qp**, because of the symmetry of the figure. The lines of triangle **mnp** represent the forces acting at joint **MNP**.

Application to a Warren Girder Uniformly Loaded (Fig. 287).—Each joint carries one-half the load of the adjacent panels. The abutments therefore carry one-half the load of a joint. The panels

being of equal lengths, the loads of the joints are equal to each other. Letter the spaces as before with capital letters.

1. Draw the polygon (st. line) of external forces **ra, ad, dg, gj, jm, mo, op, pq, qr** (**q** is center of **rp**).

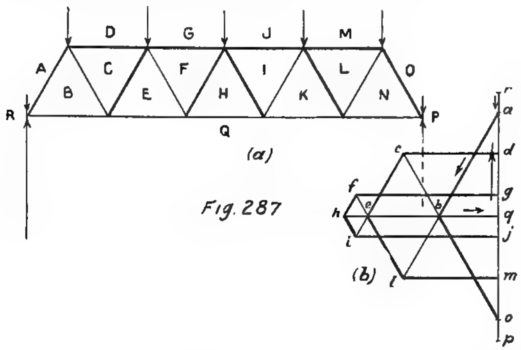


Fig. 287

2. Draw polygon of forces at joint **RABQR** as follows: **ra** is drawn already; draw **ab** parallel to **AB** and **bq** parallel to **BQ** giving point **b**; then **rabqr** is the polygon of the forces acting at that joint; that polygon is closed, showing equilibrium. **ra** and **ab** are compressions, and so will be all stresses running the same way (downward); **bq** and **qr** are tensions, and so will be all stresses running the same way (to the right and upward).

3. Draw polygon of forces at joint **ADCBA** as follows: **ad** is drawn already; draw **dc** parallel to **DC, cb** parallel to **CB** (drawn through **b**), giving point **c**; then **adcba** is the polygon of the forces acting at that joint; that polygon is closed, showing equilibrium. **ad, dc, ba** are compressions, and **cb** is a tension.

4. Draw polygon of forces at joint **DGFECD** as follows: **dg** is drawn already and so is **cd**; draw **ce** parallel to **CE** to line **eq**; draw **ef** parallel to **EF** to **gf**; then **dgfecd** is the polygon of the forces acting at that joint; that polygon is closed, showing equilibrium. **dg, gf, ec** are in compression, and **ef** is in tension.

5. Draw polygon of forces at joint **GJIHFG** as follows: **gj** is drawn already and so is **gf**; draw **fh** parallel to **FH** to **hg** (the operation generally stops here, because one-half of the frame has been analyzed and the other members will bear the same stresses as their symmetrical in the first half); draw **hi** parallel to **HI** to **ji** (parallel to **JI**); then **gjihfg** is the polygon of the forces acting at that joint.

Remark.—When there is an uneven number of panels in the upper member, the two central diagonals are neither in compression nor in tension; they are neutral, as may be seen in the diagram of stresses where these members are reduced to a point.

Application to a Howe Truss Uniformly Loaded (Fig. 288).—Mark the spaces with capital letters in the frame diagram, as before. The drawing of the reciprocal is not different from the preceding

example. For instance, the polygon of the forces acting at joint **RABQR** (a) is **rabqr** in (b); that of the forces acting at joint **ADCBA** is **adcba**; that of joint **DGFECD** is **dgfecd**; and that of joint **GHIFG** is **ghifg.**

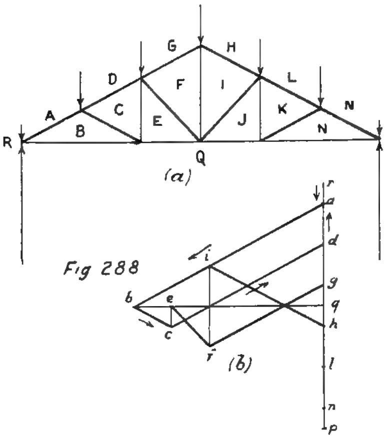


Fig 288

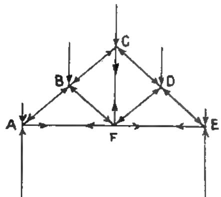


Fig. 289

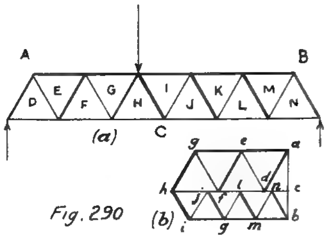


Fig. 290

Other Mode of Showing the Direction of the Stresses (Fig 289).—The members **BA, BC, BF,** etc., are in compression. The forces acting on the joints push their molecules towards the joint below as indicated by the arrows near those joints. The reactions at the supports push these same members upwards or towards the upper joints, as indicated by the arrows near these joints. The members **AF, FE, CF** are in tension; their molecules tend to pull away from the joints as indicated by the arrows near these joints.

Application to a Warren Girder with a Load Not at Center (Fig. 290).—The reactions are inversely proportional to their distance from the vertical of the load. Draw the vertical load **ab** and the reactions **bc, ca.** Draw **ad** parallel to **AD** and **cd** parallel to **CD;**

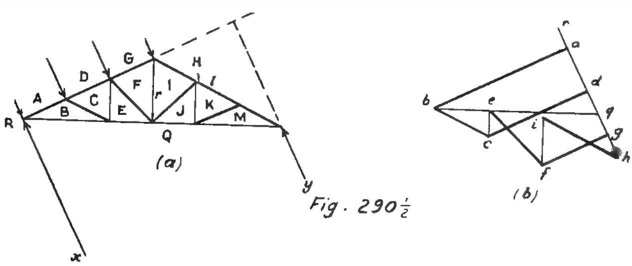


Fig. 290½

draw **de** parallel to **DE** and **ae** parallel to **AE;** draw **ef** parallel to **EF** and **cf** parallel to **CF;** draw **fg** parallel to **FG** and **ag** parallel to **AG;**

draw **gh** parallel to **GH** and **ch** parallel to **CH**; draw **hi** parallel to **HI** and **ib** parallel to **IB**; draw **ij** parallel to **IJ** and **jc** parallel to **JC**, and so continue to the end.

Wind Diagram (Fig. 290½).—The action of the wind is supposed to be normal to the surface of the roof. We will draw half of the diagram only. The reactions will be oblique and unequal, being inversely proportional to the rise **r** and length of rafter **l**.

If W is the total wind thrust on the half of the roof shown, and x and y the reactions such that $x+y=W$, we have $\frac{x}{y}=\frac{r}{l}$ and $\frac{x}{x+y}=\frac{r}{r+l}$; from which $x=\frac{rW}{r+l}$

1. Draw forces **ra, ad, dg, gh** and the reactions **hg** (=**x**) and **qr** (=**y**) respectively parallel and equal to **RA, AD, DG, GH, HQ** and **QR.**

2. Draw **ab** parallel to **AB** and **bq** parallel to **BQ**; draw **bc** parallel to **BC** and **cd** parallel to **CD**; draw **ce** parallel to **CE** and **eq** parallel to **EQ**; draw **ef** parallel to **EF** and **fg** parallel to **FG**; finally draw **fi** parallel to **FI**; it will be bisected by **bq.**

The other members would be subjected to the same strains were the wind acting on the other half of the truss.

These examples will be sufficient.

Some Forms of Frames.

The Bowstring (Fig. 291).—In which the upper members, and sometimes also the lower ones, affect the general form of an arc or bow. It has the advantage of reducing the length of the end members where the stress is greatest. Deeper girders can be used than in the Warren. When the lower member is horizontal, it is much used in through bridges.

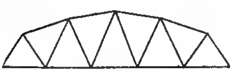


Fig. 291

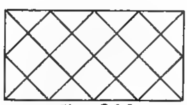


Fig. 292

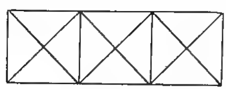


Fig. 293

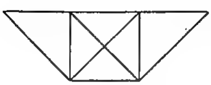


Fig. 294

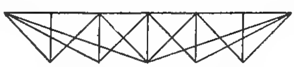


Fig 295

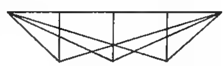


Fig. 296

Lattice (Fig. 292).—It has the slightest advantage of having the diagonal struts pinned at their junctions with the ties, and is thereby stiffened somewhat.

Cross-Panel (Fig. 293).—Mostly used with timber members.

Inverted (Fig. 294).—Much used for deck bridges of small spans.

Fink Truss (Fig. 295).—It is cheaper than the Warren.

Bollman Truss (Fig. 296).—Not as strong as the Fink. The Fink and the Bollman are also used for deck bridges.

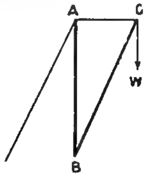


Fig 297

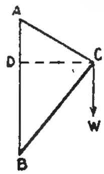


Fig. 298

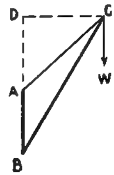


Fig. 299

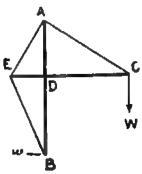


Fig 300

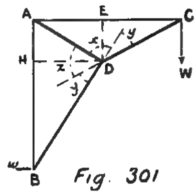


Fig. 301

Cranes and Derricks.—AB is the fixed mast, **BC** is the boom or brace, **AC** is the tie, **W** is the weight.

The mast is supported at **A** and **B**.

Stress $AC = \frac{AC}{AB} W$; stress $BC = -\frac{BC}{AB} W$;

Stress $AB = \frac{AD}{AB} W$ (*fig. 298*); stress $AB = -\frac{AD}{AB} W$ (*fig. 299*).

If the top of the mast (in **Fig. 297**) is held by inclined backstays in the plane **ABW**, then,

Stress $AE = \frac{AC}{AB} W \times \frac{AE}{EB}$; a resulting compression follows in AB

Stress $AB = -\frac{AC}{AB} W \times \frac{AB}{BE}$ (*fig. 297*).

In (*fig. 300*) Stress $CD = -\frac{CD}{AD} W$; Stress $ED = -$ Stress DC;

Stress $AC = \frac{AC}{AD} W$; horizontal reaction at B $w = \frac{CD}{AB} W$

Stress $BE = \frac{BE}{ED} w$; Stress $AE = \frac{AE}{DE} \times$(stress $CD - w$);

Stress $AB = -\left(\frac{BD}{DE} + w\right) + W$.

In (*fig. 301*), y is the angle between a boom and the prolongation of the other; DE, DH are perpendiculars.

$w = \frac{AC}{AB} W$; Stress $AC = \frac{CE}{ED} W$; Stress $CD = -\frac{CD}{ED} W$;

Stress $AB = \frac{BH}{DH} w$; Stress $BD = -\frac{BD}{HD} w$; Stress $AB = -$stress $CD \frac{\sin. y}{\sin. x} = -$stress $BD \frac{\sin. y}{\sin. z}$.

GEOMETRICAL DRAWING.

Drawing.—The art of representing real or fictitious objects by the delineation of their outlines.

Landscape Drawing.—The art of representing inanimate nature.

Figure Drawing.—The art of representing animate nature.

Academic Drawing.—The art of representing the human figure.

Geometrical Drawing or Linear Drawing, also called improperly **Mechanical Drawing** (because of the drawing instruments used), is the art of representing human constructions.

Ornamental Drawing.—The art of ideally representing animate or inanimate nature to relieve the monotony and regularity of geometrical lines in constructions.

Divisions of Geometrical Drawing.—Geometrical drawing is divided into:

Architectural drawing,

Mechanical drawing,

Topographical drawing.

Architectural Drawing—Consists in correctly representing existing or contemplated buildings and their several parts.

Mechanical Drawing—Consists in correctly representing machinery and structural work in general.

Topographical Drawing—Consists in correctly representing the surface of the ground and engineering works of various characters.

Instruments Used in Drawing.—The principal instruments are: Ruler, **T**-square, triangle, drawing pen, dividers, compasses, protractor and scale of equal parts.

Ruler or straight edge is a rectangular piece of wood, metal, hard rubber or other material. Its long sides are used to guide the pencil or pen when drawing straight lines. A perforation near one end allows the instrument to be suspended when not in use.

How to Draw a Straight Line.—A straight line is generally determined by two points. Place one side of the ruler along the two points and at the same distance from each (the eye gets gradually educated in judging equal short distances), that distance depends on the thickness of the pencil or pen point. Pass lightly a sharp lead pencil or a drawing pen filled with ink along the side of the ruler next to the points and take care, first, to just cover the points; second, to maintain an equal pressure.

Verification of the Ruler.—Draw a line along the edge of the ruler to be verified, with a very sharp point, and mark on it two points near the ends. Turn the paper upside down and draw, with the same edge of the ruler, a second line between the two points so marked. If the two lines coincide the edge of the ruler is straight.

If the two lines are too much different the ruler should be rejected. The second side of the ruler may be verified in the same manner.

T-Square.—It is a straight edge or ruler, at one end of which a shorter ruler, called head, is set at right angles. This head is made thicker than the edge, so that it may be made to slide along the drawing table or board. If the edges of the table are straight (if they are not they should be planed), the T-square will have a parallel motion.

The T-square is used to draw parallels to the long sides of the table and paper, and to act as a base for drawing perpendiculars to that direction.

T-squares are also made with a swivel head. The short ruler revolves about a center pin and may be set at any angle with the ruler; it is then clamped, and may be used to draw parallel oblique lines.

The T-square also has a perforation to suspend the instrument.

Verification.—The T-square is generally verified only for the straight edges, which may be done as before. If the head is not detachable, apply a verified straight edge to it and see whether they coincide. A convex head should be planed or rejected immediately.

Triangle.—It is a right-angled triangle of the same materials as the ruler, with a perforation for suspending the instrument. Two kinds are used—one with angles of 45°, the other with angles of 30° and 60°, respectively.

The triangle is used, first, to draw perpendiculars to a given direction; second, to draw parallels to a given direction.

Drawing Perpendiculars.—If the perpendiculars are to be parallel to the straight edge of the board, as is often the case, hold the T-square firmly in position with its head along the left-hand edge of the drawing table or board; move the triangle on one of its right sides along the T-square until the successive points are met by the other right side. Then draw a line through each one. When the perpendiculars are oblique to the border, place one of the right sides in the direction to which perpendiculars are required and another triangle or ruler against the hypothenuse to act as a base; holding this last instrument fast, move the first until the other right side of it meets the successive points; through each of them draw a line.

Drawing Parallels.—Place the hypothenuse (for instance) in the given direction; apply another triangle or a ruler against one of the right sides to act as a base; then move the first instrument along the base until the hypothenuse meets the successive points; through each of them draw a line.

One right side may be placed in the given direction, in which case the base is set against the hypothenuse. This last arrangement permits to draw a set of perpendiculars to the set of parallels by using the other right side as a guide for the drawing point.

Verification.—The sides must be verified separately as straight edges. To verify the right angle, set a straight edge as a base; rest

on it one right side of the triangle; draw a line along the other right side and mark a point on it near the further end; reverse the triangle bottom side up and slide along the base the same right side till the point marked comes in contact with the other right side; then through this point draw a line. If the two lines so drawn through the same point coincide the triangle is accurate.

Remarks.—Celluloid triangles are manufactured which have advantages over wood and hard rubber, first, in being transparent, and therefore showing much better where to begin and end an ink line; second, in not soiling the paper as much as hard rubber.

Rulers and **T**-squares are also made with celluloid edges.

Good rulers, **T**-squares and triangles are valuable instruments and should be well cared for, suspended in a dry place when not in use and wiped off before using. Avoid shocks and never use them to cut paper.

Drawing Pen.—Two equal steel blades, connected at one extremity with a handle used to guide the instrument, have their free ends brought in contact or separated by means of a connecting screw placed about half way between the point and the handle. In many pens one blade is hinged to the handle so that, by removing the connecting screw, the instrument may be cleaned more easily.

Some pens have a pivoted washer under the head of the connecting screw, by displacing which the blades are brought further apart, either for cleaning or for charging with ink. By replacing the washer the pen is set as before, either for fine or heavy lines.

Use of the Drawing Pen.—When a pencil drawing is to be preserved the lines must be inked. To that end the straight lines are gone over with ruler and pen. The ink is introduced between the blades of the pen by means of a quill dipped in the bottle or saucer; or the pen may be charged by capillary attraction, simply by moistening the end of the pen with saliva and dipping it in the liquid ink; the sides of the blades must be wiped off before using. Place a ruler along a pencil straight line of the drawing at a distance varying with the kind of pen used and which experience will teach as necessary; slide the charged pen along the edge, holding it almost vertically and with a regular pressure and motion from one end to the other.

Repairing the Pen.—When a drawing pen ceases to draw fine and sharp lines, the draughtsman may often repair it himself. Clean the blades thoroughly and bring them in contact; then rub them lengthwise upon a wet piece of slate or the back of a saucer until it is ascertained by trials that the instrument has regained its lost qualities.

A hone with oil will give much better results.

Dividers.—Two equal rods pointed at their free end and connected at the other by a screw, around the axis of which they may rotate by friction and assume any angular position. The clamping head of the screw is perforated so that by the insertion of a key the degree of friction may be regulated.

Uses of the Dividers.—1. To take distances, either from a drawing to be reproduced or from a scale of equal parts, and lay them off from a given point in a given direction. 2. To divide lines into a number of equal parts, from which use its name is derived.

In order to divide a line set one point of the dividers at one extremity of the line and open the instrument so as to divide the line as nearly in the required number of parts as possible and span the line, revolving the head of the dividers between the fingers until the other end of the line is reached. Should the point of the dividers exactly fall at the other extremity, the line will be divided as desired. Generally it will not be so, and there will be a shortage or an excess. In that case, without removing from the paper the point of the dividers further from the end of the line, open the instrument in case of a shortage or close it in case of an excess by the fraction required and try the new distance. A very few trials will give the desired result.

Compasses.—This instrument is a dividers, one arm of which is composed of two separate portions; the point has a tenon and can be inserted into the mortised stem, and steadied there by means of a screw. This arrangement permits to substitute to the movable point either a pencil holder or a drawing pen.

Uses of the Compasses.—With the point the instrument may be used as a dividers. With the pencil point or the drawing pen it is used to draw arcs or circles when the other point is placed at the center, the instrument being held vertically by the head without pressure and made to revolve uniformly. When the radius is too great a lengthening bar mortised at one end and tenoned at the other may be used to lengthen the tracing arm of the compasses.

Caution.—Metallic instruments should be kept in a dry place and wiped before and after using. No ink should be allowed to dry in the pens, and these, after being wiped, should have their blades set apart to relieve the tension of the spring on the screw.

Protractor.—A wooden, paper, horn or metallic semi-circle, the arc of which, called limb, is divided into 180 degrees, numbered both ways, 0, 10, 20, etc.; the degrees are sometimes divided into halves or quarters. Sometimes, also, a vernier permits the reading of single minutes.

Another form given to protractors is that of a rectangle, one long side of which is the diameter and the other three sides bear the graduations, the lines of which are the intersections of the sides of the rectangle with the radii that would divide the circumscribed circle into degrees.

Some protractors have a diametral ruler a little longer than the instrument and pivoted at the center; this ruler is often a scale of equal parts, and is principally used in topographical plotting.

Uses of the Protractor.—1. To measure angles already drawn. 2. To lay off given angles.

How to Measure an Angle Already Drawn.—Place the diameter upon one of the sides of the angle and the center at the apex, the instrument covering the other side of the angle. Read the number of degrees from the zero on the first side to the intersection of the limb with the second side.

How to Lay Off a Given Angle.—To lay off a given angle at a given point of a given line, place the center upon the given point and the diameter upon the given line. Starting from the zero on the given line, count on the limb the number of degrees (and fraction) required and mark a point opposite; remove the protractor and draw a line through that point and the given one.

The protractor may be used to draw perpendiculars by simply laying off angles of 90°.

Verification of the Protractor.—Draw an angle of any magnitude, say, 36°; give the instrument different positions, simply maintaining the center upon the vertex; in all these positions the difference of the two readings (each side of the angle giving one) must be 36°.

Scale of Equal Parts.—Objects of large dimensions cannot be drawn full size on paper; therefore, it is agreed to represent the unit of measurement by a smaller line, which generally is an aliquot part of the unit; the selected line is divided precisely as the unit it represents—into tenths and hundredths if the unit is decimal, as the meter or the engineer's chain, or into twelfths if the unit is a foot. This selected line, being divided, constitutes a **scale of equal parts;** its divisions are called chains, meters, feet, inches, etc., as the unit they represent, and numbered as such.

Forms of Scales.—Some are flat rulers and the two edges allow only of four scales being engraved on them, two on either side.

Some are triangular in section and have six scales, two to each edge.

How Scales Are Designated.—Architectural scales are designated thus:

16, 3, 1½, 1, ½, ¾, ⅜, ¼, ⅛, 3/16, 3/32

Scale 16 is a foot rule, exact length, divided into inches and sixteenths.

Scale 3 is 3 inches to 1 foot;	used for machinery	
" 1½ " 1½ " " 1 ";		
" 1 " 1 " " 1 ",	" " detail drawings, masonry.	
" ½ " ½ " " 1 ";		
" ¾ " ¾ " " 1 ",	" " stone work.	
" ⅜ " ⅜ " " 1 ";		
" ¼ " ¼ " " 1 ";	" " general plans for buildings.	
" ⅛ " ⅛ " " 1 ",		
" 3/16 " 3/16 " " 1 ";	" " general stone work.	
" 3/32 " 3/32 " " 1 "		

Inches and fractions, when not shown on scales, may easily be estimated.

Engineering Scales are designated thus:

			used	for
10	they	10 (or 100) ft.	details of structures	or Topographical maps,
20	indicate	20 (" 200) "		
30	that	30 (" 300) "	Sections	" Maps of large sections
40	one inch	40 (" 400) "	and	of country
50	of the	50 (" 500) "	profiles	and R.R. maps,
60	drawing	60 (" 600) "	Topographic details	" General maps.
80	represents	80 (" 800) "		

Oblique Scales (Fig. 301½).—They are generally decimal scales and more commonly flat metallic rules. A beam of eleven equidistant parallels are drawn lengthwise; they are intersected by a set of parallels, whose distance from each other is the scale unit, say, one inch to represent a foot. The portions of the top and bottom parallels intercepted by the last inch space to the left are each divided into ten equal parts; these points of division are connected obliquely, the first to the second, the second to the third and so on. Such a scale gives hundredths of an inch, which, as a scale, are hundredths of a foot. Measures are taken on this scale with the dividers according to the following rule: Place one point on the vertical division corresponding to the number of feet and on the horizontal corresponding to the hundredths; open the dividers to the oblique line corresponding to the tenths.

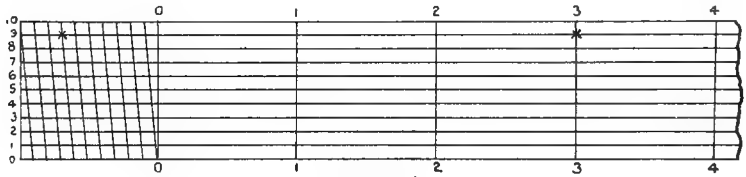


Fig. 301½

Examples: Take 3.69 feet on the oblique scale. Place one point of the dividers on the vertical 3 and horizontal 9; open the instrument to the oblique 6. An oblique architectural scale can also be constructed by drawing 9 horizontals, dividing the last foot space to the left into 12 equal parts on top and bottom parallels and connecting them obliquely as before. This scale would give the feet, inches and eighths.

With the other scales the distances are carried directly with the scale to the paper.

How Scales Vary.—The scale of a drawing varies according, first, to the size of the object to be drawn; second, to the size of the sheet of paper on which the drawing is to be drawn.

Often the scale is determined by law or by usage; in that case the size of the sheet must be selected large enough to contain the whole drawing.

Given the Size of the Object and the Scale, Find the Size of the Sheet to Be Used.—Divide the length of the object in feet by the number of feet in one inch of the scale and add, say, five inches (one inch for border and one and one-half inches for blank space); this will give the length in inches of the sheet to be used.

Given the Size of the Object and the Size of a Sheet to Be Used, Find the Scale.—Subtract from the length of the sheet enough space for border and blank (say, five or six inches) and divide the remainder, expressed in inches, by the length of the object expressed in feet; the quotient will give in inches (or fraction of an inch) the length which is to represent one foot. Or divide the length of the object in

feet by the free space in inches; the quotient will express how many feet to the inch the scale is to be.

Scale of Chords.—A straight line, divided into ninety unequal parts, numbered 0, 10, 20, etc., to 90. The distance from any point to the zero of the graduation is the length of the chord of an arc of a number of degrees equal to the reading of the point in a circle having as a radius the length of the chord of 60 given by that scale.

Use of the Scale of Chords.—It is used to draw angles less than 90°, one side and the vertex being given. From the vertex describe a curve with a radius equal to the chord of 60° Take on the scale the chord of the given angle with the dividers and set it off on the arc from its intersection with the given side.

Other Requisites.—Lead pencil, Indian ink, pencil and ink rubbers, paper, tracing cloth.

Lead Pencil.—The lead or graphite should be of uniform grade and rather hard; it should not be sharpened to a point (although many do it), but to a flat, sharp edge; when this begins to lose its edge it is rubbed on a rough surface—emery or sand paper. The lead pencil should always be held vertically.

Indian Ink.—It is a stick of Chinese or Japanese ink. To prepare it a few drops of water are poured in a saucer and this is held obliquely while the stick or cake of ink is rubbed with uniform and gentle pressure upon the upper portion of the saucer until a very dark fluid ink is obtained, which fact is ascertained by inclining the saucer in different positions, in all of which the bottom must remain black; or, again, by blowing through the mass of the liquid and failing to notice any white in the bottom of the saucer.

Indian ink must be prepared every day, and in summer even twice a day, because it will not bear washing if not fresh. Waterproof ink is largely used, but many good draughtsmen still prefer Indian ink prepared as aforesaid.

After using, the stick should be wiped dry, otherwise it may crack and break.

Rubber.—Soft rubber is used, first, to obliterate wrong pencil lines; second, to erase all the pencil lines of a drawing after they have been inked. It should be used very lightly and uniformly, principally if the drawing is to be washed.

Hard rubber is used to obliterate ink blots or wrong ink lines. In a crowded spot the space to be rubbed may be localized by means of a perforated shield.

Paper.—Many kinds of drawing papers are on the market, and it is difficult to differentiate between their respective merits. The best way to judge them is by securing samples and submitting them to the same tests.

Testing Paper.—With the same instrument and the same ink draw on all the samples several lines and the same series of fine, medium and heavy lines, broken lines, free-hand lines, etc.; draw also series of

lines with pencils of different grades, hard and soft. When they are all dry, first, look them over carefully with a magnifying glass, if necessary, to ascertain what kind of paper shows the sharpest lines; second, rub some pencil lines with soft rubber, using about the same pressure during the same time and over an equal area on each sample, and examine on which kind of paper the least trace of pencil remains; third, draw ink lines, fine and heavy, over the area just rubbed off and examine as before which samples show sharper lines; fourth, rub off some ink lines with ink rubber on all the samples, in the same manner as for soft rubber test, and note on which kinds of paper the least trace of ink remains; fifth, ascertain likewise the samples that will be least injured by an ink eraser or sharp knife used in removing ink lines; sixth, prepare a wash of light Indian ink and apply it with a brush upon a clear portion of the samples and also upon the portion already submitted to tests and note which samples show more uniformity of tint. A rubber test might also be applied to that tint when dry.

Finally, select the paper best suited to your purpose.

Bristol board is required for patent drawings.

Manilla papers are used for sketches, preliminary drawings and detail drawings, full size.

A granulous surface is preferable for wash drawings.

Before selecting sheet paper, ascertain that it is not warped.

Outward Characteristics of Good Paper.—A good paper should be even in color, texture, thickness, which latter fact is ascertained by holding it to the light, when it must present uniform transparency; this will also show whether the paper is free from metallic specks, which is undesirable.

Right Side of Drawing Paper.—It shows a more granulous surface; the back appears smoother, but generally shows the marks of the fabric of the carrier. In rolls the inside is the drawing side.

Tracing Cloth.—A thin linen or cotton fabric rendered transparent by chemical treatment and glazed generally on one side only. Its use is to copy drawings. The cloth is spread, glazed side down, over the drawing to be reproduced, and tacked; the lines of the drawing are then gone over with pencil or ink.

Purposes of a Drawing on Tracing Cloth.—First, it will bear more handling than a paper drawing; second, it is more easily carried, as it may be rolled thin; third, it may serve as a negative to make prints.

Disadvantage of Tracing Cloth.—It is much subject to the weather and generally expands, and in width more than in length. Such changes take place even when the cloth is on the board.

Lettering.—Ordinary handwriting is bad form on drawings. Round writing and Old English or Gothic are gradually falling into disfavor; but Old and Modern English are much used in engrossing.

Capillary Style (Fig. 302).—Titles of drawings and other lettering used on them are from some selected form of **Roman**, the most

common being the **capillary** or **skeleton** styles, in which all the members of a letter are fine lines of uniform thickness throughout. The thickness, however, should be such that good prints may be taken from the drawing, and it should also vary somewhat with the height of the letters.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz 123456789 123456789

abcdefghijklmnopqrstuvwxyz 123456789 123456789&

Fig 302

Block Style.—The same style of letters, with the uniform thickness increased to not more than one-fifth (1/5) the height, become what is known as **block** letters.

Book Style (Fig. 303).—This is the ordinary form of types used in books; some members of the letters are heavy lines, while others are fine, and others increase gradually from a fine to a heavy line at their middle and decrease gradually to a fine line again; this is the case in all round letters.

ABCDEFGHIJKLMNOPQRSTUY

ABCDEFGHIJKMNPQRSTUVWXZ

abcdefghijklmnopqrstuvwxyz ; 1234567890

abcdefghijklmnopqrstuvwxyz 1234567890

Fig. 303

Straight or Vertical Lettering.—When the upright members are perpendicular to the direction of the line.

Oblique Lettering.—When the upright members are drawn to an angle with the direction of the line. The obliquity or slant should be uniform, and a very pleasing result is obtained with a slant from right to left of one horizontal in four vertical.

Most Economical Style.—The style most economical, that is to say, the one which can be executed within the shortest time, care being the same, is the **oblique capillary,** both in **capitals** and **small letters,** or **lower case.**

Elongated Letters.—When the height of the letter **O** is twice its width or more.

Flattened Letters.—When the height of the letter **O** is less than its width.

Size of Letters.—The size of letters on a drawing should be made dependent: 1. On the relative importance of the designation. 2. On the size of the sheet used.

The lettering, however, should never be made more prominent than the drawing itself.

Spacing.—The spacing of words and of letters depends in a great measure upon the space left free for titles, sub-titles, etc. This spacing must be studied, so as to obtain a good effect. No rules can be given, except that the space between two angular letters, **AV,** for instance, the points of which may be placed nearly on a vertical. The space is a shade greater between an angular and a round letter, such as **AC;** a little greater between two round letters, **OC;** greater yet between a round and a straight letter, as **HO,** and greatest between two straight letters, as **IN.**

Between words, when possible, leave the space of an O.

Equipoise of Letters.—Letters must stand well; for that reason the lower half of symmetrical letters is made a little longer by placing the horizontal central line a shade above the middle of the height in the letters **B, E, F, H, P, R.** For the same reason the lower lobe of the letters **B** and **S** is curved a little more outwardly than the upper lobe; also the lower line of the letters **E** and **Z** is made to project a little more than the upper one.

We give here samples of Old English (**Fig. 304**) and a diagram (**Fig. 305**) of Modern English or Script, in which all the small letters will readily be found.

Fig 304

Fig. 305

Information Which Should Accompany Each Drawing.—Every drawing should have a title, sub-title when needed, names of parts, notes and remarks, dimensions, names of the office, date when made (or approved), serial number for filing, scale (a graphic scale is preferable), and north point if the drawing requires it.

Advice.—

1° Keep your instruments clean and in their proper place.

2° Avoid heavy pencil lines.

3° Don't erase pencil lines before the drawing is inked. The reason is that an ink line is never sharp when drawn on a rubbed spot.

4° Keep the saucers covered which contain ink or wash colors.

5° Be sure that the distances measured from or carried with the scale are correct.

6° When a number of lines meet at a point, don't cover the point with the first ink lines.

7° Heavy lines should be thicker when far apart and thinner when close together.

8° Ink lines are made finer on a paper drawing.

9° On tracing cloth the ink lines should be heavier, as well as the small lettering, in order to secure sharper prints.

Kinds of Lines (Fig. 306).—

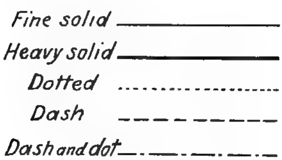


Fig 306

Fine Solid Lines represent real lines receiving direct light.

Heavy Solid Lines represent real lines not receiving direct light.

Dotted Lines (always fine) represent all hidden real lines.

Dash Lines (always fine) are used for auxiliary lines.

Dash and Dot Lines are used for center lines.

Remark.—On a drawing intended to be washed, all real visible lines should be drawn fine, solid, with a lighter ink than if the drawing was not to be colored.

Drawing of Lines.

For definitions of lines, geometrical terms, properties of figures, etc., see **Geometry, Trigonometry, Mechanics,** etc.

Drawing of a Perpendicular.—A line (or direction) is always given. The drawing of perpendiculars with **T**-square and triangles has been explained already.

(a) Point Given on the Line.—When the perp. is to pass through a point on the given line we may proceed in different ways.

First Method (Fig. 307).—When the given point is far from the extremity of the line, set off, with the compasses armed with pencil point, equal distances on either side from the given point and. from the two points thus obtained as centers, describe (with a greater radius than just used) two arcs intersecting each other on the side of the

line where the perp. is to be; then connect the point of intersection with the given point.

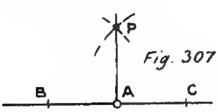


Fig. 307

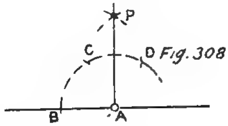


Fig. 308

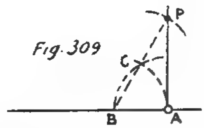


Fig. 309

Second Method (Fig. 308).—With any radius, describe from the given point as a center and on the side of the line where the perp. is to be, an arc of a circle, on which lay off twice the radius used beginning at the line; from the two points just obtained on the arc describe two other intersecting arcs, with the same radius and further from the line; connect the point of intersection with the given point.

Third Method (Fig. 309).—When the point is near the end of the line. From the point describe a quarter of a circle; from its intersection describe with the same radius another quarter of a circle passing through the given point; from the point of intersection of the two arcs, and still with the same radius, describe a third arc further from the line; connect the point of intersection of the line and first arc with the point of intersection of the first two arcs and produce it till it intersects the third arc, which point, connected with the given point, will give the required perpendicular.

Fourth Method (Fig. 310).—From any point beside the line and on the side of it where the perp. is to be, draw a circle passing through the given point; connect its intersection with the line with the center and produce it till it intersects the arc; this point connected with the given one will be the required perpendicular.

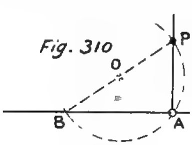


Fig. 310

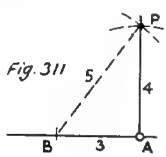


Fig. 311

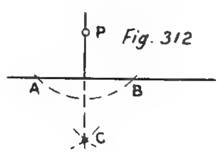


Fig. 312

Fifth Method (Fig. 311).—Draw from the given point as a center in the direction of the required perpendicular an arc of a circle with a radius of four units of any scale (the larger the better); from the same point cut the given line either on the right or the left with an arc of a radius equal to three units of the same scale; finally, from the point thus determined as a center, and with a radius equal to five units of the same scale, cut the arc first drawn and connect the point thus obtained with the given point for a perpendicular. You will thus have drawn a triangle, the sides of which are 3, 4 and 5, and as $3^2 + 4^2 = 5^2$, the triangle is right-angled.

(b) Point Given Outside the Line—First Method (Fig. 312).—From the point, **P**, as a center, describe an arc, **AB**, that will cut the given line in two points, **A** and **B**. From these points as centers and

with equal radii draw two intersecting arcs on the other side of the line from the given point and connect their intersection, **C,** with the given point, **P.**

Second Method (Fig. 313).—Draw from the point **P** an oblique **PA** to the line and bisect it at **O.** From this point **O** as a center, draw an arc **PBA,** passing through the given point **P** and connect its intersection with the line **B** to the given point **P.**

Drawing of a Parallel.—A line (or direction) is always given.

The drawing of parallels with **T**-square and triangles has been explained already.

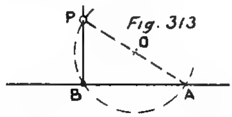


Fig. 313

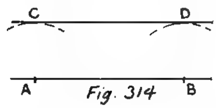


Fig. 314

Given the Distance From the Parallel to the Line (Fig. 314.)—Take the given distance with the compasses, and with it as a radius draw two arcs of circles on the side of the line where the parallel is to be, and from centers selected as far apart on the line as convenient. With ruler or triangle draw a line that shall just touch (or be tangent to) the arcs and it will be the required parallel.

The Parallel Is to Pass Through a Given Point.—Use one of the following methods:

First Method (Fig. 315).—From the given point **P** as a center and with a radius **PA** as large as convenient, describe an arc of a circle **CA** on the same side as the point is and cutting the line at **A.** From **A** as a center and with the same radius describe another arc **BP** passing through the given point **P** and also cutting the line at **B.** Take with the compasses the distance from this last point **B** to the given point **P** and lay it off from **A** to **C** on the arc first drawn and from its intersection **A** with the line. Finally, connect the last point obtained **C** with the given point **P** and the line will be the required parallel.

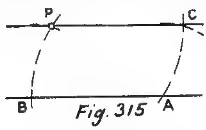


Fig. 315

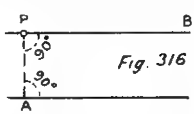


Fig. 316

Second Method (Fig 316).—Draw a perp. **PA** from the point to the line and erect on it a perpendicular **PB** at the given point **P;** it will be the required parallel.

Third Method—Parallel Rule.—The properties of a parallelogram have been utilized in the construction of the parallel rule, which consists of two equal rules connected with two equal arms pivoted at their junction points. To use it, place one ruler (generally the lower one) with its inside edge along the given line and move the other ruler until it meets the given point, through which draw a line.

If the distance is longer than the arms, span it in several stages, moving one while holding the other one fast.

Rectangular System of Co-ordinates—Abscissa, Ordinate (Fig. 317).—The line **xx′**, to which a series of perpendiculars are drawn, is called the axis of **X's**; one of the perpendiculars **yy′**, supposed to be

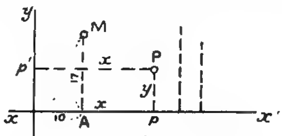


Fig. 317

known and to which all the other perpendiculars (parallel to each other) may be supposed to refer, is called the axis of **Y's.** These two lines form a rectangular system of co-ordinates, and the intersection **O** of the two axes is the origin of the co-ordinates.

Any point **P** given in position in the plane of these axes may be determined by perpendiculars **Pp — Pp′** to each axis. The perpendicular **Pp′**, drawn from a point to the axis of **Y**'s, is the abscissa of that point; and the perpendicular **Pp** drawn from the same point to the axis of **X's** is the ordinate of that point. Noting that the distance **Op** from the foot of the ordinate to the axis of **Y's** equals the abscissa **Pp′** of the point, the abscissa of a point is often defined as the distance from the origin to the foot of the ordinate. The abscissa of a point is represented by **x** and its ordinate by **y.** Any point in the plane of the axes will have one set of co-ordinates, and one only; therefore, all the points of that plane are determined in position.

Find a Point Whose Co-ordinates are Given, say,

$$x = 8$$
$$y = 19$$

Lay off, with the scale of the drawing, a length **OA** equal to 8 units on the axis of **X's** and from the origin **O**; at the point **A** erect a perpendicular **AM** to the axis of **X's** (or draw a parallel to the axis of **Y's**) and make it equal to 19 units of the same scale. The point **M** will be determined.

Multiple Lines.—A line which is to be a number of times the length of another.

First Method (or Graphic Method) (Fig. 318).—Take the length of the line with the dividers and lay it off from the given point the required number of times in the given direction.

Fig 318

Or take the line with the compasses and lay distances off with successive radii of one, twice, four times, etc., the line.

Second Method.—Measure the line with the scale if length is not known; multiply the length by the given multiple and lay off the result as above stated.

Arc and Angle Made Equal to a Given Arc or Angle—First Method (Fig. 319).—In case of arcs they are supposed to be of equal radii, and the origin of the required arc is given. In case of angles one side and vertex of the required angle are given. From the vertices **O** and **A** of the given and required angles draw arcs **MN-BC** with equal radii; the question is reduced to making an arc, from the given side as origin, equal to the given arc (that between the sides of the given angle). Take with the dividers the distance between the points of intersection **M** and **N** of the arc and sides of the given angle **O** and lay it off on the other arc from its origin **B**; then connect that point with the given vertex **A** on the given side. The distance thus taken and carried is the length of chord of the given arc. That chord might be scaled off and the distance carried with the same scale from the origin of the required arc.

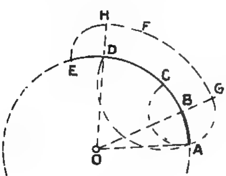


Fig. 320

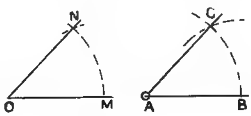


Fig. 319

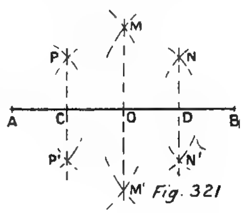


Fig. 321

Second Method.—Measure the given angle with the protractor, then make an angle equal to it on the given line and at the given point. In the case of arcs, draw the central angle of the known arc and the radius of the given extremity of the other arc and proceed as for angles.

Multiple Arcs and Angles (Fig. 320).—The chord of the given arc **AB** is carried on the required arc (or on the one drawn from the vertex of the required angle) the number of times stated.

The protractor may also be used.

Division of Lines—Straight Line Divided in 2, 4, 8, etc., Equal Parts (Fig. 321).—From the extremities **A** and **B** of the line describe, with equal radii, arcs intersecting above and below the given line at **M** and **M'**; connect the points of intersection with a ruler and mark on the given line the point **O** where that perpendicular would cut the line; that point **O** will bisect the given line. Bisecting each half of the line in like manner will divide it in 4 equal parts, and each quarter being likewise bisected will divide the line into 8 equal parts.

Dividing a Line Into Any Number of Equal Parts (Fig. 322).—Draw any line from one extremity of the given line, and on it lay off with a scale or with the dividers (it is better to use the dividers, as

points can be pricked which, however light, are always visible) as many equal distances as the line is to be divided. This done, connect the furthest point with the other extremity of the line (or simply apply a triangle along these points) and draw parallels to it through each of the other points.

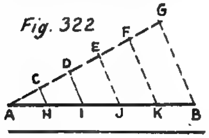


Fig. 322

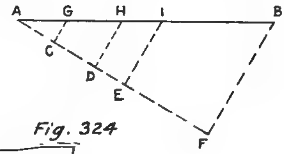


Fig. 324

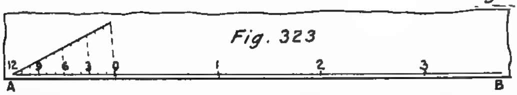


Fig. 323

Drawing of a Scale of Equal Parts (Fig. 323).—The draughtsman is sometimes obliged to draw his own scales. Take a long strip of paper and draw a fine ink line along one of the sides and as close to it as possible. The unit of the scale is generally known; lay it off with dividers from one extremity **A** of the line as many times as possible; now divide the last unit to the left, according to the preceding methods, into as many equal parts as the unit has subdivisions (12 if the unit is a foot). Write zero (**O**) at the right end of the first left-hand division, then successively and from left to right, **1, 2, 3,** etc., to the end. From **O** towards the left, if the subdivisions are inches, write **3, 6, 9, 12** at the 3rd, 6th, 9th and 12th inch of the scale.

Division of a Line Into Proportional Parts (Fig. 324).—Proceed as before; draw a line at the extremity **A** of the one to be divided, forming an angle with it, and on this, instead of equal, lay off with a scale proportional parts to the given lines, if the lengths of these are known, or lay off the lines themselves taken with the dividers; connect the last point to the other extremity of the line to be divided and draw parallels as before explained.

$$\frac{AG}{AC} = \frac{GH}{CD} = \frac{HI}{DE} = \frac{IB}{EF}.$$

Interpolating a Given Number of Equidistant Parallels Between Two Parallels (Fig. 325).—Place a scale between the given parallels

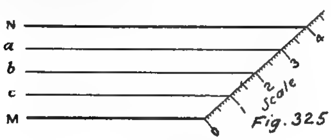


Fig. 325

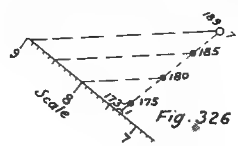


Fig. 326

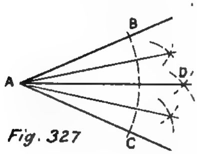


Fig. 327

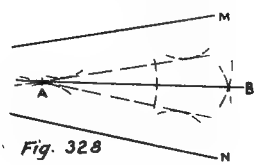


Fig. 328

so that **O,** being on one of them, **M,** the number indicating how many equal spaces there are to be (one more than the number of intermediary parallels), shall be on the other given parallel **N;** mark with a sharp pencil, or prick with a needle, points at every intermediary division of the scale; finally through each point draw the parallels required **a, b, c.**

In a similar manner parallels whose distances are to be proportional to given lines may be drawn.

The distance on the scale between the given parallels is the sum of the given lines.

Spacing Contours Between Two Points of Known Elevations (Fig. 326).—An example will suffice. Let the elevations be 173.1 and 189.7 and the contours 5 ft. apart vertically. Draw any straight line through 189.7, and connect the two points; place a scale with the reading 73.1 (or 3.1) on point 173.1 and the reading 89.7 (or 19.7) on the line drawn through point 189.7; when this position of the scale is obtained mark points (in pencil only) on the reading 75, 80 and 85, through which draw parallels to the first line passing through point 189.7; the intersections of these parallels with the line connecting points 173.1 and 189.7 will be the points of passage of contours 175, 180 and 185. Prick these points and erase the pencil lines at once.

Bisection of an Angle (Fig. 327).—In case of an angle, draw from the vertex **A** an arc **BC** between its sides. From the extremities of the arc and with equal radii draw arcs intersecting on the side opposite the vertex and connect that point with the vertex. By dividing each half in two, each quarter in two, etc., the given angle may be divided into 4, 8, 16, etc., 2^n equal parts.

Bisection of an Angle when the Sides Cannot Be Produced (Fig. 328).—Draw within the angle equidistant parallels to the sides which shall intersect, and bisect their angle.

Bisection of an Arc (Fig. 329).—The center may not be easily determined to draw the central angle. In that case simply draw, from the extremities of the given arc, arcs of circles of equal radii intersecting each other on each side of the arc and join the points of intersection, which line will bisect the given arc. By dividing each half again in two, each quarter in two, etc., the given arc may be divided into 4, 8. 16, etc., 2^n equal parts.

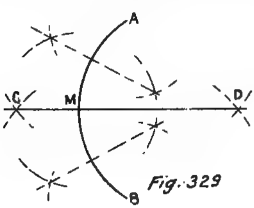


Fig. 329

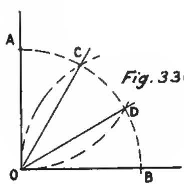


Fig. 330

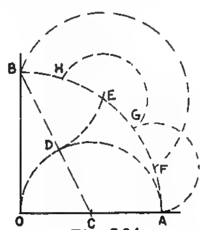


Fig. 331

Division of an Arc Into Any Number of Equal Parts.—This may be pretty accurately done by trials. Take with the dividers a dis-

tance which you judge to be the required fraction of the arc and carry that distance on the arc from one of its extremities. If the last point falls upon the other extremity the problem is solved. If not, without removing the point of the dividers nearer the origin of the arc, enlarge or shorten the distance, as the case may be, by a fraction of the difference and try the new distance. A very few trials will divide the arc as required.

Trisection of a Right Angle (Fig. 330).—Draw from the vertex an arc between the sides and carry the radius from each point of intersection upon the arc and within the angle. These points **C** and **D** will divide the arc and therefore the angle (by joining the points to the vertex) into three equal parts.

Bisecting each third would divide the right angle into six equal parts.

Division of a Right Angle Into Five Equal Parts (Fig. 331).—Draw from the vertex a quadrant between the sides of the angle with as large a radius as convenient. Bisect the radius on one of the sides and draw a semi-circle on that radius. Join the center of the semi-circle with the extremity of the radius on the other side and from this last point as a center, with a radius equal to the outside portion of the secant, draw an arc which will span, on the quadrant, just 1/10 the full circumference. Lay off that 1-10 twice upon the quadrant and the remainder will be 1/5 the quadrant.

Drawing of a Circle Through Three Points (Fig. 332).—Connect the points with two lines (these lines may simply be supposed drawn without actually being so), which will be chords of the required circle. Bisect the chords **AB, BC** and the intersection of the perpendiculars will be the center **O**. The radius will be the distance from the center to any one of the three points, say **OA.**

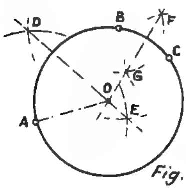


Fig. 332

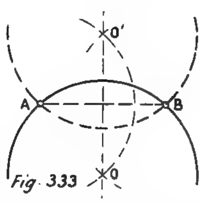


Fig. 333

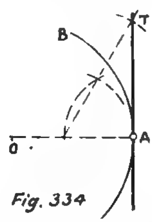


Fig. 334

Drawing of a Circle Through Two Points, Radius Being Given (Fig. 333).—Join the two points and bisect that chord. From one of the points **A** as a center and with the given radius **R**, cut the bisecting line at a point **O**, which will be the center. As the bisecting line may be cut in two points **O** and **O'**, one on each side of the chord, the problem admits of two solutions.

Questions of Tangency.

Drawing of a Tangent to an Arc—First Case—The Point Is Given on the Arc (Fig. 334).—Draw the radius of the given point **A** with one side of a triangle (or simply place the triangle in that direction). Ap-

ply another triangle to the hypothenuse to act as a base; then move the first triangle till the other right side meets the point **A** and draw through it a line **AT**, which will be the required tangent.

Or erect, at the end of the radius of the point, a perpendicular by one of the preceding methods.

Second Case—The Point Is Given Without the Arc (Fig. 335).—Draw through the point a line which shall just touch the arc. Very often this method will be accurate enough.

When the point of contact is to be absolutely determined, join the given point **A** with the center **O** and, on that line **AO** as a diameter, draw a circle which, by its intersections **B** and **C** with the given arc, will give the points of contact **B** and **C.**

There are always two solutions.

Condition of Tangency of a Straight Line and a Circle (or Arc).—The straight line must be perpendicular at the extremity of the radius of the point of contact. Conversely, an arc is tangent to a straight line when its center is upon the perpendicular erected at the point of contact.

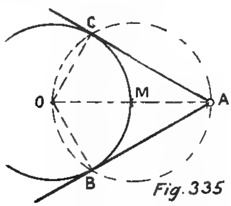


Fig. 335

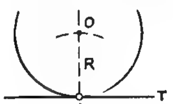


Fig. 336

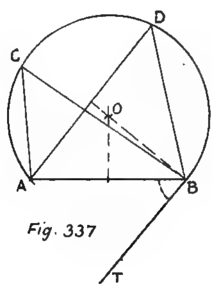


Fig. 337

Drawing of a Circle of Given Radius and Tangent to a Given Line at a Given Point (Fig. 336).—Erect a perpendicular **AO** to the line **T** at the given point **A** and upon it lay off the given radius **R** from the given point **A**; the center **O** will thus be obtained.

Drawing of a Segment Capable of a Given Angle.—(That is a portion of a circle such that all the angles inscribed therein shall be equal to a given angle.) The chord of the segment is generally given. (If not, assume it.)

First Method (Fig. 337).—At one extremity of the chord make an angle equal to the given one. Bisect the chord with a perpendicular; erect a perpendicular upon the other side of the angle and at the vertex; the intersection of these two perpendiculars will give the center of the arc.

Second Method (Fig. 338).—At one end **B** of the chord draw any line **BM** and assume a point **M** on it; at that point make an angle **M**

with the line equal to the given angle and draw a parallel **AC** to line **BM**; through **B** draw **BC** parallel to **MN**. **A, B** and **C** are three points which determine the required arc **ACB.**

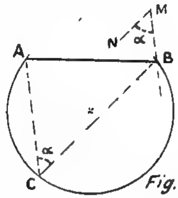


Fig. 338

Condition of Tangency of Two Circles (or Arcs).—The two centers and the point of contact must be on a straight line.

Common Tangents to Two Circles—First Case—External Tangents (Fig. 339).—Let R be the greater radius and r the smaller; O the center of the greater circle and o that of the smaller. From center O draw an auxiliary circle with radius **R-r;** draw tangents to it **oa-ob** from **o;** draw the radii **oa-ob** of contact and produce them till they intersect the larger circle at points **A-B,** which will be the required points of contact in the greater circle; draw through **o** parallels **oC-oD** to these radii, which will determine the points of contact **C-D** in the smaller circle. Connect the points of contact and these lines **AC-BD** will be the required tangents.

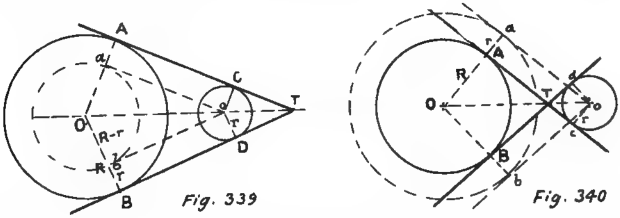


Fig. 339 *Fig. 340*

Verification.—The tangents must meet upon the line of the centers **Oo.**

External Center of Similitude.—The point **T** (Fig. 339) of the line of centers where the common external tangents meet is called the external center of similitude.

Second Case—Internal Tangents (Fig. 340).—From the center **O** describe an auxiliary circle with radius equal to **R + r** and draw tangents to it **oa-ob** from **o;** draw the radii of contact **Oa-Ob** and parallels to them **oc-od** through **o;** these lines will determine the points of contact **c-d,** which connect with **A** and **B.**

Verification.—The tangents must meet upon the line of the centers **Oo.**

Internal Center of Similitude.—The point **T (Fig. 340)** of the line of centers **Oo,** where the common internal tangents meet, is called the internal center of similitude.

Data Necessary to Determine a Circle:

1. Center and radius—or center and a point.
2. Radius and two points.
3. Three points.
4. Two points and a tangent.
5. Two points and one tangent circle (a).
6. Three tangents.
7. One point and two tangents.
8. One tangent circle and two tangents (a).
9. One point and two tangent circles (a).
10. Three tangent circles (a).
11. One point, one tangent circle and one tangent (a).
12. Two tangent circles and a tangent (a).

(a) These given circles are to be tangent to the required circle and may or not be tangent to one another or to the given lines.

Drawing of a Circle Tangent to a Line (Straight) and Passing Through Two Points—First Case (Fig. 341).—One of the given points **A** is on the given line **CD** and is therefore the point of contact.

Erect a perpendicular to the tangent at that point; connect the two points **A** and **B** and bisect that line with a perpend. The intersection **O** of the two perpendiculars is the center of the required circle.

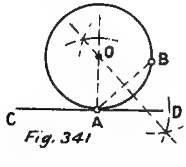


Fig. 341

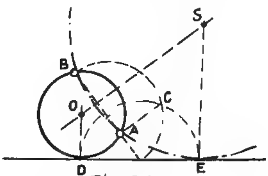


Fig. 342

Second Case (Fig. 342).—Both points **A-B** are without the line **L**—and evidently on the same side. Join the two points and continue the line till it meets the tangent at **T**; upon this total line **BT** as a diameter draw a semi-circle and erect a perpendicular **AC** to the diameter through the point **A** nearest the tangent. From the end **T** of the diameter, on the tangent, as a center and with the distance **TC** from that point to the intersection **C** of the perpendicular with the semi-circle as a radius, describe an arc **DCE** that will cut the tangent in two points **D-E**, which will be the points of contact of the required circles, for there are two solutions. The problem is reduced to drawing a circle that shall be tangent to a line **L** at a given point **D** or **E** and that shall pass through two given points **A** and **B**. Erect a perpen-

dicular to the tangent at the point of contact **D** or **E**, and bisect the distance **BA** of the given points by means of a perpendicular. The intersection **O** or **S** of these two perpendiculars is the center.

There is one solution only when the line of the two points is parallel to the tangent. Bisecting the line of the points will determine the point of contact on the tangent and the problem is reduced to the first case.

Drawing of a Circle Tangent to a Circle and Passing Through Two Points.—First Case (Fig. 343).—One of the points **A** is upon the given circle **O** and is therefore the point of contact. Join the center **O** and the point of contact **A** and produce that line. Bisect the line of the two points **A-B** by a perpendicular, which, meeting the radius of the point of contact, will determine the center **o**.

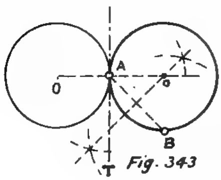


Fig. 343

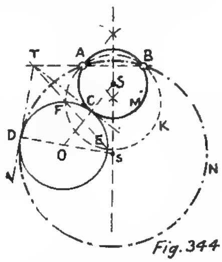


Fig. 344

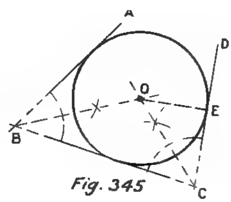


Fig. 345

Should the line bisecting the chord of the points be parallel to the radius of contact, the unknown radius will be infinite and the required circle will be the tangent **T** to the given circle at the point of contact.

Second Case (Fig. 344).—The given points **A-B** are both without the circle **O.** Through the two points **A-B** draw any circle **K** that shall cut the given one and draw the chord **EF** of intersection, which produce; draw also a chord through the points **A-B,** which produce also. From the intersection **T** of these chords draw a tangent to the given circle which will give the point of contact **C.** The problem is thus reduced to the first case. There are two solutions, because two tangents **TC-TD** can be drawn from **T** to circle **O**: One **M** in which the given circle is tangent externally to the required one, and the other **N** in which it is tangent internally.

Drawing of a Circle Tangent to Three Lines (Straight) (Fig. 345.)—Bisect two of the angles **B-C** formed by the three lines which will determine the center **O.** The radius is the length of perpendicular **OE** drawn from the center to any one of the lines as **CD.** If two of the lines are parallel, the same construction holds good; then the distance between the parallels is the diameter of the circle.

Drawing of a Circle Passing Through One Point and Tangent to Two Lines (Straight)—First Case (Fig. 346).—The point **A** is given upon one of the lines **BC.** Bisect the angle **B** of the two lines and erect a perpendicular **AO** upon the line on which the given point is and at that point, which will determine the center **O.**

Second Case (Fig. 347).—The point is given without the lines (and obviously between them). Bisect the angle of the lines and let fall a perpendicular upon it from the given point **A** which you produce by the same distance, thus obtaining a second point **E** symmetrical to the first with regard to the bisecting line. The question is then reduced to drawing a circle determined by two points **A-E**—the given one **A** and its symmetrical **E** and tangent to a line **C**—either of the given ones. There are always two solutions **O** and **S.**

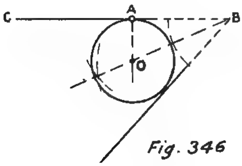


Fig. 346

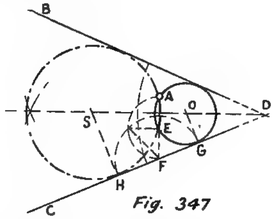


Fig. 347

Drawing of a Circle Tangent to a Circle and Two Lines (Fig. 348).—Draw two parallels to the given lines and at a distance equal to the radius of the given circle. Find the center of a circle that shall pass through the given center and be tangent to the two new lines; this center will also be that of the required circle and its distance from the given circle will be the required radius. There are four solutions.

Drawing of a Circle Through a Given Point and Tangent to Two Circles.—First Case (Fig. 349).—The given point is on one of the given circles. Let **O** and **o** be the given centers and circles and a point **A** given on **O.** Join **OA** and produce it; lay off radius **o** from **A** towards **O**; connect the point of intersection with center **o** and bisect this line; the intersection of this line with line **OA** will give the center. There are two solutions.

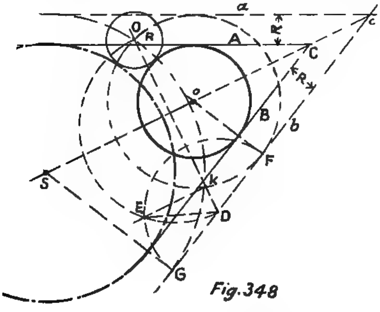


Fig. 348

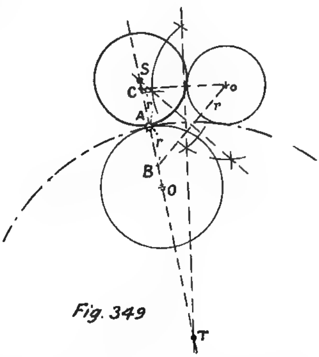


Fig. 349

Second Case (Fig. 350).—The given point is without the given circles. Find the external center of similitude **T** of the two given circles **O** and **o** (point of intersection of the external tangents) and join it to the given point **A**; produce that line. Draw also the line

of the centers **O-o.** Draw an auxiliary circle that shall pass through the internal points of intersection of the given circles with the line of centers and also through the given point **A.** This circle will cut the line drawn from the center of similitude **T** to the given point **A** at another point **A′**, which belongs to the circle required. The question then is reduced to the drawing of a circle that shall pass through two points **A** and **A′** and be tangent to a circle (any one of the two given ones).

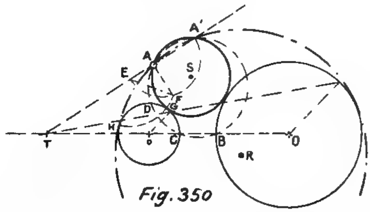


Fig. 350

If the auxiliary circle were tangent to the line **TA,** the question would be reduced to the drawing of a circle tangent to a given circle and to a given line at a given point.

There are four solutions, according as the given circles are; first, external to the required one; second, internal, and, third, either one internal and the other external.

When one of the given circles is to be internal, the internal center of similitude is to be determined and joined to the given point and the auxiliary circle will pass through the points of intersection of the center line with the given circles lying on the same side from the center.

One of the solutions may be a straight line.

Drawing of a Circle Tangent to Three Given Circles (Fig. 351)—

Let $R > R_1 > R_2$ be the radii of given circles O, O_1 and O_2. From center O draw an auxiliary circle with radius equal to $R - R_2$; from center O_1 draw a second auxiliary circle with radius equal to $R_1 - R_2$. The question is reduced to drawing a circle passing through a point (the center O_2 of the smallest circle), and tangent to two circles (the auxiliary ones just drawn).

There are generally eight solutions.

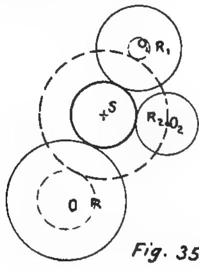


Fig. 351

When the given circles are tangent two and two, there are two solutions only, except when the smallest circle is tangent to the common tangent of the other two, in which case there is but one solution.

These eight solutions are exemplified in the following table:

Solution.	External.	Internal.	Solution.	External.	Internal.
1°	O_1 O_2 O_3	— — —	5°	— — O_3	O_1 O_2 —
2°	— — —	O_1 O_2 O_3	6°	O_1 O_2 —	— — O_3
3°	O_1 — —	— O_2 O_3	7°	O_1 — O_3	— O_2 —
4°	— O_2 —	O_1 — O_3	8°	— O_2 O_3	O_1 — —

Drawing of a Circle Tangent to One Line, One Circle and Passing Through a Point—First Case (Fig. 352).—The point is on the line. Draw an auxiliary line parallel to the given one and distant from it the length of the radius **R** of the given circle; draw a perpendicular to it from the given point. The center of the circle that shall be tangent to the auxiliary parallel, at the foot of the perpendicular and that shall pass through the center of the given circle will be the center of the required circle, and its radius will be the distance of that center from the given point on the given line. There are two solutions.

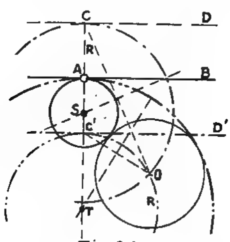


Fig.352

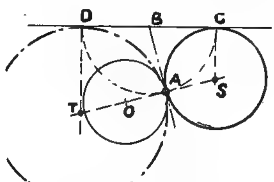


Fig. 353

Second Case (Fig. 353).—The point is on the circle. Draw a tangent to the given circle at the given point and produce it to an intersection with the given line. From that intersection set the tangent off to the given line and the point of contact of the given circle and the line will be obtained. A perpendicular to the line at the point of contact intersecting the radius joining the point on the given circle will give the center of the required circle.

There are two solutions.

If the tangent to the given circle at the given point were parallel to the given line, that radius produced to the line would give the point of contact; the distance from this point to the circle would be the diameter of one solution and its distance to the other end of the diameter of the point would be the diameter of the other solution.

Third Case (Fig. 354).—The point is without both line and circle (but obviously on the same side of the line as the circle). Draw an auxiliary line parallel to the given one and distant from it

the length of the given radius. The required center will be the intersection of two parabolas. First, with the point as the center and the line as the directrix of one; second, with the center of the given circle as center and the parallel as directrix of the other. There are two solutions.

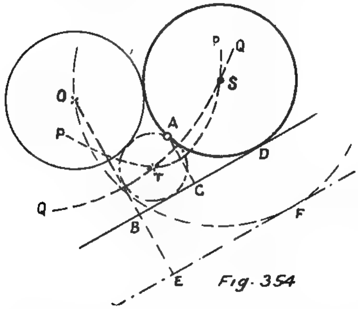


Fig. 354

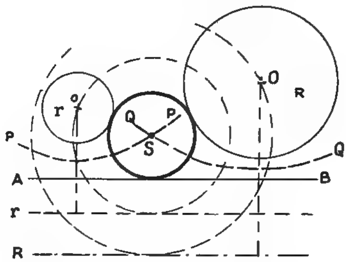


Fig. 355

Drawing of a Circle Tangent to One Line and Two Circles (Fig. 355).—Let **R** and **r** be the radii of the given circles whose centers are **O** and **o.** Draw auxiliary lines parallel to the given ones and distant from it the lengths **R** and **r**—call these parallels **R** and **r.** The required center will be the intersection of two parabolas: First, with **O** as center and parallel **R** as directrix of one; second, with **o** as center and parallel **r** as directrix of the other. There are four solutions.

Arches.—An arch is generally understood as a curved covering for an opening in a structure. That opening may be the distance between two piers of a bridge.

Spring Line is the horizontal line connecting the tops of the uprights.

Drawing of a Semi-Circular Arch, the Span Being Given (Fig. 356).—The tops **A-B** of the side walls or uprights must be level or have the same elevation. Bisect the spring line **AB,** which will give the center **O.** The radius is half the span. The rise equals the radius.

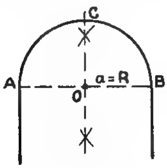


Fig. 356

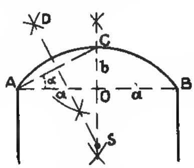


Fig. 357

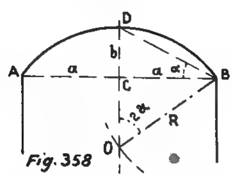


Fig. 358

Drawing of a Segmental Arch, Span and Rise Being Given (Fig. 357).—Bisect the span **AB** by means of a perpendicular **OC,** which will be the central line. Lay off the rise on the central line from the middle point **O** of the span, which will give the soffit **C.** Connect the soffit with one end of the span **A** and bisect that chord **AC** by a perpendicular **DS,** which, by intersecting the central line, will give the required

center **S.** The radius is the distance from the center to one extremity of the span.

Let $2a$ be the span, b the rise and α the angle formed by the chord and span.

Then $R = \dfrac{a^2+b^2}{2b}$ or $R = \dfrac{a}{\sin.2\alpha} = \dfrac{b}{2\sin^2\alpha}$

Drawing of a Segmental Arch, Span and Radius Being Given (Fig. 358).—Bisect the span **AB** for the center line. From one end **B** of the span as a center and the given radius **R**, strike the center line at a point **O** which will be the required center.

b is calculated from formula $b = R - \sqrt{R^2 - a^2}$

and α from $tg.\alpha = \dfrac{b}{a}$, or again $\sin.2\alpha = \dfrac{a}{R}$

Segmental arches require heavier abutments and are therefore used for small openings only.

For large openings, as those of a bridge, tunnel, etc., it is important that the abutments be tangent to the curve of the arch at the spring. Therefore, when the rise is less than half the span

$$b < a$$

the line of intrados is made with an uneven number of arcs tangent two and two, the extreme ones being tangent to the vertical line of the abutments and consequently having their centers on the spring line.

The sum of the angular measures of all the arcs is always 180°

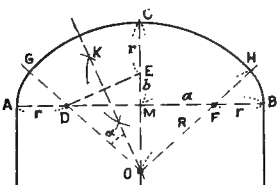


Fig. 359

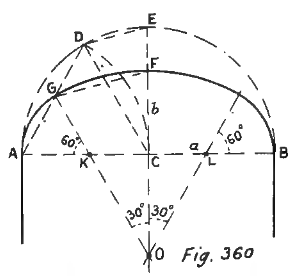


Fig. 360

Drawing of an Arch with Three Centers, Given the Span a and Rise b—First Method (Fig. 359).—Bisect the span **AB** for the center line and lay off the rise **MC** for the soffit **C.** Assume (if it is not given) the extreme radius

$$r < b$$

and lay it off 1° on the span from its extremities (that will give the centers **D-F** of the extreme arcs); 2° on the rise downwards from the

soffit **C** and connect two of the points thus obtained, **E-D,** for instance. Bisect that line **ED** by a perpendicular **KO,** which will, by its intersection with the central line, give the center **O** of the middle arc. Connect this center **O** with the other two **D-F** for lines of centers and of contact. Describe the extreme arc **AG-BH** to the line of centers and finally describe the middle arc **GCH** from its center **O** with a span of the compasses reaching to the points of contact, which are the ends of the extreme arcs.

The middle arc would be reduced to a straight line in case the extreme radius **r** had been assumed equal to the rise **b.**

α being 1/4 of the central angle, is determined by the equation

$tg.\,\alpha = \frac{b-r}{a-r}$. The larger radius $R = \frac{a - r(1 - \sin.2\alpha)}{\sin.\,2\alpha}$.

Second Method (Fig. 360).—If the angular measure of the three arcs is to be the same, each will be 1/3 180° = 60°, and the following construction may be used: Bisect the span for the center line and lay off the rise for the soffit **F.** Describe an auxiliary semi-circle on the span and on the same side as the arch, on which circle lay off the radius **AD** from the extremity of the span **A;** connect that point **D** to the middle **C** and the end **A** of the span (forming an equilateral triangle) and to the intersection **E** of the center line with the auxiliary circle. Draw a parallel to this last line through the soffit **F** (and in the same quadrant) to an intersection **G** with the side of the equilateral triangle; finally, through this point **G** draw a parallel to the radius **CD** of the auxiliary circle till it intersects the center line at a point **O** which is the center of the middle arc. The center of the extreme arc is the point **K** where the same line intersects the spring line and the point of contact the point **G** from which it was drawn.

The radii are given by formulas:

$$r = \tfrac{1}{2}(a+b) - (a-b)\sqrt{3}$$
$$R = \tfrac{1}{2}(3a-b) + (a-b)\sqrt{3}.$$

This arch has a fine appearance when the rise is but a little smaller than half the span.

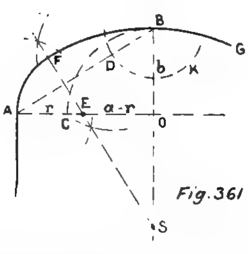


Fig. 361

When the rise is small, compared to half the span, it is better to assume the extreme radius **T** by trials until a pleasing curve is obtained.

Third Method (Fig. 361).—Bisect the span for the center line **SOB**

and lay off the rise $OB = b$ for the soffit **B**. Join the soffit **B** with the extremity **A** of the spring line and draw from it an auxiliary arc **K** with a radius $BD = AC = a - b$ equal to the difference between the half span **a** and the rise **b**; this arc cuts the hypothenuse of the triangle at **D**. Bisect the portion **AD** of the hypothenuse further from the soffit by a perpendicular **FS**, which will intersect the spring line at a point **E** which is the center of the extreme arc **AF**, and the center line at a point **S** which is the center of the middle arc **FBG**.

The radii are given by the formulas:

$$r = \frac{(a^2+b^2) - (a-b)\sqrt{a^2+b^2}}{2a}$$

$$R = \frac{(a^2+b^2) + (a-b)\sqrt{a^2+b^2}}{2b}$$

This method, always applicable, gives a pleasing curve. Mr. Bossut proved that such an oval is the one in which the difference of curvature of the arcs is the least.

Arches are very often drawn with a greater number of centers in order to bring the intrados nearer an elliptical form. The bridge of Neuilly (France), a model of elegance, has 11 centers, The following method gives a satisfactory curve, and the problem can be instrumentally solved; besides, it is applicable to any number of centers:

Drawing of an Arch with Nine Centers. (Fig. 362).—All the angles of the curves are equal. The radii increase by a constant quantity.

1° Bisect the span for the center line **AOA′**; lay off the rise for the soffit **B**.

2° From the middle **O** of the span, with the rise **OB** as a radius, describe an auxiliary semi-circle BHH_1 upon the center line, and another semi-circle $OO'O_1$, with the same radius, upon the spring line as a diameter from the point **H** where the first semi-circle cuts the spring line.

3° Divide the quadrant **BH** included between the spring line and the soffit into nine equal parts and mark every second point **I-J-K-L** from the spring line up. Connect these points with the center. Each of these angles (or arcs, except the last, which is ½ of the others—half the central arc) equals $\frac{180}{9} = 20°$; or, cumulating them from the spring line, 20°, 40°, 60° and 80°

4° Draw perpendiculars to the spring line **Ii, Jj, Kk, Ll** from each of the points so marked on the quadrant. These perpendiculars are the sines of the cumulative angles, and the distances from their feet to the center are the cosines of the same angles.

5° Lay off the sines on the center line one after another from the soffit **B** down to **P**; call their sum **N**. Lay likewise the cosines on the spring line from the center **O** to **Q**; call their sum **M**. The distance from the first semi-circle H_1 to the extremity **P** of **N** is $N - 2b$; the distance from the second semi-circle **H** to the extremity **Q** of **M** is $M - 2b$ Lay off this last distance of $M - 2b$ at the lower end of **N**, and in addition to it from **P** to **R**, then the distance H_1R from the first semi-circle H_1 to the lowest point **R** will be $(N - 2b) + (M - 2b) = M + N$

— **4b.** Lay off on the center line and from the lowest point **R** towards the soffit the length **RS** = **M** (sum of the cosines).

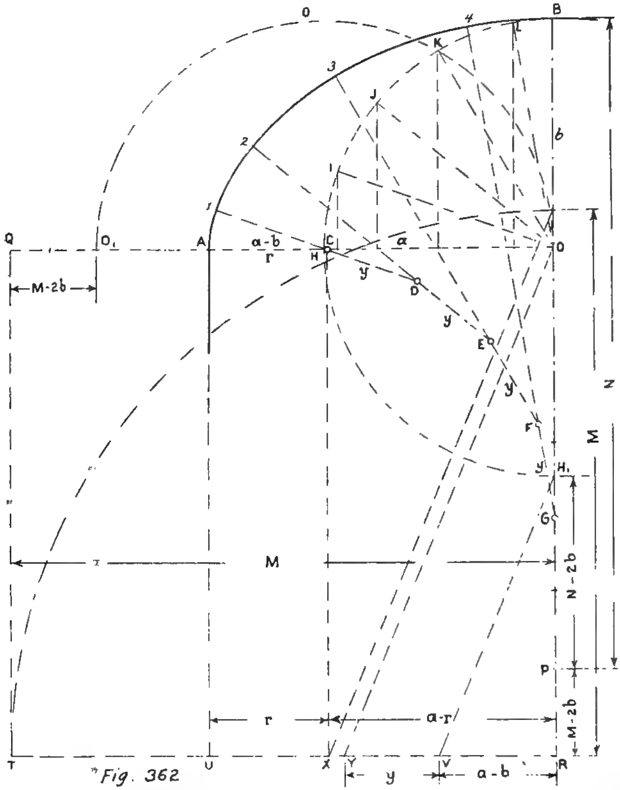


Fig. 362

6° Through the lowest point **R** draw a parallel **RT** to the spring line, on which lay off **RU** = half the span **a**, **and RV** = the difference between it and the rise, or **a — b**.

7° Connect the end **V** of **a — b** just laid off with the lower end **H₁** of the central semi-circle (where it cuts the center line) and draw a parallel to this line from **S** the upper end of the length **M** (laid off on the center line) and another from the center **O** of the same semi-circle. These parallels will intersect the parallel to the spring line in two points **X-Y**; the distance from the further one **X** to the extremity **U** of **a** (as laid off on the parallel to the spring line) is the extreme or smallest radius **r**, which lay off by a parallel **XC** to the center line, and from that point **C** draw the extreme arc **A1**. The distance from **Y** to **V** is the increment of the successive radii, which will be, in succession, **r** plus **y**, **r** plus **2y**, **r** plus **3y** and **r** plus **4y**.

8° In order to get the second center, draw, through the center **C** last obtained of the extreme arc, a parallel to the radius **OI** of the central semi-circle determining the angle of 20° (from the spring line) and lay on it, in opposite direction from the arch, and from the center **C** of the extreme arc, **a** distance **CD** equal to **y**, which will give the second center **D**. That line **DC1** will also limit the extreme arc. Draw the second arc from that second center **D**. Through the second center **D** draw a parallel **ED2** to the second radius **OJ** of the central semi-circle determining the angle of 40° (from the spring line) and lay on it **DE** = the same distance **y** for the third center **E**.

Continue in the same manner for the fourth and fifth centers **F-G**, drawing through the last center obtained parallels to the radii determining the angles of 60° and 80° and laying off the distance **y** for an increase in each case. As a verification the fifth and last center **G** must fall on the center line.

The extreme radius **r** and the increment **y** are given by the formulas:

$$r = a - \frac{M(a-b)}{M+N-4b} \qquad y = \frac{b(a-b)}{M+N-4b}$$

$$\text{in which} \left\{ \begin{array}{l} M = \cos.20° + \cos 40° + \cos.60° + \cos.80° ; \\ N = \sin.20° + \sin 40° + \sin 60° + \sin.80° . \end{array} \right.$$

For any number $2n+1$ of centers,

$$\left\{ \begin{array}{l} M = \cos.\alpha + \cos.2\alpha + \cos.3\alpha + \cdots\cdots + \cos.n\alpha \\ N = \sin.\alpha + \sin.2\alpha + \sin.3\alpha + \qquad + \sin.n\alpha \end{array} \right\} \text{ in which } \alpha = \frac{180°}{2n+1}$$

An even more satisfactory arch may be obtained by uniformly increasing the angles from the center to the ends by a few degrees.

For instance, in the example above, the central angle would be β; the next $\beta + 3°$; then successively $\beta+6°$, $\beta+9°$; $\beta+12°$. We have then:

$$\beta + 2(\beta+3°+\beta+6°+\beta+9°+\beta+12°) = 180°$$

$$\text{or} \quad 9\beta + 60° = 180° \quad \text{and} \quad \beta = 13°\,20'$$

The problem would be solved as above.

Conic Sections and **Odontoidal Curves** (relating to gear-teeth). See **Geometry.**

DRAWING OF PLANE FIGURES.

Right-Angled Triangle—Given the Two Sides (Fig. 363).—Draw a right angle on which lay off the lengths given **b** and **c** and connect those points for the hypothenuse **a**.

$$\text{We have} \quad a = \sqrt{b^2+c^2}, \quad \text{Area } S = \frac{bc}{2}.$$

Right-Angled Triangle—Given One Side and the Hypothenuse (Fig. 364).—Draw a right angle, on one side of which lay off the given

side **b**; from the end of **b**, with the compasses opened to the length **a** of the hypothenuse, cut the other side of the right angle.

We have $c = \sqrt{a^2 - b^2}$.

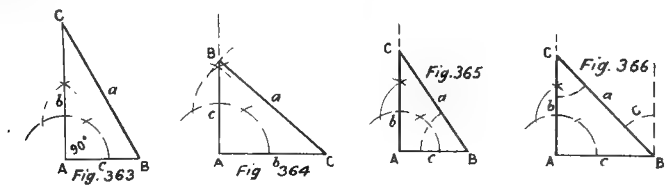
Right-Angled Triangle—Given One Side and One Angle (Fig. 365).—1° **The Adjacent Angle.**—Draw a right angle, on one side of which lay off the given side **c**, at the end of which make an angle equal to the given angle **B**.

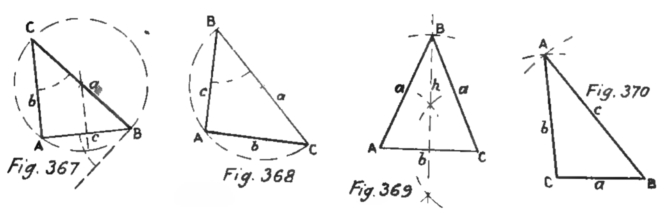
The other side $b = c\,tg.\,B$;
and the hypothenuse $a = \dfrac{c}{\cos.\,B}$

2° **The Opposite Angle—First Method (Fig. 366).**—Draw a right angle, on one side of which lay off the given side **c**; at the end of that line draw upon **c** an auxiliary perpendicular. Make at that same point and with that perpendicular an angle equal to **C** (between the two perpendiculars).

The other side $b = c\cot.\,C$,
and the hypothenuse $a = \dfrac{c}{\sin.\,C}$.

Second Method (Fig. 367).—After laying off **c** on one of the sides of the right angle, construct on it a segment capable of the angle **C**; its intersection with the other side will give the vertex **C** of the opposite angle.





Verification.—The hypothenuse must pass through the center.

Right-Angled Triangle—Given the Hypothenuse and One (Acute) Angle (Fig. 368).—Draw a semi-circle upon the hypothenuse **a**, at one end of which make an angle equal to the given angle, say, **B**; the intersection of the other side of that angle with the semi-circle will be the vertex **A** of the right angle.

$b = a \sin.\,B,\ c = a \cos.\,B.$

Isosceles Triangle—Given Base and Height (Fig. 369).—Draw base **b** to a scale and bisect it with a perpendicular, upon which lay

off the given height **h**. Connect the point obtained to the extremities of **b**.

$$tg.A = \frac{2h}{b} \;; \qquad a = \sqrt{h^2 + \frac{b^2}{4}} = \frac{b}{2\cos.A} = \frac{h}{\sin.A}$$

Other problems can be worked out by considering one-half of an isosceles triangle as a right-angled triangle.

Triangle—Given the Three Sides (Fig. 370).—Draw one side, say, **a**; from one end of it as a center and another side **b** as a radius describe an arc; from the other end of the first side as a center and the last side **c** as a radius, cut the arc first drawn; that will be the unknown vertex, which join to the extremities of **a**.

$$\sin A = \frac{2}{bc}\sqrt{p(p-a)(p-b)(p-c)} \text{ and similar formulas for the other angles;}$$

$$\text{Area } S = \sqrt{p(p-a)(p-b)(p-c)} \quad \text{in which} \quad p = \frac{a+b+c}{2}$$

Triangle—Given Two Sides and the Length of the Projection of the Third Side Upon One of the Two (Fig. 371).—Draw a line equal to one of the given sides **a**; from one end **C** of **a** describe an arc of a circle with a radius equal to the second side **b**; from the other end **B** of **a**, carry on it the length **m** of the given projection and erect at that point **D** a perpendicular **DA** upon **a**; its intersection **A** with the arc will give the third vertex.

The third side $c = a^2 + b^2 - 2a\ (a - m)$.

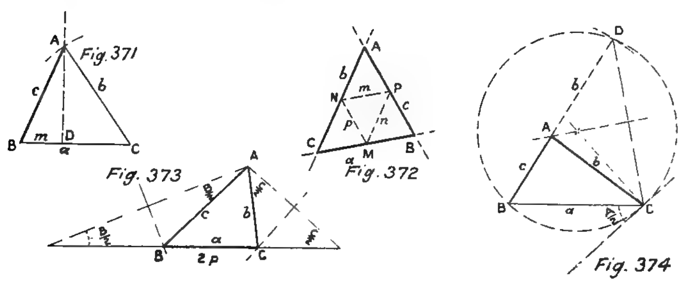


Fig. 371 Fig. 372 Fig. 373 Fig. 374

Triangle—Given the Middle Points of the Three Sides (Fig. 372).—Connect the three given points, thus forming an auxiliary triangle. with sides **m, n** and **p**, and draw through each point a parallel to the opposite side.

$$a = 2m, \; b = 2n, \; c = 2p.$$

Triangle—Given the Perimeter 2p and the Angles A, B and C (Fig. 373).—Draw a line equal to the perimeter **2p**. Make at one end of that line an angle equal to ½ **B** and at the other end an angle equal to ½ **C**; at the point where the two lines intersect draw upon each side and within the triangle angles also equal to ½ **B** and ½ **C**, thus

forming isosceles triangles on those two sides as bases. The internal sides of these isosceles triangles, with the central portion determined on the line first drawn (2p), will form the required triangle.

$$a = \frac{2p \sin. A}{\sin. A + \sin. B + \sin. C}, \text{ etc.}$$

Triangle—Given One Side a, the Opposite Angle A, and the Sum b + c of the Other Two Sides (Fig. 374).—Draw a line equal to side **a**, upon which describe a segment capable of an angle equal to ½**A**. From one end **B** of **a** as a center cut this arc with a radius equal to **b + c** and draw the line; connect the point **D** with the other end **C** of **a**; bisect the line **DC** with a perpendicular which will, by its intersection with **b + c**, determine the third vertex **A**, which connect with the other end **C** of **a**, thus closing the required triangle.

$$\sin\left(C + \frac{A}{2}\right) = \frac{(b+c) \sin. \frac{A}{2}}{a}, \text{ from which } C \text{ then } B, b \text{ and } c$$

Triangle—Given One Side a, the Opposite Angle A, and the difference b — c of the Other Two Sides (Fig. 375).—Upon the side **a** describe a segment capable of an angle equal to $90° + \frac{A}{2}$, and cut it from one end **C** of **a** as a center, with **b — c** as a radius, and draw the line **CDM**; connect that point **D** with the other end **B** of **a** and erect a perpendicular, bisecting that line **BD**, which will, by its intersection **A** with **b — c** produced, determine the third vertex **A**, which connect with the other end **B** of **a**, thus closing the required triangle **CAB**.

$$\cos.\left(B + \frac{A}{2}\right) = \frac{(b-c) \cos. \frac{A}{2}}{a}, \text{ from which } B \text{ then } C, b \text{ and } c.$$

Triangle—Given the Feet of the Three Perpendiculars (Fig. 376).—Draw an auxiliary triangle **MNP** through the three given points and bisect its angles; draw perpendiculars to the bisecting lines through the given points, and these will form the required triangle **ABC**.

The sides and angles of the interior triangle are known; therefore, the corner triangles are also known by one side and two adjacent angles.

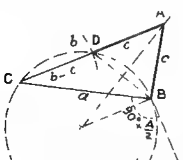


Fig. 375

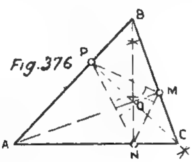


Fig. 376

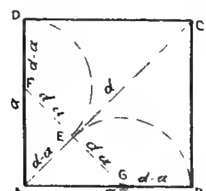


Fig 378

Fig. 379

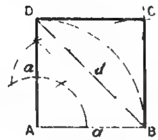


Fig. 377

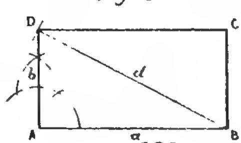


Fig. 380

Square—Given the Side a (Fig. 377).—Draw to a scale the given line **a**. Erect a perpendicular at one extremity **A** and make it equal to the given side **a**, by means of an arc described from that extremity **A** as a center and **a** as a radius. From the extremities **B-D** of that arc, and with the same radius, describe two other arcs whose intersection **C** will be the fourth vertex, and connect this to the extremities **B-D** of the other two sides.

Diagonal $d = a\sqrt{2} = 1.4142\,a$; Area $= a^2$

Square—Given the Difference Between the Diagonal d and the side a (Fig. 378).—Draw a right angle **A** and bisect it **AC**. On the bisectrix, carry **d-a** from **A** to **E** and erect a perpendicular **FG** at that point till it intersects the sides of the right angle. From these intersections **F-G** carry again **d-a** to **B** and **D** on the two sides and away from the vertex; that will determine the length of two sides **AB-AD** of the square (evidently equal), which you complete as above.

$$a = (d-a)(\sqrt{2}+1) ;$$

$$a = 2.4142\,(d-a) ;$$

$$d = 3.4142\,(d-a) .$$

Rectangle—Given Two Sides (Fig. 379).—Draw a right angle **A** and on each side of it carry one of the given sides **a** and **b**.

From the end **B** of **a** and with **b** as a radius describe an arc of a circle; from the end **C** of **b** and with **a** as a radius describe another arc, cutting the first at **D**; that will determine the fourth vertex **D**. This you connect with the extremities **B-C** of **a** and **b**.

Diagonal $d = \sqrt{a^2+b^2}$. Area ab .

Rectangle—Given One Side and the Diagonal (Fig. 380).—Like right-angled triangle, one side and the hypothenuse of which are given; then complete the figure.

$$b = \sqrt{d^2 - a^2} .$$

Parallelogram—Given Two Sides and the Angle (Fig. 381).—Make an angle equal to the given one, then proceed as for a rectangle, the two sides of which are given.

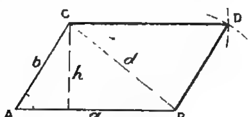


Fig. 381

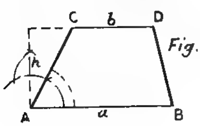


Fig. 382

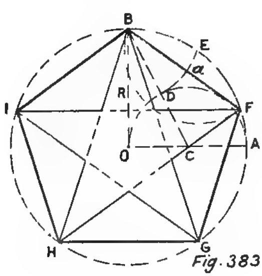


Fig. 383

The height of the parallelogram may be determined **h** = **b** sin **A**, also the projection of **b** on **a** and consequently the projection of the diagonal **d** on **a**. **d** is the hypothenuse of a determined right-angled triangle.

Trapezoid—Given an Angle, the Height and the Two Parallel Sides (Fig. 382).—Draw the given angle, on one side of which lay off the given side **a**, at any point of which draw a perpendicular, which make equal to height **h**; through its extremity draw a parallel to **a**, which make equal to the second side **b**, starting at the intersection **D** with the second side of the angle; that parallel will give the third and fourth vertices. Complete the figure.

The Diagonals may be calculated as before.

$$\text{The area } S = \frac{a+b}{2}h$$

Regular Pentagon—Radius Given (Fig. 383).—Draw the circle **O** and in it two radii **OA, OB** at right angle. Draw an auxiliary circle on one of them **OA** and connect its center with the extremity **B** of the other radius; from that extremity **B**, carry the distance **BD** from it to the auxiliary circle twice in succession **BE-EF** upon the given circle. Take the total arc of those two distances **BF** and carry it around the given circle; it will go exactly five times. Connect these points consecutively.

$$\text{The side } a = \frac{R}{2}\sqrt{2(5-\sqrt{5})};$$

$$\text{The area } S = \frac{5R}{2}\sqrt{2(5+\sqrt{5})}.$$

Stellated Pentagon.—By joining these points two and two, we obtain a regular five-pointed star or stellated pentagon.

$$\text{The side } a = \frac{R}{2}\sqrt{2(5+\sqrt{5})}.$$

Regular Decagon.—Carry the distance **BD** on the circle; it will go exactly ten times.

$$a = \frac{R}{2}(\sqrt{5}-2)$$

Regular Hexagon—Radius Given (Fig. 384).—Draw the circle **O** with given radius **OA** and carry said radius on it, from a point **A** (generally given on it) and in succession; the radius will go exactly six times. Connect the points consecutively.

$$a = R \quad ; \qquad S = \frac{3R^2}{2}\sqrt{3}$$

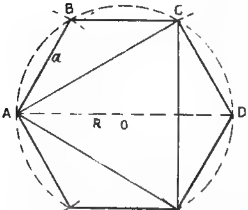


Fig. 384

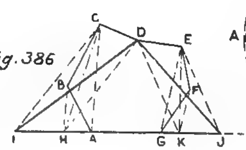


Fig. 386

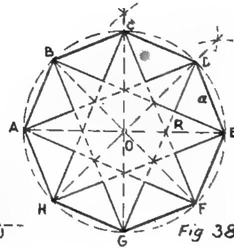


Fig 385

By joining the points two and two, the inscribed equilateral triangle is obtained, the side of which is

$$a = R\sqrt{3} \; ; \quad \text{and the area} \quad S = \frac{3R^2}{4}\sqrt{3}$$

Regular Octagon—Radius Given (Fig. 385).—Draw the circle with given radius and two diameters at right angles; bisect the right angles and connect consecutively the points thus determined on the circle by the eight diameters.

$$a = R\sqrt{2-\sqrt{2}} \; ; \qquad S = 2R^2\sqrt{2}$$

By joining the points two and two, the inscribed square is obtained (see above).

By joining points three and three, the stellated octagon is obtained, the side of which

$$a = R\sqrt{2+\sqrt{2}} \, .$$

Transform a Polygon Into an Equivalent Triangle (Fig. 386).—Isolate a triangle **ABC** in the polygon by means of a diagonal **AC**. Produce one of the sides **AG** adjacent to the diagonal and not isolated; draw through the isolated vertex **B** a parallel **BH** to the diagonal **AC** till it intersects the effected prolongation at **H** and connect that point with the other end **C** of the diagonal. The two triangles having the diagonal as a common side are equivalent; therefore, the new polygon is equivalent to the first and contains one side less. Continue with the new polygon as with the first until a triangle **IDJ** is obtained.

Drawing a Polygon Similar to a Given One; or, Enlarging or Reducing a Given Figure (Fig. 387).—If the dimensions of the given figure are given in numbers, draw to a reduced (or enlarged) scale (in the given ratio) one side of the required polygon and make an angle, say, at the left end, equal to its homologous in the given figure; lay off the second side with the reduced (or enlarged) scale, then the second angle and so continue till the figure is closed.

It is generally convenient to draw the first side parallel to the one to be reduced (or enlarged); the angles are then made equal by parallel lines.

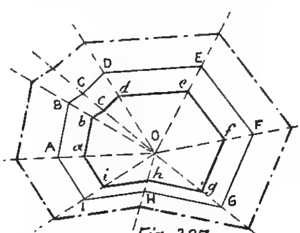


Fig. 387

Or, again, a point **O** is marked within the figure and at the center of it (approximately), from which radial lines **OA-OB** are drawn to all the vertices; these are scaled with the scale of the drawing and carried with the scale of reproduction; or the proportional dividers may be

set to the required proportion when one side is used to take the length and the other to lay them off. The points are then connected consecutively. In case of an enlargement the radial lines are produced beyond the given figure.

PROJECTIONS.

Descriptive Geometry.—Representation of geometrical figures (lines, surfaces and solids) by means of their **orthographic projections.**

Hypotheses—Horizontal Plane of Projections (Fig. 388).—We suppose an indefinite horizontal plane **H** drawn through space. It will divide space into two regions: one above, or superior, which we will represent by **S**; the other below, or inferior, **I**.

Vertical Plane of Projections (Fig. 389).—We suppose a second indefinite plane perpendicular to the first; it will be vertical **V** and, considered alone, will divide space into two regions: one in front, or anterior, **A**; the other beyond, or posterior, **P**.

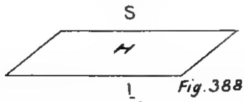


Fig. 388

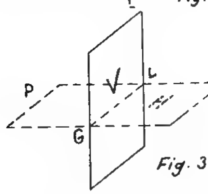


Fig. 389

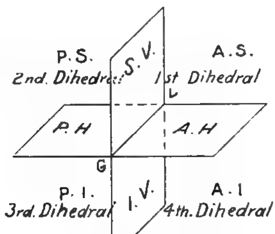


Fig. 390

Ground Line.—The intersection of the two planes of projections is a straight line and is called the ground line, marked **GL** or **xy**.

Portions of the Planes on Either Side of the Ground Line (Fig. 390).—The portions of each plane of projection separated by the ground line get their names from the region into which they extend with regard to the other plane; thus we have:

The **anterior-horizontal** plane, **A.H.**

The **posterior-horizontal** plane, **P.H.**

The **superior-vertical** plane, **S.V.**

The **inferior-vertical** plane, **I.V.**

Portions of Space Determined by the Planes of Projections.—The four regions or dihedrals into which space is divided by the two planes of projections receive their names from the portions of the two planes which form them, naming the horizontal plane first. Thus the first dihedral will be that formed by the **A.H.** plane and the **S.V.** plane; taking the two first letters of these abbreviations, we are led to call that region **A.S.** or anterior-superior region. We then have:

1st dihedral or region **A.S., anterior-superior**;

2d dihedral or region **P.S., posterior-superior.**

3d dihedral or region **P.I., posterior-inferior.**

4th dihedral or region **A.I., anterior-inferior.**

Where Objects Generally Are Supposed to Be.—Objects to be projected are generally supposed to be in the **A.S.** region, or first dihedral. But they sometimes extend into one of the others.

A **Solid** is a finite portion of indefinite space, and is defined by its **Surface**, which is the limit which separates it from surrounding space.

Some solids are considered as being generated by certain surfaces moving in accordance with certain laws.

Surfaces are bounded by **lines** which are the intersections of adjacent surfaces in a solid. They are also sometimes considered as generated by the motions of lines.

Lines are determined by **points**, which are insections of lines. They are also sometimes considered as generated by the motions of **points.**

Therefore, points determine lines, lines determine surfaces and surfaces determine solids.

Projections of a Point (Fig. 391)—1° Horizontal projection.—The foot **m** of the perpendicular **Mm** let fall from the point **M** in space to the horizontal plane.

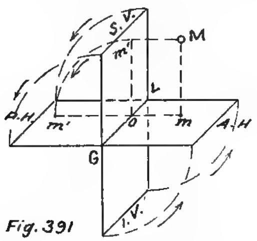


Fig. 391

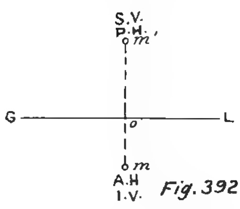


Fig. 392

2° **Vertical Projection.**—The foot **m'** of the perpendicular **Mm'** let fall from point **M** in space to the vertical plane.

These two perpendiculars are called the **projecting lines** and the plane which they determine is called the **projecting plane.**

The projecting plane, containing a perpendicular to each plane of projection is itself perpendicular to each plane of projection and consequently to their intersection, the **G.L.**

The intersections **mo, m'o** of each plane of projection with a projecting plane **Mo** are perpendicular to the **G.L.**

Those intersections **mo-m'o** are equal to the projecting lines **Mm-Mm'** as opposite sides of a rectangle. Therefore, we say that

The distance from a point to the horizontal plane of projection is measured by the distance of its vertical projection from the ground line, and

The distance from a point to the vertical plane of projection is measured by the distance of its horizontal projection from the ground line.

Plan.—The horizontal projection of a point, line, surface or solid.

Elevation.—The vertical projection of a point, line, surface or solid.

Hypothetical Revolution of the Vertical Plane.—Suppose a poinf **M** in the **A.S.** region of space (**Fig. 391**); **m** its horizontal and **m'** its vertical, projections. Let **mo** be the intersection of the projecting plane, with the **A.H.P.** and **m'o** its intersection with the **S.V.P.** Let now the **V.P.** revolve about the ground line until its superior portion comes in contact with the **P.H.** plane; then will its inferior portion come in contact at the same time with the **A.H.** plane. At that moment there will be drawn upon a single plane surface (the **H.P.**) an indefinite straight line, which is the ground line **G.L.** (**Fig. 392**) and a perpendicular to it **mom'**, giving the distances from the point in space: **mo** to the **V.P.** and **m'o** to the **H.P.**, and this figure will be the **descriptive** representation of the point **M** in space by its plan **m** and its elevation **m'**.

The distance **m'o** of the vertical projection from the ground line is the distance of the point in space **M** from the horizontal plane of projection. The distance **mo** of the horizontal projection from the ground line is the distance of the point in space **M** from the vertical plane of projection.

Positions of a Point in Space.—With regard to the planes of projection a point may have one of nine positions in space (**Fig. 393**). It may be situated:

1° **a-a'** in the **A-S.** region;

2° **b-b'** in the **P-S.** region;

3° **c-c'** in the **P-I.** region;

4° **d-d'** in the **A-I.** region;

5° **e-e'** on the **A.H.** plane;

6° **f-f'** on the **S.V.** plane;

7° **g-g'** on the **P.H.** plane;

8° **h-h'** on the **I.V.** plane;

9° **i-i'** on the **G.L.**

The figure shows these several positions.

Projections of a Str. Line Are Str. Lines (Fig. 394).—All the projecting lines of the points of a given line will be in the projecting plane and meet the plane of projection at their common intersection,

which is a str. line, and is the projection of the given line. Therefore:

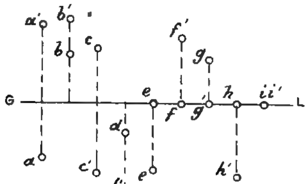


Fig. 393

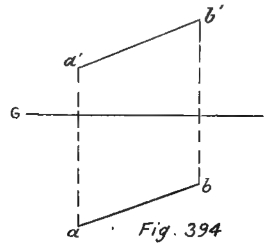


Fig. 394

The horizontal projections **a-b** of 2 points **A-B** of a line being given or obtained, connect them with a str. line for the horiz. projection **ab** of the line.

The vertical projections **a′-b′** of the same points being given or obtained, connect them with a str. line for the vert. projection **a′b′** of the line.

When the projections of any two points of a line **AB** are known, the line is absolutely determined by its projections **ab-a′b′**, as it is determined in space.

Principal Positions of a Str. Line (Fig. 395).—A line may be:

1° Parallel to the **H.P.**

2° Parallel to the **V.P.**

3° Parallel to both the **H.P.** and **V.P.**, or parallel to the **G.L.**

4° Perpendicular to the **H.P.**

5° Perpendicular to the **V.P.**

6° Perpend. to the **G.L.**

7° In a profile plane (one perp. to both **H.P.** and **V.P.**, like a projecting plane).

8° Oblique to both planes and meeting the **G.L.**

9° In any position in space.

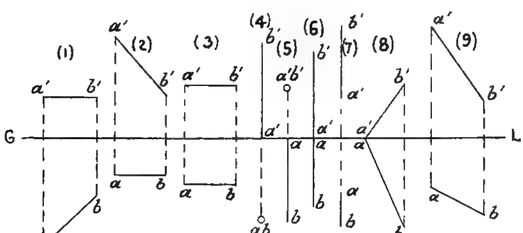


Fig. 395

When a Line is parallel to the H.P. (Fig. 395), its V. projection is parallel to G.L. (1).

This results from the fact that all the points of the line in space being equally distant from the **H.P.**, the distances of their V. projections from the **G.L.** are equal.

When a Line is parallel to the V.P., its H. Projection is parallel to the G. L. (2)

When one Projection of a Line is parallel to the G.L. the Line in Space is parallel to the other Plane.

When a Line in Space is parallel to both H. and V. Planes, both its Projections are parallel to the G. L. and conversely. They are therefore parallel to each other. **(3)**

When a Line is perpendicular to the H.P. its H. Projection is a Point and its V. Projection is perpend. to the G. L. (4) In fact, all the projecting lines to the **H** plane are in the same vertical which is the line itself.

When a Line is perpend. to the V. P., its V. Projection is a Point and its H. Proj. is perpend. to the G.L. (5)

When a Line is situated in a Profile Plane (or projecting plane), both its Projections are perpend. to the Ground Line. (6) (7) In position **(6)** the line in space meets the **G.L.** In position **(8)** the line in space is oblique to both planes and meets the **G.L.** Position **(9)** shows the projections of a line entirely in the first dihedral.

Traces of a Line.—The intersections of a line with the planes of projection.

Horizontal Trace.—Intersection of a line with the **H.P.**

Vertical Trace.—Intersection of a line with the **V.P.**

When a Line Has One Trace Only.—When it is parallel to one of the planes of projection; then it has no trace in that plane.

When the Two Traces Coincide.—When the line in space meets the **G.L.**

To Determine the Traces of a Line (Fig 396).—The horizontal trace, for instance, belongs to the **H.** projection of the line, and being in the **H.** plane, its **V.** projection is on the ground line; hence the rule:

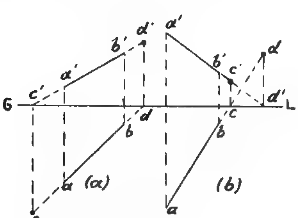


Fig. 396

Produce the **V** projection to the **G.L.** and erect on this a perp. to the **H.** projection for the **H.** trace.

Produce the **H.** projection to the **G.L.** and erect on this a perp. to the **V.** projection for the **V.** trace.

In Fig. (**b**) the **H.** trace is in the **P.H.** plane.

Find the Projections of a Line, the Traces of Which Are Given.—Drop from each trace a perp. to the **G.L.** and connect with other trace.

Traces of a Plane.—When a plane is oblique to both planes of projection, it will meet or intersect them if sufficiently produced.

Horizontal Trace of a Plane.—Intersection of a plane with the **H.** plane.

Vertical Trace of a Plane.—Intersection of a plane with the **V.** plane.

These traces are straight lines.

The Traces of a Plane Meet at the Same Point, because the intersection of three planes is a point.

How Planes Are Indicated.—By a capital on the **H.** trace, the same capital accented on the **V.** trace and a Greek letter at their meeting point on the **G.L.** (Fig. 397.)

When a Plane Has One Trace Only.—When it is parallel to one plane of projection it has no trace in that plane. The other trace is then parallel to the **G.L.**

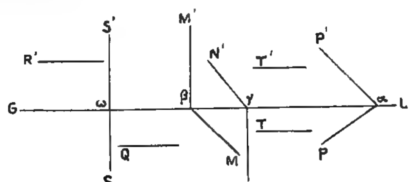


Fig. 397

When a Trace Is Perpendicular to the G. L.—When a plane is perpend. to one of the planes of projection, its trace in the other plane of projection is perpendicular to the **G.L.**

When Both Traces Are a Single Perp. to the G.L.—When the plane is a profile plane.

When the Two Traces Are Parallel to the G.L.—When the plane in space is itself parallel to the G.L.

Principal Positions of a Plane.—A plane may be:

1° Parallel to the *H.P.* as *R'*; 2° Parallel to the *V.P.* as *Q.*; 3° Parallel to the *G.L.* as *TT'*; 4° Perpend. to the *H.P.* as *MβM'*; 5° Perpend. to the *V.P.* as *NγN'*; 6° Perpend. to the *G.L.* as *SωS'*; 7° Any position in space as *PαP'*.

Conditions Which Determine a Plane.—1° Three Points not in a line. 2° Two parallel lines contained in it. 3° Two intersecting lines contained in it. 4° A line and a direction. 5° A point and a direction (it might be required to be parallel or perpend. to a given plane).

The planes mostly used are profile planes and planes perpendicular to either plane or projection.

Revolution of a Plane Perp. to the H.P. and of a Point in It (Fig. 398)—1° **Revolution around its V. Trace.**—Any point **A** in it will describe in space an arc of a circle parallel to the **H.** pl. and projected in it full size at **aa₁.**

with center α ;

the **V.** projection of that arc is **a′A** parallel to the **G.L.** The point **A** is the point in space brought in the **V.** plane and determined by the arc **aa₁**, the horizontal **a′A** and the perpend. **a₁A** to the **G.L.**

2° **Revolution Around Its H. Trace.**—A point **B** in space will move in an arc of a circle, the plane of which

is perp. to the H trace βQ and is therefore projected on bB perp. to βQ; and its distance from the center b of that arc is $= b_1 b' = \beta b'_1 = \beta b'_2 = b'_2 B$; this last a paral. to βQ.

Revolution of a Plane Perp. to the V.P. and of a Point in It (Fig. 399).—Same cases as above and similar construction, changing **V.** into **H.** and **H.** into **V.**

Revolution of Any Plane (Fig. 400).

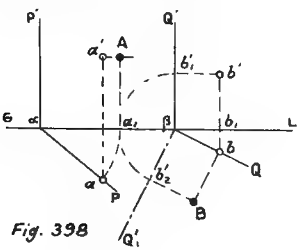


Fig. 398

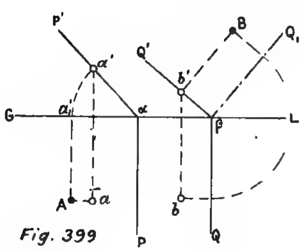


Fig. 399

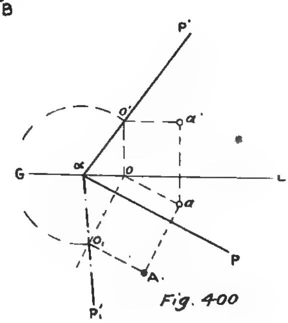


Fig. 400

Let $P\alpha P'$ be a plane and aa' a point given in it, which fact may be ascertained by drawing through A a horizontal $ao-a'o'$ ($a'o'$ parallel to $G\cdot L$. and ao parallel to $P\alpha$) in plane $P\alpha P'$; then must oo' be perp. to $G.L$. When this is so the plane $P\alpha P'$ and the point A may be brought into the $H.P.$ by drawing oo_1 perp. to $P\alpha$ and cutting that perp. from α as a center with a radius $= \alpha o'$. The horizontal in space becomes o_1A in the $H.P.$ and aA is perp. to $P\alpha$.

Traces of a Line in a Profile Plane (Fig. 401).—Revolve the plane and the two points about its H. trace; ascertain the traces of the line and bring them back into first position.

Distance Between Two Points or Length of Line Joining Them (Fig. 402).—The simplest method is to imagine the line revolving about the vertical of one of the points, say, **aa'**, until it becomes parallel to the **V.** plane **ab_1**; the **V.** projection of arc **bb_1** (whose center is **a**) is the **H.** line **b'B**; **ab-a'b'** becomes **ab_1-a'B** and **a'B** equals the line in space or is the distance between **aa'** and **bb'**.

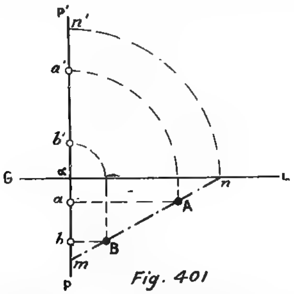


Fig. 401

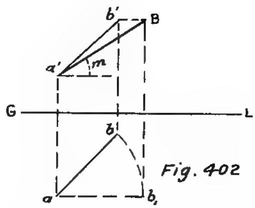


Fig. 402

In that fig. **m** is the angle which the line in space makes with the **H.P.** A like construction determined by a revolution of the figure about the horizontal **a'** would also give the required distance of the points and give, besides, the angle which the line makes with the **V.P.**

Side View of a Point (Fig. 403).—The projection of a given point **aa'** upon a certain profile plane $P\alpha P'$

and revolved onto the **V.P.** as at **a''**, is the side view of the point **A**, of which **a** is the plan and **a'** the elevation.

The Projections of a Perpendicular to a Plane Are Perpendicular to the Traces of That Plane (Fig. 404).—The projecting planes of the line are, respectively, perpendicular to the **H.P.** and the

given plane $P\alpha P'$, the other to the $V.P.$ and the plane $P\alpha P'$; they are therefore perp. to their intersections the $H.$ and the $V.$ traces of the given plane $P\alpha P'$.

The point of intersection of the perp. and the plane is obtained by imagining, through the given line, and auxiliary plane $Q\beta Q'$ perp. to the $V.P.$ and determining its intersection $cd-c'd'$ with $P\alpha P'$. That intersection will give the point $b-b'$ where the line meets the plane.

The true distance **a'B** is obtained as explained above.

Changing the Planes of Projection (Fig. 405).—A different V. plane will intersect the **H.P.** along a different ground line **G_1L_1**. The point **A** remaining at the same distance from the **H.P.**, the distance **a'o** from its vertical projection to the **G.L.** will still be the same; it will also be situated upon the perpendicular **aa''** to the new **G_1L_1**. Hence the construction; draw a perpend. **aa''** to the new **G_1L_1** and take **$o_2a'' = o_1a_2 = o_1a_1 = oa'$**.

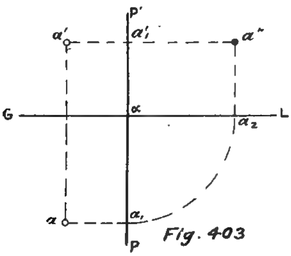


Fig. 403

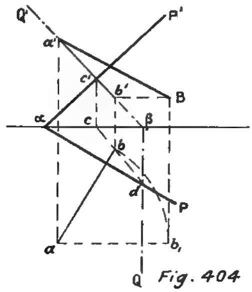


Fig. 404

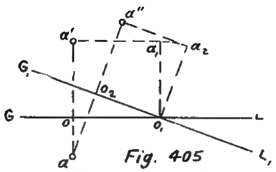


Fig. 405

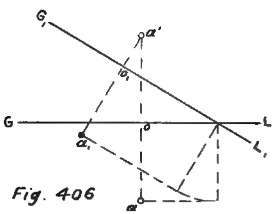


Fig. 406

A similar construction would give the new **H.** projection of a point if the **H.P.** were changed (**Fig. 406**).

Both planes might be changed, which would require the changing **1°** of the **V.P.**, **2°** of the **H.P.**

Projections of Surfaces.—Any polygon may be decomposed into triangles, which may be treated separately.

Any plane surface terminated by an irregular contour may also be decomposed into triangles, some sides of which will be curved. The vertices situated on the curved contour may then be connected with a continuous curve.

Surfaces are often situated in profile planes, or in planes perpendicular to the **H.** or the **V.** planes of projections.

Exact Size of a Triangle Situated in a Profile Plane (Fig. 407).—Let **abc — a'b'c'** be a triangle given by its projections in a

profile plane $P\alpha P'$. Revolve said plane around its H trace $P\alpha$.

Each vertex will in space describe a quadrant parallel to the **V.P.** and

projected full size on it. Hence the construction shown in the fig. **ABC** is the required triangle full scale size.

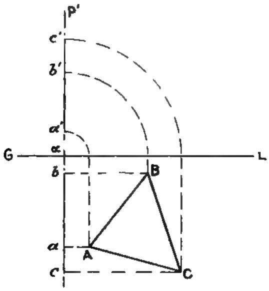


Fig. 407

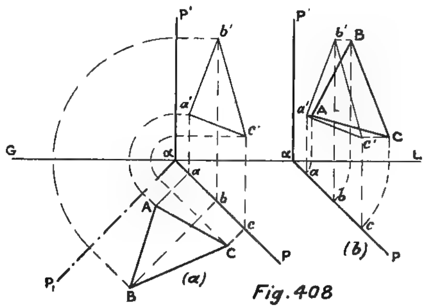


Fig. 408

Exact Size of a Triangle Situated in a Plane Perp. to the H.P. (Fig. 408.)

Revolve plane PαP' about

its **H.** trace and onto the **H.P.** with the vertices of the given triangle **abc — a'b'c'**, thus obtaining **ABC** as the real size of the triangle.

By revolving PαP' about its V. trace,

We obtain the same result in **ABC.**

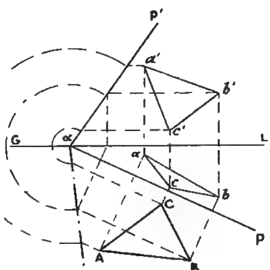


Fig. 409

Exact Size of a Triangle in an Oblique Plane (Fig. 409).—The several vertices must be in the given plane and that fact is ascertained as already explained.

The plane PαP' and the three points are then brought in the H.P. at ABC by means of horizontals in PαP' drawn through each vertex.

To draw the projections of a figure to be situated in a plane and

to comply with certain conditions, bring the plane PαP' which is to contain

it in the **H.P.** and in that position, draw the figure in it that will answer the question and revolve back the plane and the figure to the original position when the required projections will be obtained.

(by means of horizontals of $P\alpha P'$ through each vertex).

Theorem—Any Projection of a Circle on a Plane Is an Ellipse.— This was proved by Courcelle. The major axis is the projection of that diameter which is parallel to the plane of projection (largest projection of any diam. exact size) and the minor axis is the projection of that diameter which is perpendicular to it (smallest projection of any diam.).

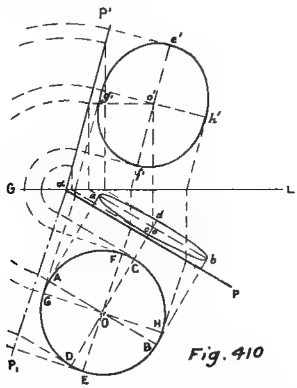


Fig. 410

Projections of a Circle Situated in a Plane—Given Center and Radius (Fig. 410).—

Ascertain that the given center oo' is in the given plane $P\alpha P'$ (by a horizontal through oo' in plane $P\alpha P'$). Revolve $P\alpha P'$ in the $H.P.$ and

oo' at **O.** from which point draw the circle with the given radius. A number of points of that circle might be brought back

by a counter-revolution of $P\alpha P'$

and connected by a continuous curve; but it is better to determine the axes of the ellipses which will be the projections of the circle, because these axes will define the ellipse geometrically.

1° A diameter **AB,** parallel to the **H.P.** will be the major axis **ab** of the ellipse horizontal projection, and a diameter **CD,** perpendicular to it will be the minor axis **cd.**

2° A diameter **EF** parallel to the **V.P.** will be the major axis **e'f'** of the ellipse vertical projection, and a diameter **GH,** perpendicular to it, will be the minor axis **g'h'.**

3° The vertical projection of **C** and **D** would be the highest and lowest points, and the horizontals through these points would be tangent to the ellipse vertical projection.

Projections of a Cube Resting on the H.P. with a Face Parallel to the V. Plane (Fig. 411).—(There must of necessity be two faces so parallel.) The base of the cube is a square **abcd**; the arrises terminating at these points and joining the opposite face are verticals and their **H.** projections are the points **a, b, c** and **d** themselves, so that **abcd** is also the **H.** projection of the upper face, **efgh**; hence two letters at each angle. The base being on the **H.P.**, the **V.** projections of **a, b, c** and **d** will be on the ground line at **a′**, **b′**, **c′** and **d′**.

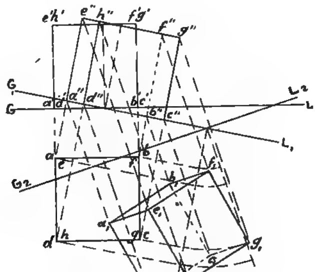


Fig. 411

ac, bf, ch and dg being perpend. to the **H.P.**, their **V.** projections **a′e′**, **b′f′**, **d′g′** and **c′h′** will be perpend. to **G.L.** and equal in size to **ab**, for instance. These projections will therefore be two equal squares. In order to obtain another view, we may change the **V** plane of projection along **G_1L_1** and determine a new **V** projection **a″b″c″d″e″f″g″h″**. A change of **H.P.** along **G_2L_2** would give a new **H.P.** **$a_1b_1c_1d_1e_1f_1g_1h_1$**.

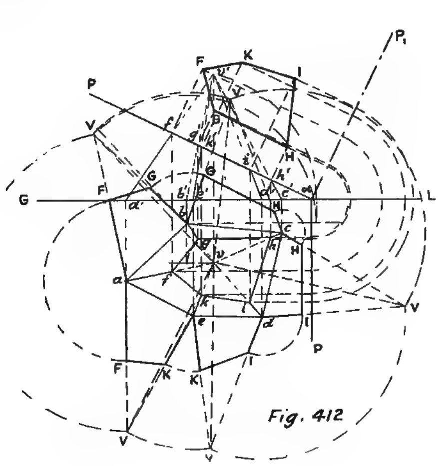


Fig. 412

Section of a Pyramid Resting on the H.P. by a Plane Perpend. to the V. P. (Fig. 412).—The pyramid is irregular; its vertex is **vv′** and its base **abcde—a′b′c′d′e′**;

the secant plane $P\alpha P'$ gives directly

the **V** projection **f′g′h′i′k′** (because it is perpend. to the **V. P.**) which in turn determines the **H** projection **fghik** of the section. We suppose the top portion of the pyramid removed; hence the dotted lines.

Envelope of the Frustrum.—Revolve each face around its **H** trace, upon the **H** plane. To that end, taking **vab** as an example, draw the perpend. **vV** upon **ab** and take **oV**=the height of triangle **abV** (this is the hypothenuse of a right-angled triangle, the sides of which are the vertical **v**, given at **bv′**, and the **H** projection **vo**); connect **Va** and **Vb** upon which perpendicular drawn from **f** and **g** to **ab** will give the points **F** and **G.** This face being thus obtained, the others are simply determined by drawing perpendiculars from **v** to the sides of the base, **bc, cd, de, ea** and intersecting **vV** from **a** as a center and **aV** as a radius, then drawing **Va, Ve**; an arc from **a** with **aF** as a radius will give on **aV** the point **F** of the section, and perpendicular **kK** to **ae** will give **K**; and so continue to the last face. The exact section **FGHIK** is obtained

by revolving $P\alpha P'$ about its V trace $\alpha P'$ and onto the $V.\ P.$

Parabolic Section of a Right Cone Resting on the H.P. by a Plane Perpend. to the V. P. (Fig. 413).—The section of a cone by a plane is an ellipse when the plane meets all the elements (successive positions of the generatrix) of the cone on the same side of the vertex, which condition is fulfilled when the angle which it makes with the axis of the cone is less than the angle made with the same axis by the generatrix.

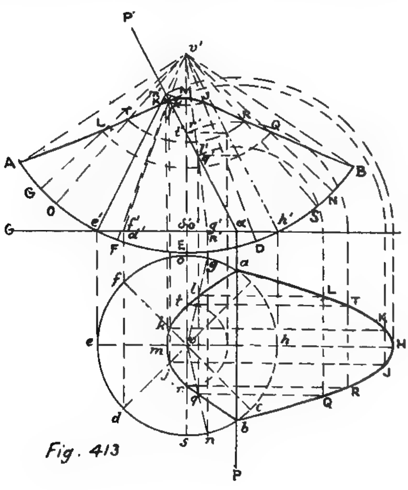


Fig. 413

The section is a parabola when the plane meets all but one of the elements; it is parallel to that element and the angle it makes with the axis equals that made with the same axis by the generatrix.

The section is a hyperbola when the plane meets all the elements, some on one side and the rest on the other side of the vertex. (The elements, supposed indefinite in length describe two equal cones, called nappes, opposite through the vertex) or when the plane intersects the two nappes; that condition is fulfilled when the angle which the secant plane makes with the axis of the cone is greater than the angle made with the same axis by the generatrix.

In the present case $P\alpha P'$ is parallel to the element $vh - v'h'$.

The best method is to draw elements dividing equally the circumference of the base; their **H** projections will be radii and their **V** projections are easily determined by the vertex v′ and the **V** projections of the ends of the radii, which are all on the **G.L.** The intersections of the **V** projections

of these elements with $P\alpha P'$ are on $P'\alpha$

and from these the **H** projections are obtained on the corresponding radii. By connecting the **H** projections of the points so obtained with a continuous curve, a hyperbola will result. The exact size of the section is then brought into the **H.P.**

by revolving $P\alpha P'$ about its H. trace $P\alpha$;

the several points will move on parallels to **GL** and come at distances from **ab**

equal to distances of their V. projections from α.

Envelope of the Frustrum.—The envelope of the whole cone is a sector with an element as a radius and of a length of arc $= 2\pi R$.

Now the same arc, in the sector, will have for value $\frac{\pi g n}{180}$; hence the equation $2\pi R = \frac{\pi g n}{180}$; from which $n = 360^\circ \times \frac{g}{R}$ which is the measure of the angle of the sector. These considerations lead to the following method.

Draw from v' as a center and with g as a radius an arc of a circle limited on either side by an angle $= 180^\circ \frac{g}{R}$ made with the vertical $v'v_1$.

Divide this arc into as many equal parts as the circle on the **H** plane was divided and connect the points of division to **v′**; these lines will be the several elements in full size. Through the **V** projection of the points of intersection of the different elements draw parallels to **G.L.** to the extreme element **v′h′**, and through each of the points thus determined draw an arc of a circle from center **v′** stopping at

the corresponding element of the envelope; then connect these last points with a continuous line.

In the figure, the cone is supposed to have been opened along elements **vh-v'h'** and spread out on either side of **v'$\mathbf{v_1}$** on which the opposite element **ve-v'e'** is laid.

Plane Drawn Tangent to a Cone Through a Point in Its Surface (Fig. 414).—**v-v'** is the given cone and **m'** the **V** projection of the given point. Any plane tangent to **v-v'** at **m'** will have the element **v'd'** as element of contact. The **H** projection of **v'd'** is **vd** and also **v$\mathbf{d_1}$**; hence two solutions.

The plane tangent to *vd-v'd'* will have its *H.* trace *Pα* tangent to the circle base at *d*.

That plane will also contain the horizontal **md-m'd'**, the vertical trace **d'** of which will determine.

the *V.* trace *αP'* of the required tangent plane *PαP'*. A second solution *QβQ'* results from the point *m,m'* and is determined by the tangent *Qβ*

to the circle base and the **V** trace **c'** of the horizontal **$\mathbf{m_1}$c-m'c'** drawn through **$\mathbf{m_1}$m'**.

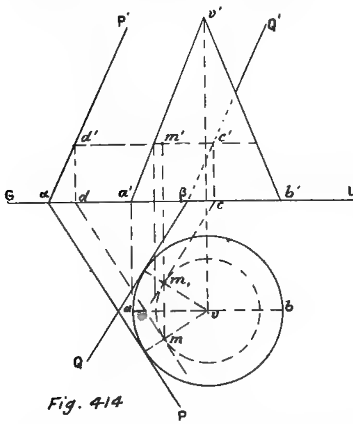


Fig. 414

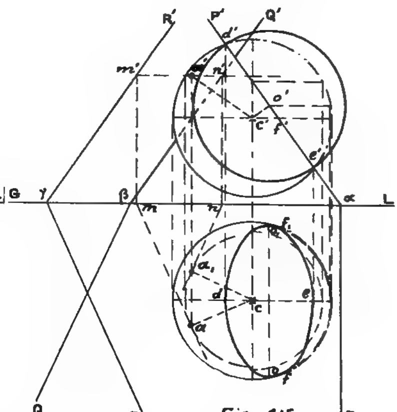


Fig. 415

Point on a Sphere-Section of a Sphere by a Plane Perpendicular to the V.P.—Plane Tangent to the Sphere Through a Point of it (Fig. 415).—**c-c'** is the given sphere and **a'** the **V** projection of a point **A** on its surface. To locate the **H** projection of **A** notice that it must be situated, first, on the perpendicular **a'a** to **GL**; second, on the small circle **cb-$\mathbf{c_1}$'b'**; there are two such projections **a** and **$\mathbf{a_1}$**.

PαP' is a given sectional plane perp. to the *V. P.*

The section in the sphere is a small circle shown in full at **o'**, the diameter of which is **d'e'**. To locate its horizontal projection, which is an ellipse, notice that its major axis is the horizontal diam-

eter $\mathbf{oo_1}$ and its minor **de.** The points $\mathbf{f\text{-}f_1}$ common to the ellipse and circle are the **H** projections of **f′** on the great circle **c-c′**

and on the secant plane $P\alpha P'$.

A plane drawn through **aa′** or $\mathbf{a_1a'}$ will have to be perpendicular to radius **ca-c′a′** or $\mathbf{ca_1\text{-}c'a'}$ and its traces will be perpendicular to the corresponding projections of the radius. The plane will, besides, contain the horizontal **am-a′m′** or $\mathbf{a_1n\text{-}a'n'}$

and its *V.* trace $\gamma R'$ or $\beta Q'$ will pass through the *V.* trace of that horizontal; finally draw γR or βQ perpend. to ac or a_1c
The required plane is then $R\gamma R'$ or $Q\beta Q'$

Note—The problems treated herein are a few of the numerous questions which the Draughtsman may have to solve; but they exemplify methods which are general and therefore adapted to most cases.

ARCHITECTURE.

Orders of Architecture.—Orders of architecture are types of ancient buildings determined by fixed rules deduced from the critical examination of monuments of antiquity.

Principal Orders of Architecture.	Tuscan ;	distinguished by	*simplicity ;*
	Doric ,	"	*triglyphs in the frieze ,*
	Ionic ;	"	*large volutes in capital ;*
	Corinthian ;	"	*acanthus leaves and small volutes in capital ;*
	Composite or Roman ;	"	*Ionic volutes and Corinthian leaves.*

Principal Parts of All Orders.—Pedestal, column and entablature.

Each of These Parts Is Itself Composed of Three Members, as Indicated in This Table:

Order of Architecture.	*Entablature*	*Cornice ; Frieze ; Architrave ;*
	Column	*Capital , Shaft ; Base ;*
	Pedestal	*Cornice or Cap ; Dado or Die ; Base*

Unit of Measurement.—The unit of measurement is called the **modulus (M)** and is equal to the lower radius of the shaft. Its subdivisions are called **minutes (m)**, and the modulus contains 12 minutes in the Tuscan and Doric Orders, and 18 minutes in the others.

Proportions of the Parts—How Modulus Is Determined (Fig. 416).—The total height being given, divide it into 19 equal parts. The pedestal contaings 4 of these parts, the column 12 and the entablature 3.

Divide the length found for the column into

14	parts for the	*Tuscan*	order; the length of the column is then	7 diameters;
16	" " "	*Doric*	" " " " "	8 "
18	" " "	*Ionic*	" " " " "	9 "
20	" " "	*Corinthian & Roman*	orders; " " " "	10 "

Total Height of the Orders.—

Tuscan $\frac{14}{3} + 14 + \frac{14}{4} = 22^M\ 2^m$;

Doric $\frac{16}{3} + 16 + \frac{16}{4} = 25^M\ 4^m$;

Ionic $\frac{18}{3} + 18 + \frac{18}{4} = 28^M\ 9^m$;

Corinthian & Composite $\frac{20}{3} + 20 + \frac{20}{4} = 31^M\ 12^m$

Principal Moldings (Fig. 417).

First—**Flat Moldings:**

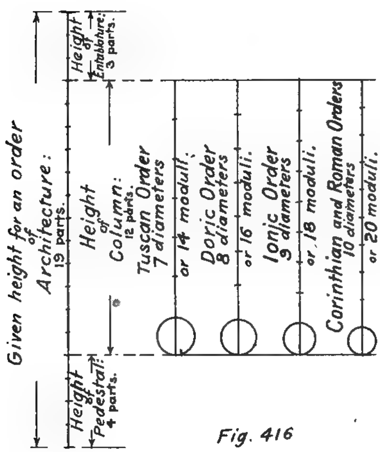


Fig. 416

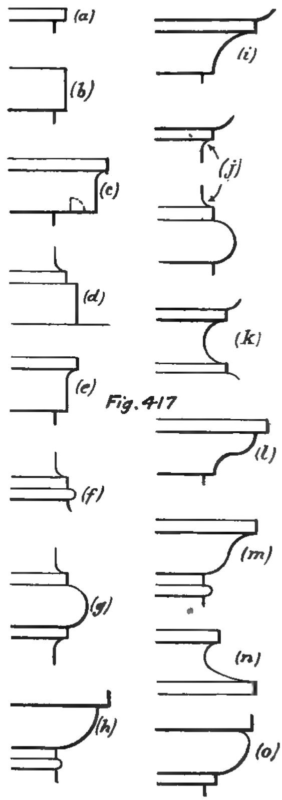


Fig. 417

List or Fillet (a).—Thin; its projection equals its height.

Band (b).—Wide, flat; its projection same as the fillet.

Larmier or Corona (c).—Drip stone with great projection at the brow of a cornice; a channel, called **weeper**, is provided on the under side; the edge is the drip.

Plinth (d).—A slab or block at the lower end of pedestal and base; projects like the fillet.

Tailloir or Abacus (e).—Top band in the Tuscan Cap; projects like the fillet.

Second.—**Circular Moldings:**

Bead (f).—Thin, semi-circular molding about the thickness of a fillet.

Torus (g).—Semi-circular molding much larger than the bead.

Quarter-Round, Echinus or Ovolo (h).—Convex molding of ¼ circle.

Cavetto or Cove (i).—Concave molding of ¼ circle.

Apophyge or Conge (j).—Concave curve on the top or at the bottom of a column or shaft.

Gorge (k).—A concave semi-circular molding.

Cyma or Cyma-Recta (1).—A molding formed with two ¼ circles tangent to each other; convex above. concave below.

Cyma-Reversa (m).—A molding formed with two ¼ circles tangent to each other; concave above, convex below.

Scotia or Trochilus (n).—A concave molding with unequal curves. One end projects more than the other.

The circular moldings, as just explained, are the Roman moldings; they can be drawn with arcs of circles.

The corresponding Greek moldings are more like conic sections and therefore more elegant.

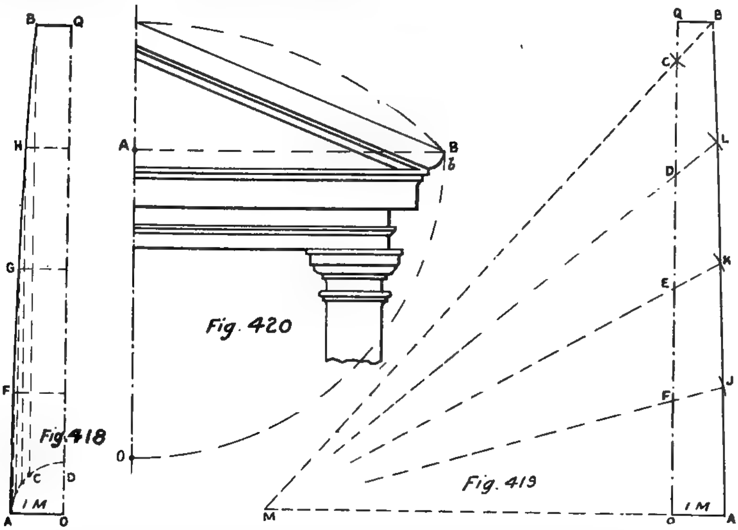
For instance, the **Echinus (o)** in the cap of the **Poestum,** a **variety** of **Doric,** is a portion of an ellipse.

Entasis.—A column is generally straight for one-third of its height; from that point the diameter is slightly reduced to the top, where it is about 4 minutes less. But that reduction is not regular and the profile is not a straight line but a convex curve called **entasis.**

How to Determine the Entasis.—

First Method (Fig. 418).—At the point **O** = **1/3** the height of the column describe a ¼ circle with radius = 1 modulus. Lay off the width **QB** and take **DC** = **QB.** Divide the remaining arc **AC** into a number of equal parts and the distance **OQ** into the same number of equal parts. Parallels drawn through these points will give the points **F, G, H,** etc., of the profile.

Second Method (Fig. 419).—Draw the horizontal **OM** at one-third the height. Lay off **QB** as given or chosen and describe arc **C** from center **B** with radius $=$ **OA** $=$ **1 M.** Join **BC,** which will give **M.** Divide **CO** into any number of equal parts and draw arcs **J, K, L,** from centers **F, E, D,** with radii $=$ **1 M.**



Determination of the Apex of a Fronton (Fig. 420).—After determining the nose of the quarter-round **b,** describe arc **BO** from center **A** with radius **AB,** then arc **BC** from center **O** with radius **OB.**

$$AC = R(\sqrt{2} - 1) = .4142\,R \quad \text{in which } R = AB$$

Distance Between Column Centers—Tuscan Order.

Columns without pedestal supporting entablature	**6 M 8 m.**
Columns without pedestal and pilasters with arches	**9 M 6 m**
Columns with pedestal and pilasters with arches	**12 M 9 m**

Similar proportions for other orders.

Fluting.—Flutes are longitudinal parallel channels cut in the column surface. The edges of the flutes are sometimes sharp, as in the Doric-Poestum. The flutes then are shallow curves in section. In the Roman orders, the edges are flat and generally one-fifth the distance between centers of adjacent flutes at any height. The flutes and edges taper in proportion to the tapering of the column.

SHADES AND SHADOWS.

Direct Light.—It emanates from the luminous center.

Reflected Light.—It is returned by illuminated surfaces.

Shade.—Surfaces of a body which are opposite the light.

Shadow.—Portions of illuminated surfaces deprived of the light which they should receive because of the interposition of bodies placed between them and the luminous center.

Line of Separation.—Contour of the shade or shadow.

Hypotheses (Fig. 421).—1° Luminous rays are parallel to each other.

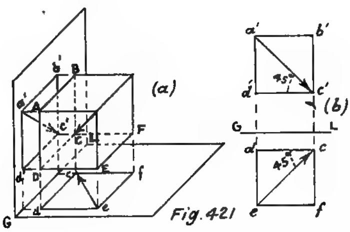


Fig. 421

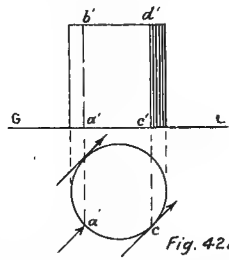


Fig. 422

2° Luminous rays are supposed to light bodies from left to right and in a direction parallel to the diagonal of a cube the faces of which would be parallel to the planes of projection (**a**).

Luminous rays form with the two planes of projection an angle = 35° 16′, and their projections an angle of 45° with the **G.L.** (**b**).

How to Determine the Shade of a Solid.—The line of separation is determined by the contact of the luminous rays tangent to the surface of the body.

See in (Fig. 422) shade of a cylinder.

Remark.—In rounded bodies the lighted portions are not luminous alike. The most intensely lighted is half way between the extreme tangent rays, as at **a-a′b′** in the figure. The darkest spot would be the element diametrically opposite.

Rules for Washing.—If a drawing is to be washed the following rules are generally followed:

1° Any lighted plane surface parallel to one of the planes of projection receives a uniform light flat wash.

2° When several plane surfaces parallel to one another are echeloned, the nearest to the spectator receives the lightest wash, the second a little darker one, and so on to the last; the grading depending on the distance.

3° When a lighted plane surface is oblique to a plane of projections, it receives an uneven wash, the elements of which follow the rule of distance.

4° Shaded surfaces follow like rules; the intensity decreases as the distance from the eye increases.

5° Of several lighted plane surfaces the lightest one is that which is more perpendicular to the direction of light.

6° On curved surfaces (Figs. 423, 424) the lightest portion is that point or element receiving a perpendicular ray; the shading is then increased in both directions to the element or point directly opposite. However, it is customary in a round surface to decrease the shading from the line of separation to the edge of the body when the shade is not entirely visible. This apparent lightening of a portion of surfaces more in the shade is supposed to be due to reflected light.

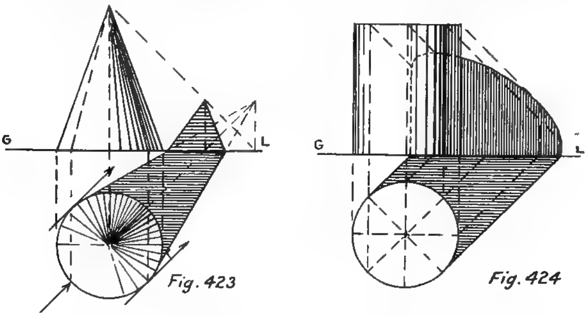


Fig. 423 Fig. 424

Shadows follow similar rules.

7° The contour of a shadow should not be terminated by too sharp a line; a lighter thin band should be provided.

8° The projecting arrises of lighted surfaces should be left white for a short width; this is the light fillet.

Lines of a wash drawing should be lighter in color than a drawing which is to be simply black and white.

Shading with the pen is done according to the same rules.

Heavy Lines.—When a drawing is to be washed or pen shaded, it is not good taste to draw heavy lines. For a line drawing, all lines on the shadow side of the object should be heavy lines (Fig. 425). When several heavy lines are close to each other they are not made so heavy as isolated ones.

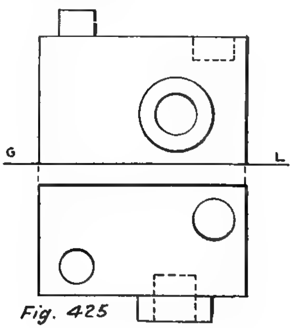


Fig. 425

Note.—It is customary, in many offices, to suppose the light coming from the top left-hand corner at an angle of 45°, whether the drawing be an elevation or a plan. We cannot approve the practice.

PERSPECTIVE.

Perspective.—The art of representing solids as they appear from a fixed point.

Hypotheses.—1° A transparent plane called **Tableau, Perspective Plane,** or more generally **Picture Plane,** is supposed to stand vertically between the spectator and the object. The intersection of this plane with the ground supposed to be the **H** plane is the **Ground Line** of the perspective.

2° Lines are supposed to issue from the eye, or center of vision, and be directed to the visible points of the object; they are **visual rays.**

Line of Horizon.—A horizontal line drawn through the eye.

Perspective of a Point.—The trace of the visual ray of the point upon the picture-plane.

Perspective of a Line.—If a plane be drawn through a straight line and the eye, its intersection with the picture-plane between the extreme rays is the perspective of the line.

Or, as this intersection is straight, we say that the perspective of a straight line is the straight line joining the perspective of its extremities.

General Definition of the Perspective of a Line.—The perspective of a line may be defined: 1° The aggregate of the perspectives of its points. 2° The intersection of the picture-plane with the conic nappe drawn through the eye and the line.

In (Fig. 426) **H** and **V** are the planes of projections and **G M N L** the picture-plane perpendicular to both; **O** is the position of the eye and **A** that of a given point in space. **OA** is the visual ray and its trace **P** in the picture-plane is the perspective of **A.** The question is to determine that point **P.** Let **aa′** be the projections of the point and **oo′** those of the eye. The plane containing **AO** and **Aa** also contains **Oo** and intersects the **H.P.** at **ao,** the **H** projection of **AO**; it intersects the picture-plane in **bP.** The plane containing **AO** and **Aa′** also contains **Oo′** and intersects the **V.P.** at **a′o′**, the **V** projection of **AO**; it intersects the picture-plane in **cP.** **Pb** is a vertical (parallel to **Aa** and **Oo**) and **Pc** is a horizontal (parallel to **Aa′** and **Oo′**). $\mathbf{O_1}$ is the projection of the eye **O** on the picture-plane and is the **point of view** of the perspective and **ON** is the **line of horizon.**

Now the figure shown on the picture-plane can be easily reproduced graphically on the **V** plane of projection by revolving the picture-plane either to the left or to the right; but the projections (called dispositions in perspective) being in this case to the left of the picture-plane, the perspective might be on top of the projections; therefore it is better to revolve it to the right, where there is no obstruction. Now by revolving the picture-plane about its **V** trace

LN would reverse the figure, the points nearest **NL** which are to the right of the others would be brought to the left of them in the **V.** plane. Therefore we revolve the picture-plane about a vertical, say **GM,** until it becomes parallel to the **V** plane, when what it contains will be projected in exact size. This leads to the following graphic construction.

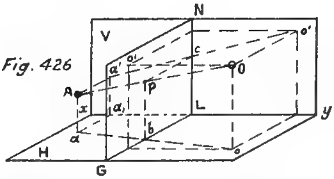


Fig. 426

Find the Perspective of a Point (Fig. 427).—The point is given by its projections **aa′**; the picture plane **GN** is given (or assumed) and the position of the eye **oo′** is also given by its projections (so many feet to the right or left of the point, so many feet above the ground, and so many feet away from the picture-plane—or from the point). Connect **oa** and **o′a′**, **b** and **c** will be the measures of the perspective of the point. Revolve the picture-plane **GLN** about the vertical **G** until its trace **GL** becomes parallel to **xy** at $\mathbf{GL_1}$. In that motion, the points $\mathbf{o_1}$ and **b** describe quadrants $\mathbf{o_1o_2}$ and $\mathbf{bb_1}$, whose **V** projections are horizontals **o′O** and **cP.** Finally draw $\mathbf{o^2O}$ and $\mathbf{b_1P}$ perpendicular to **xy** (or parallel to **GLN**). **P** is the perspective of point **aa′** with regard to the point of view **O.** **Oo′** is the line of horizon.

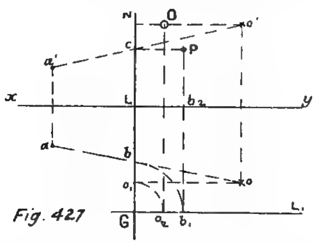


Fig. 427

Note.—In practice the quadrants $\mathbf{o_1o_2}$ and $\mathbf{bb_1}$ are replaced by lines drawn at 45°.

Perspective of a Pyramid (Fig. 428).—Find the perspective of each point visible from **oo′**; for instance, **ss′** will have its perspective measures at $\mathbf{s_1s'_1}$ which point is brought in $\mathbf{s_2}$ then **S,** and so on for the others.

Theorems—

1. The perspective of a vertical is vertical.

2. The perspectives of parallels to the picture-plane are parallel to the ground line.

3. The perspectives of perpendiculars to the picture-plane are converging to the point of view.

4. The perspectives of a beam of horizontal parallels converge to a point on the line of horizon, called the **vanishing point** of the beam.

5. The perspective of a line passing through the vertical of the eye converges to the **point of view.**

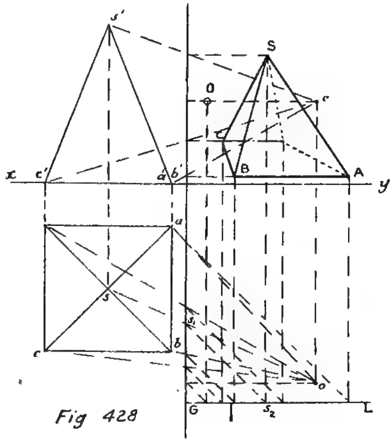


Fig 428

Distance Points—

6. The vanishing points of beams of horizontal parallels making angles of 45° with the picture-plane are two points on the line of horizon called **distance points,** and they are equally distant from the point of view, that distance being the distance from the eye to the picture-plane.

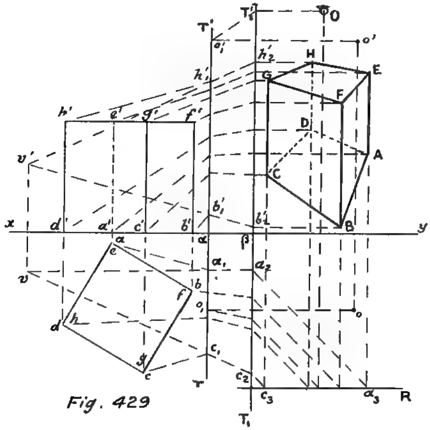


Fig. 429

Enlargement of a Perspective (Fig. 429).—It is sometimes desirable to have as large a perspective as possible. To obtain such an enlargement, select a

secondary *tableau* $T_1\beta T_1'$ parallel to the first one $T\alpha T'$

After obtaining on the original tableau the measures of perspectives of the several points (from b'_1 to h'_1 and from a_1 to c_1), and set on the **V** trace of the auxiliary tableau any height $b'_2h'_2$ which may be given or desired, and join $h'_2h'_1$, $b'_2b'_1$, thus determining a point v'. Select **v** and on the vertical $v'v$ such as to have

a convenient distance $\beta\alpha_2$, after which join to v all the points on $T\alpha$ and to v' all the points on $\beta T'$, producing these lines to an intersection with $T_1\beta T_1'$. Revolve $T_1\beta$ to a position R parallel to the V plane

drawing through the points lines of 45°. Draw through the points thus brought in **R** perpendiculars to **xy** until an intersection with parallels to **xy** drawn from the corresponding points

on $\beta T_1'$ which will fix the perspective figure.

TOPOGRAPHICAL DRAWING.

Topographical Drawing.—Plotting and drawing, from the Surveyor's or the Engineer's notes, the lines that have been run and measured in the field.

These notes are kept in various ways which the Topographical Draughtsman masters readily.

Surveyor's Survey.—The notes for the survey of a polygonal field are summarized in the general form:

N. 85° E.	1386.0
S. 29° 15′ E.	632.0
S. 1° 45′ E.	200.0
S. 85° W.	1490.0
N. 15° 30′ W. . . .	789.0

We may here remark that the beginning of the first course is the most westerly point, and the end of the third course the most easterly point of the survey. Also that the end of the first course is the most northerly point, and the beginning of the last course the most southerly point. But nothing indicates how long the field is on a north-south line, and how long on a west-east line; and that knowledge is generally useful in determining the size of paper to use when the scale is given; it also serves to better center the drawing on a given sheet.

What the Draughtsman Has to Calculate.—The Draughtsman has to calculate the latitudes and departures or the northings or southings and the eastings or westings of the several sides. This is done either by the Traverse Table or by Tables of Logarithms.

Use of the Traverse Table.—The Traverse Table gives the **Latitude** (difference of latitude) and **Departure** (difference of longitude), called also **Northings** or **Southings** for the first, and **Eastings** or **Westings** for the second, for all angles varying by a minute (in some tables the angles are given only to ¼ degree), between 0° and 90°, and for all lengths from 1 to 100; and as these lines are all proportional to each other, a northing (for instance) for a distance of 73 will give that for a distance of 730 or 7300 by simply multiplying it by 10 or 100.

Example: Taking the first line in the example, open the Table at 85° and look for the latitude of 1300 (opposite 13), then that of 86 and add them together for the latitude of 1386, which is 120.5. Do the same for the departure, which is 1380.67. The latitude is a northing and the departure an easting, as indicated by the bearing N. 85° E. The same research for all the sides will give a result which is to be entered in the proper column headed **N, S, E** or **W** of the table of calculations given below.

Sum of the Northings and Southings to Be Equal; Also of the Eastings and Westings.—If the figure is a closed polygon, the sum of the **N's** must equal the sum of the **S's**, and the sum of the **E's** must equal the sum of the **W's.**

Meridian Distance.—When the area is to be calculated, the column head **M.D.** (meridian distances) is formed by adding the Eastings in succession, then subtracting the westings. The last result must be **O.**

Double Meridian Distances.—The column headed **2M.D.** (double meridian distances) is formed from column **M.D.** by adding a number to the preceding one.

Calculating Areas.—The numbers in column **2M.D.** are then multiplied by the corresponding latitude found in **N** or **S**, and the products are entered in column **N.A.** (north areas) if the latitude is a **N.** or in column **S.A.** (south areas) if the latitude is a **S.**

Subtract the sum of **N.A.** from the sum of **S.A.**; take ½ the difference, which will be the required area in square feet. Divide by 43560 for area in acres.

Co-ordinates (Fig. 430).—Distances from points to a north-south and west-east lines.

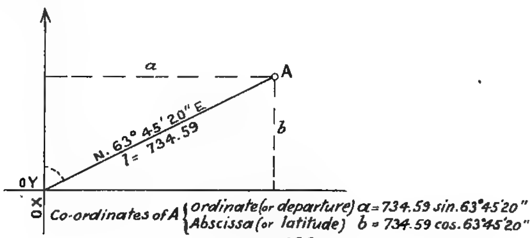


Fig. 430

Ordinate.—The departure equals length of line multiplied by the sine of the bearing.

Abscissa.—The latitude equals length of line multiplied by the cosine of the bearing.

Arrangement of the Operation

N 63° 45′ 20″ E. 734.59

$\bar{1}.952\ 7516$	$\bar{1}.645\ 6204$
$2.866\ 0450$	$2.866\ 0450$
$2.818\ 7966$	$2.511\ 6654$
658.86	324.84

Use of Tables of Logarithms.—In this example l=length of line= 734.59. In this example **B**=Bearing=63° 45′ 20″.

The direction of the co-ordinates is given by the initials **N** and **E**, so that the departure will be an easting and the latitude will be a northing.

In the first vertical column are, first, the log. sin. 63° 45′ 20″; second, the log. 734.59; third, their sum, and, fourth, the number corresponding: 658.86, which is an **E.**

In the second vertical column are, first, the log. cos. 63° 45′ 20″; second, the log, 734.59; third, their sum, and, fourth, the number corresponding: 324.84, which is a **N.**

Thus might be calculated by logarithms, instead of by the Traverse Table the numbers in columns **N, S, E. W** of the given traverse.

Cumulative Co-ordinates.—If the given polygon were attached to a system of survey at the first point, for instance, that point would, in the system, have certain co-ordinates or a northing (or southing) and an easting (or westing) from the origin. The abscissas are generally entered in a column marked **X**; they run eastward when positive and westward when negative. The ordinates are also entered in a column marked **Y**; they run northward when positive and southward when negative.

Suppose that the co-ordinates of the first point are **x** = 2135.82 and **Y** = 268.39; write them opposite that first point in columns **X** and **Y.** To obtain the other numbers in **X**, add the corresponding **E** (or subtract, if it is a **W**). To obtain the other numbers in **Y**, add the corresponding **N** (or subtract if it is an **S**).

The results are cumulative co-ordinates and they give the perpendicular distances to the original **N-S** line and to the original **E-W** line of the system.

Table of Calculations.

Calculations.

1386.	N 85° E.	632.	S. 29°15' E.	200.	S. 1°45' E.
3.141 7632	3.141 7632	2.800 7171	2.800 7171	2.301 0300	2.301 0300
$\bar{1}$.998 3442	$\bar{2}$.940 2960	$\bar{1}$.688 9723	$\bar{1}$.940 7634	$\bar{2}$.484 8479	$\bar{1}$.999 7974
3.140 1074 993 81	2.082 0592	2.489 6894	2.741 4805	0.785 8779	2.300 8274
1380.73	120.80	308.81	551.42	6.1077	199.91

1490.	S. 85° W.	789.	N. 15°30' W.
3.173 1863	3.173 1863	2.897 0770	2.897 0770
$\bar{1}$.998 3442	$\bar{2}$.940 2960	$\bar{1}$.426 8988	$\bar{1}$.983 9105
3.171 5305 217 88	2.113 4823	2.323 9758	2.880 9875
1484.33	129.86	210.85	760.30

B.	D.	N.	S.	E.	W.	X	Y
N. 85° E. 84°59'57"	1386.0	120.8		1380.73 1380.54		2 135.82	268.39
S. 29°15' E. 29°14'53"	632.0		551.42 551.39	308.81 308.77		3 516.36	389.19
S. 1°45' E. 1°45'02"	200.0		199.91 199.90	6.11		3 825.13	– 162.20
S. 85° W. 85°00'04"	1490.0		129.86 129.85		1484.33 1484.54	3 831.24	– 362.10
N. 15°30' W. 15°30'05"	789.0	760.3 760.34			210.85 210.88	2 346.70	– 491.95
	4497.0	881.1	881.19	1695.65	1695.18	2 135.82	268.39
corrected		881 14	881.14	1695.42	1695.42		

The example selected shows a difference of .09 in the **N** and **S**, and a difference of .47 in the **E** and **W**; therefore the traverse does not close absolutely. The error is about ½ ft. on a circuit of nearly 4,500 ft. or 1/90 ft. per hundred feet, which is considered very accurate field work.

Adjustment of a Traverse.—In order to adjust the traverse, increase the sum of the **N's** and decrease that of the **S's** by ½ their difference; or in the example given, increase the **N's** by .04 and decrease the **S's** by .05 and pro rata to their lengths. Likewise decrease the sum of the **E's** by ½ .47, or say .23, and increase that of the **W's** by .24 also pro rata to their lengths, thus obtaining the corrected latitudes and departures written under the calculated ones, which are used to calculate the columns **X** and **Y** containing the corrected co-ordinates.

Angle Adjustment.—Corrections for angles can be made by calculating the bearings by their corrected tangents

$$\frac{138.54}{120.8} \quad \frac{308.77}{551.39} \quad \frac{6.11}{199.9} \quad \frac{1484.54}{129.85} \text{ and } \frac{210.88}{760.34}$$

These corrected bearings given under the measured ones in the example, show how closely the work was done in the field, the greatest difference being only 7″.

Plotting the Traverse.—The total **N**'s, 881.14 ft. and the total **E**'s, 1695.42 ft. may guide in the selection of a suitable sheet of paper. A **N-S** line for meridian 3000 is drawn 831.24 ft. from the right limit of the plot. A perpendicular to this is next drawn 389.19 ft. below the top limit of the plot for the **E-W** line zero; and all these distances to the scale of the drawing.

Place the scale on the **E-W** line, called axis of **X**'s, in such a way that 1000 shall be on the 3000 ft. meridian. and mark, from **O** of the scale, the distances 135.82 and 346.70; displace the scale along the **X**'s line and bring **O** to the 3000 ft. meridian; then mark off the distances 516.36, 825.13 and 831.24.

At the points so marked draw parallels to the axis of **Y**'s (**N-S** line or meridian), above if the **Y** is positive, as 268.39 and 389.19, and below if it is negative, as —162.20, —362.10, —491.95; and scale off the distances, which will give the corners of the Traverse. Connect them by straight lines and the polygon is drawn.

Basic Lines.—The lines of a Traverse are often given only as bases for the definition of real points on the ground, such as the sides of a road, fences, houses, rivers, streams, or a **contour line** (line all the points of which are at the same elevation above a certain datum). In this case the Transit notes are generally kept in the way shown below.

Form of Transit Notes.—On top of the page.

1. Name of line (generally a letter).

2. Date of survey.

3. Names of transitman and chainman.

4. In the first left-hand column are written the stations at the angles. (A station is a chain length, or 100 ft.)

5. In the second column the distances between angle points are entered, which are equal to the differences of the stations.

6. In the third column are the deflection angles right (**R**) or left (**L**), from the foresight (**F.S.**), which means the continuation of the line previously run on the ground; it is **R** or **L** from the forward motion and is so taken because that deflection is generally an acute angle. When the angle would be obtuse, the deflection may be taken from the back site **(B.S.)**, also **R** or **L**, and should be so entered in the book.

7. The deflection angles are sometimes doubled in the fourth column; then the average gives a better approximation.

8. In the fifth column are the magnetic bearings; they are taken to check the work.

Calculated Bearings—

9. In the sixth column are the bearings deduced from the bearing of the first line, for instance, by adding or subtracting the successive deflections, as the case may be.

Calculating Courses—Example.—The original line is **N.** 22° 05′ 20″ **E.**, and the deflections of the successive lines are 31° 04′ **R**; 14° 09′ 10″ **R**; 64° 12′ 40″ **R**; 21° 00′ 20″ **L** of **B.S.**; 64° 02′ 40″ **L.**

We find the successive bearings as follows:

N. 22° 05′ 20″ E.
31° 04′ R.
N 53° 09′ 20″ E.
14° 09′ 10″ R.
N 67° 18′ 30″ E.
64° 12′ 40″ R.
131° 31′ 10″
S. 48° 28′ 50″ E. } Their sum is 180°
N W. ← Bearing of the B. S.
21° 00′ 20″ L.
N. 69° 39′ 10″ W.
64° 02′ 40″ L.
133° 41′ 50″
S. 46° 18′ 10″ W. } Their sum is 180°

The **Rule** is: Add a deflection to the **R** to a course (bearing) **N.E.** or **S.W.**; if the sum is more than 90°, snbtract from 180° it will become **S.E.** or **N.W.** Subtract a deflection to the **R** from a course **S.E.** or **N.W.**; should it become negative, the excess is a **S.W.** or **N.E.** Subtract a deflection to the **L** from a course **N.E.** or **S.W.**; should it become negative, the excess is a **N.W.** or **S.E.** Add a deflection to the **L** to a course **N.W.** or **S.E.**; if the sum is greater than 90°, subtract from 180° it will become **S.W.** or **N.E.**

Diagrams.—The second page is reserved for sketches, diagrams of details, ties, etc.

Page of a Transit Book (from actual survey) (Fig. 431).

To plot this line, the Traverse was calculated and adjusted at all intersections with lines already established, which gave the corrected **X** and **Y** of all the angle points. These angle points are plotted by means of their co-ordinates, and the several stations and intermediate points noted are marked off with the scale. The offsets are plotted on perpendiculars and to the **R** or **L** as indicated on the sketch. It will be noticed that, beginning at stations 128 + 99.1 the stream runs between two stone walls for a certain distance.

Levels.—The elevations of the angle points and all intermediate stations where levels were taken are written in pencil. It is sometimes agreed that the point showing station on the plan shall be taken

as the decimal point of the elevation, when written in ink, as shown on above sketch at Sta. 129.

LINE B'13 Aug. 28 1885

Sta	Dist.	Defl Ang	Double Def.	Mag. C	Cal Course
133					
132+55.8 ⊙	128 7	36°17'L	72°34'	S 77¾ E N. 77½ W	N 78°53'30"W.
132 +50					
+87					
131+53.8 ⊙	102 0	9°35'R	19°10'	S 41° E N 41°¼ W.	N 42°36'30"W.
131 +32					
+66					
130+07.5 ⊙	146.3	90°50'15"L	181°40'30"	N 50°¾ W. (a)	N 52°11'30"W.
130					
+98					
+47					
129					
128+99.1 ⊙	108.4	35°36'15"R	71°12'30"	S 40° W N 40° E.	N.38°38'45"E.
+70					
+40					
128					
127 B'13					

NOTE. (a) The Magnetic Bearing (Course) was evidently forgotten.

{X Transitman
{Y Chainman

Notes

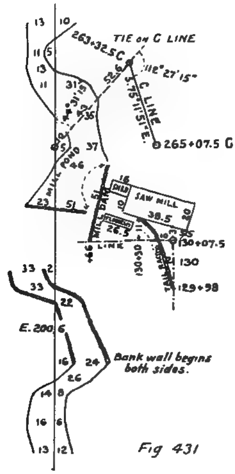


Fig 431

Form of Level Notes.—On top of the page are written the name of the line (generally a letter), the date and the names of the leveler and rodman.

Stations (Sta.)—In the first left-hand column are entered (generally from top to bottom) the numbers of the stations and intermediate points (plus points) at which elevations were taken (where rod was placed).

Back Sights (B-S.), or readings of the rod on the point the leveler comes from, are entered in the second column.

Height of Instrument (H-I.) is entered opposite the turning points (points were two readings, a **F.S.** and a **B.S.** are taken on the same position of the rod) in the third column.

Foresights (F.S.) or readings of the rod on the point the leveler goes to, are entered in the fourth column.

Elevations (El. or E.), or heights of the points above (or below when negative) a given or assumed datum, which is a level plane of elevation zero. It may be mean low or high water, or normal level of a lake, a river at a certain point. Often it is simply chosen low enough to make sure that all possible elevations shall be +.

In another column, or on the second page, are entered remarks and notes.

(For page of a Level Book, see **Book IV, The Leveler).**

Cross Sections.—Cross sections are lines of levels which run generally at right angle **R** and **L** of a transit line, from a station or a plus point, the elevation of which was previously determined in the leveling of the line. The angle at which the **X** sections are taken is always entered in the field book as 90°**R**; **90°L; 135°L,** etc.

(For Form of Cross Section Notes, see **"The Leveler."**)

Plotting of Cross Sections.—At the stations as given in the book make on the plan, to the **R** or **L** of the line, the angle indicated, and on the line thus obtained either lay off the distances recorded and write down the corresponding elevations, or, if certain elevations only are required, as those of contours every two or five ft., interpolate them either by calculation or by scale, and lay them off; then write down their elevations.

Stadia Work.—From designated stations on a line—these stations are established from the angle points or a preliminary or secondary traverse—stadia points are given by either deflection angle and distance, or by azimuth angle (angle made to the **R** of a **N.-S.** line and varying from 0° to 360°) and distance. Lay them off as indicated in the notes and record the elevations. In that connection a protractor with a pivoted diameter divided to a scale will be found convenient.

Contours (Fig. 432).—When the elevations at which contours are to be drawn have all been interpolated between the known points on the line and cross sections, connect them by continuous curves, which will be the contours of those elevations.

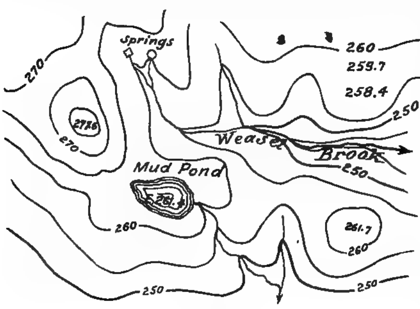


Fig. 432

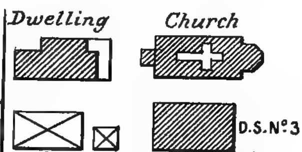


Fig. 433

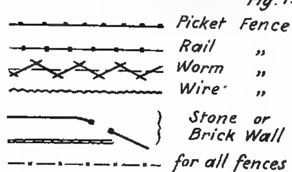


Fig. 434

Head all contours in streams, if there be any in the valley.

Don't cross twice a shore line with the same contour.

Slightly curve contours in roadways, allowing for side ditches.

Contours may form islands or basins, but a portion of the side of an island cannot be contiguous to the side of an adjacent island of same elevation; but two such islands may be tangent.

An island and a basin of same elevation cannot be contiguous for any length. Such conditions are unnatural.

In the annexed diagram contour 195 forms islands at **A** and **B** and a basin at **C.**

Representation of Objects—Buildings (Fig. 433).—Some offices require brick and stone houses to be filled in with hachures and frame buildings with two cross lines; where others require all public stone or brick buildings and all frame dwellings to be hatched; only barns and outhouses to be crossed. A church is distinguished by a cross. Buildings have generally a shade line and the designation of their purport.

Fences (Fig. 434).—All wooden fences may simply be shown by short dash lines; sometimes differences are made with picket fences, rail fences, worm fences (astride on the property or plot line), wire fences, stone or brick walls. Or, again, all fences are shown by dashes separated by a small cross.

Road Lines (Fig. 435).—On general maps, two parallel solid fine lines.

On detail maps shown with their fences, if there be any, or with dotted lines.

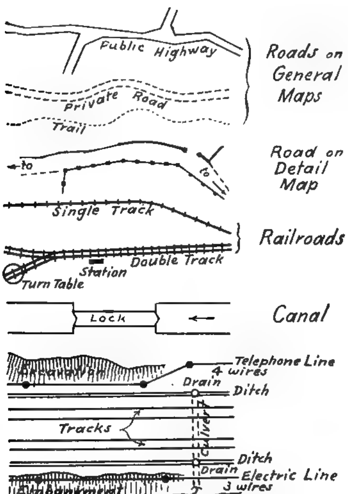


Fig. 435

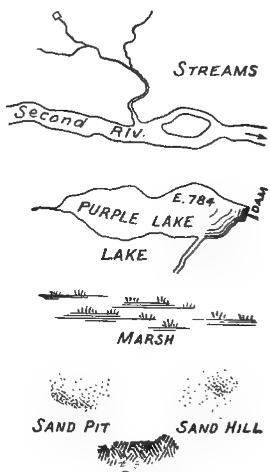


Fig. 437

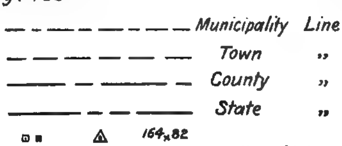


Fig. 436

Railroads.—Single Track—With a single line and cross-ties.

Double Track—With a double line and cross-ties.

Canals.—Two parallel lines narrowed down for locks and angular lines for gates.

Excavation.—If not shown by contours may be represented by hachures running from the berm towards the road.

Embankment.—May be represented by hachures running from the road down.

Trolley Line.—May be shown with two sets of double lines.

Electric Pole Lines.—Electric lines may be shown with solid small circles for poles, connected with a fine line. The number of wires should be mentioned, as well as the name of the line, and the indication: Telegraph Line, Telephone Line, or Electric Light Line, as the case may be.

Property Lines (Fig. 436).—May be solid, with a heavier line for outside boundaries or groups of properties.

When the line runs in the center of a stream or a road, it should be dotted, because such line cannot be absolutely determined.

Sometimes a dash line, with a dash and dot, for outside boundaries.

Political Lines—City or Borough Line.—Equal dashes, alternating fine and heavy.

Town Line.—Equal heavy dashes.

County Line.—Alternate long and short heavier dashes.

State Line.—One long and two short still heavier dashes.

Survey Marks—Monument.—A diamond with point at center, marked **MON.**

Triangulation Station.—A triangle with point at center, marked **Trig.**

Bench Mark.—A cross marked **B.M.,** followed by the elevation.

All these details, as well as ditches, drains, etc., may be shown only on maps of a large scale, say, 100 ft. to the inch or less.

Features—Streams (Fig. 437).—For small brooks, a single wavy line, fine at the source or spring and increasing in thickness to the mouth. If the scale of drawing allows the stream may be shown with its two banks, the thickness of which lines must be the same all the way.

Rivers.—Must be shown with both banks duly located in the survey. The direction of the flow must be indicated by an arrow.

Lakes.—Lakes or large bodies of water are shown by their bank as located by field notes, with their surface elevation duly recorded.

Water Lines.—In elaborate drawings lines are drawn along the shores of rivers, lakes, bays, etc.; they are generally parallel to the shore and closer together near it; they widen towards the center. They are called water lines and are always fine.

Marshes.—Are represented with groups of fine parallel lines, on which short bunches of grass are drawn.

Sand Pits.—Are shown with fine dots about evenly distributed.

Sand Hills.—Are shown with fine dots closer together at the top.

Rocky Bluffs.—When contours cannot be shown as being too close together, short hachures are used, breaking direction.

Trees (Fig. 438).—Isolated trees are shown with an irregular, wavy line, simulating the shape of the tree as seen from above.

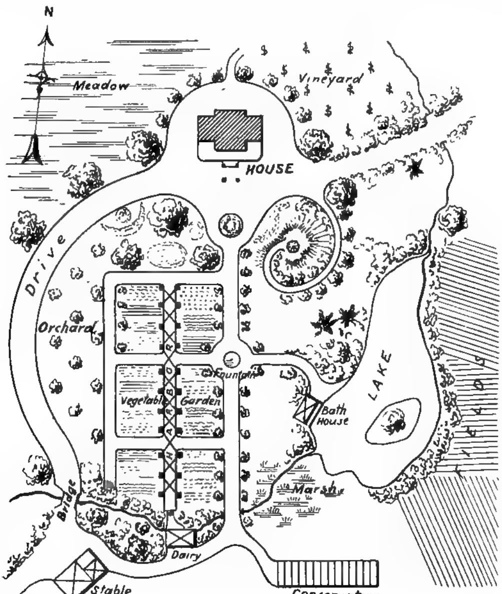


Fig. 438

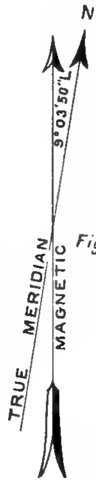


Fig. 439

Fir trees, like an irregular star. They are sometimes shaded.

In orchards trees are regularly arranged in squares or quincunx.

Woods.—Irregular, wavy lines throughout the wooded area, a little more shaded on two sides (bottom and right hand), with some trees here and there a little more marked.

Vineyards.—Vines arranged in quincunx, each shown with a vertical line for the pole and a wavy line around it for the vine.

Ornamental Gardens.—The paths and alleys are dotted and the plots may show borders, clumps of shrubs, flower beds, etc., if the scale of the drawing will allow.

Graphic Scale.—When a drawing is completed, a graphic scale should always be drawn (not merely indicated), with the same scale used in the plotting. It would be even better to draw two scales perpend. to each other, one lengthwise, the other crosswise. The reason for this is that paper or cloth will expand or contract according to the weather, and it may become set with an unequal expansion in width than in length. In that case areas taken with planimeter may be adjusted to the two scales.

North Point.—Always draw a north point (Fig. 439) on a topographical map and indicate it either **Magnetic** or **True Meridian**; if the magnetic deflection is known, the two meridians should be shown and the deflection written in the angle which they form. Write **N** on the north side of the line. A map should always be oriented in such a way that the north shall point more to the top of the drawing.

Border.—A map should not take the whole surface of the sheet of paper, because the edges may wear and tear by handling. It should leave a space large enough all around to allow a border (a simple thick line) to be drawn, forming a rectangle. The blank space around the border should vary, being wider on large maps than on small sheets, where ½ inch may be enough. Good taste should direct. The thickness of the border line should also vary with the size of the sheet used.

Planimeter.—An instrument designed to measure plotted areas which are limited by irregular boundaries; as, for example, a contour line forming an island or a basin, a contour line cut by a straight line, etc., or an area bounded by geometrical lines when absolute accuracy is not desired.

Description of the Planimeter.—The instrument consists of two arms, one graduated and of variable length, the other of fixed length and one extremity of which is jointed by ball and socket to the center of a weight to secure the permanency of position of the instrument when used, and the other extremity is pivoted to a standard or frame carrying the graduated arm in a groove where its length is determined by the zero of a straight vernier which it carries. The standard carries a small vertical shaft with a pinion engaging in a worm, the axis of which is permanently attached to the center of a vertical wheel, the rim of which acts as a traveler and the periphery of which is divided into 100 equal parts graduated every 10 divisions 1, 2, ...9, 0. This wheel moves along a circular vernier of same diameter, carried by the frame, and permits the reading to one-tenth of a division of the wheel. To the top of the pinioned vertical shaft a disc divided into 10 equal parts is horizontally attached; it revolves in front of the slide on which is a reference mark opposite the point of contact of frame and disc. The pinion and worm are so geared that a complete revolution of the wheel only causes the disc to revolve one-tenth of a revolution, or one graduation, so that the vernier will give the units of a reading, the small divisions of the wheel the tens, the large divisions of the wheel the hundreds and the divisions of the disc the thousands.

The graduated arm is divided into a number of half-centimeters, each divided into ten equal parts, which in turn may be divided in 10

by the straight vernier. At the end of this arm is attached a vertical pointer. In the box of each instrument a table gives the setting (number to be set opposite the straight vernier) of the arm for different scales: for example, for a drawing to the scale of 1″ to 100′ the arm length is given as 138.6 in the second column, and the reading after the operation wil be a number of 100 sq. feet and is therefore to be multiplied by 100.

When the graduated arm is set, a clamp-screw will maintain it in that position.

How to Set the Planimeter.—Set the weight (ordinarily a pound disc) in a convenient position outside the area to be measured (the best place for this fixed axis of the instrument is the top of the area). Set the graduated arm or rod to the reading given in the table for the particular scale of the drawing, by bringing that number opposite the zero of the straight vernier, and clamp it to that position.

Fix the ball of the plain rod in the socket at the center of the weight and clamp it there by means of the collared arm. Ascertain now if it is possible to span the whole area at one operation, and, if not, subdivide it into sections by pencil lines.

How to Use the Planimeter—

First Method.—Mark a point (a corner is generally selected) upon the boundary and place the pointer there. Read the notation of the instrument: first the disc, say 6, then in succession, the large divisions of the wheel, say 3 (the zero of the circular vernier between 3 and 4), the small division, say 7 (the zero of the vernier between 7 and 8), and lastly the vernier at the coincidence, say 5; the starting notation in this case would be 6375, which you mark down in book. Take lightly but firmly hold of the pointer and move it to the right along the bends and turns of the boundary until you come back to the point of starting. Take a second reading of the instrument and record it. Suppose the second reading to be 9031, the difference 2656 multiplied by the number of square feet given in third column of table, say 100 sq. ft. for a 1″ to 100 ft. scale, will give the number of square feet in the measured area: 265,600 sq. ft. in this case.

Were the second reading 1348, for instance, a number smaller than the first reading, it would be an indication that the disc made more than a revolution and that reading is in reality 11,348.

Often an area is gone over twice or three times with the planimeter, in which case the difference between the final and first readings should be divided by 2 or by 3.

It is never necessary to set the instrument at zero.

Caution.—Any tracing or paper drawing is apt to shrink or expand, and irregularly in length and width, so that it is not always advisable to set the instrument as indicated in table for the given scale. A little more accurate method is the following:

Second Method.—As the rods of the instrument are apt to expand or contract, also for the sake of spanning a greater area at an operation, the following method is sometimes preferred: Pull out the graduated arm as long as possible and clamp. Draw a square on the scale

and planimeter it. Calculate what area one planimeter unit represents. Planimeter the areas required and finally multiply the readings by the value of the planimeter unit. Or, again,

Third Method.—If lengths of straight lines are given in figures, take one running lengthwise and another running crosswise, and construct with them a rectangle the area of which you calculate;

for instance 729.61×633.04 $315\,950.3144$ sq. ft.
Planimeter this rectangle. Let the reading be $7\,178$
If $7\,178$ planimeter units $=$ $315\,950.3$ sq. ft.
1 plan. unit $\dfrac{315\,950.3}{7\,178} = 44.01$ sq. ft.

Now proceed to planimeter any area wanted in that map and multiply the number of planimeter units (difference of readings) by 44.01; the product will be the square feet.

Planimeter as Little as Possible.—If a regular polygon or a series of regular polygons are or can be inscribed within the surface the area of which is desired, the area of these polygons should be calculated and the rest or the outlying surface to the irregular boundary alone should be planimetered.

Approximation of Results.—It may safely be assumed that with proper attention the possible error is less than one-half of one per cent.

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THE ASSISTANT ENGINEER

BY

PROF. JEAN P. GENTHON
Grad. Univ. of France
Assistant Engineer, Aqueduct Commissioners
Member of The Municipal Engineers of the City of New York

BOOK VIII

THE ENGINEER

WRITTEN FOR

THE CHIEF

Journal of the Civil Service

PUBLISHED BY
THE CHIEF PUBLISHING CO.
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1909

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THE ENGINEER

Book VIII

PREFACE

This series is designed for the young man who, desirous of making engineering in the Public Service his career, wishes to take a Civil Service examination. If successful, his name will be placed on the eligible list and he will in time see open to him the doors of a Government, State or City Department.

The plan of this work is as follows:

The Assistant Engineer must know not only his duties but those of the men under his charge. Therefore each position is taken or each rung of the engineering ladder is ascended in turn beginning with the most easily obtainable, and in that work are explained to the man occupying that position or striving for it.

1°. The requirements for the Government, the State and County of New York, and the City of New York Civil Service, with the scope of the examinations, the ratings and questions given at previous examinations.

2°. The scientific requirements or what the candidate should know.

3° The technical requirements, or knowledge and use of the instruments, and duties in the field and in the office.

Although the requirements for the higher positions demand a knowledge of higher mathematics, the author has had the same experience as Mr. Wm. F. Shunk. In his long practice there had never come before him a question which could not be satisfactorily solved by elementary mathematics.

It will be noticed that certain technical examination questions for a position may seem to belong properly to the grade next higher. This is due to the necessity where the examiners are placed of raising occasionally the standard of an examination in order to draw out the very best material among the always increasing number of applicants. We therefore recommend the prospective candidate to study a little further than would seem necessary.

J. P. GENTHON.

July 1, 1907. New York.

PRESS OF
THE CHIEF
NEW YORK

PRELIMINARY CHAPTER.

GENERAL QUALIFICATIONS REQUIRED.

The principal qualifications required of a young man who wishes to enter the Public Service in an engineering department are:

1°. Aptitude for mathematics.

2°. Habit of observation.

3°. Good memory.

4°. System.

5°. Readiness for work.

APTITUDE FOR MATHEMATICS.—He may not have at the start more than a common school education, but he must constantly increase his mathematical stock and keep on studying in order to fit himself for the next higher grade or position.

HABIT OF OBSERVATION.—This habit may be in the man—I have seen it in children—If not, it has to be coaxed and cultivated. Keep your eyes open when a new problem or a new solution are presented; when a new material or a combination of materials or new appliances and processes are used for old or new purposes. Notice the several kinds of labor, and of labor-saving devices employed. Remark the professional discussions which arise before you and take part in them. When in doubt, ask questions.

GOOD MEMORY.—Habit of observation strengthens memory. which faculty may be improved to a high degree. Remember names and faces of persons; ways and means you have seen made use of by others to attain certain results; principal formulas employed in your line of engineering; stations of remarkable points, and the like.

SYSTEM.—This quality means arrangement, classification, organization and will show itself in the following instances: field-book clean and clear; calculations well arranged, entered in blank-books, always checked and summarized when necessary; plans and drawings on regular sizes of paper according to the classes to which they belong, with figures and letters of standard form and size, the proper titles, scale, assigned number, border and a uniform margin; regular steps taken to reach an end, as dividing a work into such sections and employing on them such force as to complete it in the shortest time, in the most economical way and the most homogeneous manner; a place for everything and everything in its place.

READINESS FOR WORK.—Be ready, when called upon, not only to perform your own work but to assist others. Help in checking figures and calculations, in cleaning and packing instruments, in filing drawings and papers. Give information or advice, go for supplies. Finally be of even and genial temper and all around you will feel better but none more than yourself.

ORGANIZATION OF AN ENGINEERING CORPS.

All departments the duties of which are the erection of public works have a Chief Engineer who prepares the work and directs its

construction. The Chief Engineer is assisted and advised by a Deputy Chief Engineer and one or more Consulting Engineers.

When the work is very extensive, as a railroad, a system of highways, of sewers for a large city, a canal, an aqueduct, it is divided into approximately equal portions called divisions, and to each of these a Division Engineer is assigned whose duty it is to prepare the work within that division and to direct its construction, subject to the orders and approval of the Chief Engineer.

A division may be subdivided into sections with an assistant Engineer in charge having under him as aids, transitmen, levelers, topographers and inspectors.

The Transitman is assisted by chainmen and an axeman.

The Topographer is assisted by flagmen, a rodman and a chainman.

The Leveler is assisted by a rodman, sometimes by chainmen and an axeman.

In the office of the Chief Engineer are Assistant Engineers, Draughtsmen (Topographical, Mechanical, Architectural) and Tracers or Copyists.

There may be like positions in a Division Engineer's Office.

Clerks and Stenographers, although employed in an Engineering department, are not included in the Engineering nomenclature which is summarized in the following table:

ENGINEERING CORPS.

Chief Engineer.			Draughtsmen.	Topographical. Mechanical. Architectural. Tracer or Copyist
	Division Engineers.		Inspectors.	
		Assistant Engineers.	Transitmen.	Chainmen. Axemen.
			Topographers.	Rodmen. Flagmen. Chainmen.
			Levelers.	Rodmen. Chainmen. Axemen.

WHERE POSITIONS ARE OPENED.

GOVERNMENT.

All positions in the Engineer Department at Large are under the War Department with headquarters at

Baltimore, Md.	Memphis, Tenn.	St. Louis, Mo.
Boston, Mass.	Milwaukee, Wis.	St. Paul, Minn.
Buffalo, N. Y.	Mobile, Ala.	San Francisco, Cal.
Charleston, S. C.	Nashville, Tenn.	Savannah, Ga.
Chattanooga, Tenn.	New London, Conn.	Seattle, Wash.
Chicago, Ill.	New Orleans, La.	Sioux City, Iowa.
Cincinnati, Ohio.	Newport, R. I.	Tampa, Fla.
Cleveland, Ohio.	New York, N. Y.	Vicksburg, Miss.
Detroit, Mich.	Norfolk, Va.	Washington, D. C.
Galveston, Tex.	Philadelphia, Pa.	Wheeling, W. Va.
Grand Rapids, Mich.	Pittsburg, Pa.	Wilmington, Del.
Jacksonville, Fla.	Portland, Me.	Wilmington, N. C.
Little Rock, Ark.	Portland, Oreg.	Yellowstone Park, Wyo
Louisville, Ky.	Rock Island, Ill.	

where examinations may be taken except that of Assistant Engineer, which is taken at Washington, D. C.

Draftsmen are on demand in nearly all branches of the Federal Service. Civil Engineers are also certified to the Reclamation Service and the Quartermaster's Department at Large.

Inquiry may be made to the United States Civil Service Commission at any of the above-named towns for dates of examinations, application blanks, etc.

NEW YORK STATE AND COUNTY.

Examinations may be taken at

Albany.	Ithaca.	Ogdensburg.
Amsterdam.	Jamestown.	Olean.
Auburn.	Kingston.	Plattsburg.
Binghampton.	Lockport.	Poughkeepsie.
Buffalo.	Malone.	Rochester.
Elmira.	Newburg.	Utica.
Hornelsville.	New York.	Syracuse.
		Watertown.

The Commission receives applications for any position at any time.

Apply to "State Civil Service Commission," Albany, N. Y.

NEW YORK CITY.

Borough Manhattan.—Topographical Draughtsman.

Borough The Bronx.—Inspector of regulating, paving and grading; mechanical and topographical draughtsmen.

Borough Brooklyn.—Axeman; chainman; rodman; inspector of regulating, paving and grading; inspector of sewer construction; transitman and computer; assistant engineer.

Borough Queens.—Rodman; transitman; topographical draughtsman; assistant engineer.

Borough Richmond.—Axeman; rodman; leveler; transitman; topographical draughtsman.

Department of Water Supply, Gas and Electricity.—Engineer corps (all grades).

Department of Parks.—Engineer corps.

Department of Bridges.—Engineer corps.

Department of Docks and Ferries.—Engineer corps.

Department of Sewers.—Engineer corps.

Department of Highways.—Engineer corps.

Department of Buildings.—Inspector of plumbing, light and ventilation, of masonry and carpentry, of steel construction, of elevators.

Department of Finance.—Engineer corps.

Department of Education.—Inspector of heating and ventilation, of buildings; draughtsmen.

Board of Aqueduct Commissioners.—Engineer corps.

Board of Water Supply.—Engineer corps.

The old **Board of Rapid Transit** is now attached to the **Public Service Commission** for the First District and the men of its engineer corps are subject to the State Civil Service.

For information and blank applications apply to "Municipal Civil Service Commission," 299 Broadway, New York City.

Notices of coming examinations are posted in the public room of their office.

These notices, as well as those for the State and Government Service, appear regularly in "THE CHIEF."

BOOK VIII
THE ENGINEER

Assistant Engineer—He who is assigned to the charge of a certain engineering work.

Who His Superiors Are.—He reports to the Division Engineer or to the Chief Engineer.

CIVIL SERVICE REQUIREMENTS.

Federal Civil Service—Panama Canal.

Title—Assistant Civil Engineer.

Age Limits—25 to 50 years at time of examination.

Salary—$200, $225 and $250 per month.

Written Examination.

Subjects.	Relative Weights.
1. Pure and applied mathematics (elementary problems in mensuration, solution of plane triangles, and theoretical and applied mechanics, involving a fair knowledge of pure mathematics to and including calculus)	15
2. Construction and care of instruments (comprising transit, including stadia work, level, plane table, rods, chain, tape, current meters, etc.)	10
3. Theory and practice of surveying (comprising surveying, leveling, and other field work required in civil engineering and not covered in subjects 1 and 2)	10
4. Design and construction (involving elementary knowledge of designing and constructing highways, railroads, dams, retaining walls, foundation work, trusses, etc.)	25
5. Training and experience	40
Total	100

Five years' practical experience in use and care of instruments in surveying.

After one year's satisfactory service, appointee will be eligible for promotion as an **assistant engineer** without further examination.

New York State and County Civil Service.

Title—Assistant Engineer.

Salary—$5 to $6 a day.

Examination.

Subjects.	Relative Weights.
1. Theoretical and practical questions, including highway construction, strength of materials, canal and water supply construction, hydraulics of canal and water supply engineering, topographic surveying and mapping	10
2. Experience and personal qualifications	7
3. Education	3
Total	20

Specimen Questions.

Theoretical and Practical Questions—(Sheet 1.)—Directions: Slide-rules may be used. Logarithm tables will be furnished by the examiners.

1. Give in detail the best method of mixing Portland concrete "by hand." Give the reasons why Portland cement is generally preferred to natural cements.

2. Describe fully and in detail the proper method of preparing the subgrade for a macadam highway. (a) Across a level, sandy farm. (b) In a hilly and rocky country. (c) Across a swampy country with a subsoil of clay.

3. What are the most important requirements for which provision must be made in the construction of a macadam highway?

4. In the construction of a macadam highway, how would you prevent: (a) "Ravelling"; (b) sinking of the stone into the subgrade; (c) rapid powdering or disintegration of the stone; (d) a muddy or dusty road?

5. Explain in detail how you would take a series of accurate levels for determining the flow of water in a level country. How do you determine accurately the depth of the water over the crest of a weir?

6. A beam 12 feet long is held in a horizontal position, one end resting against a vertical wall; the other end, which is to sustain a weight of 600 pounds, is supported by two guy ropes fastened at the top of the wall, 16 feet above the beam, one 5 feet to the right and the other 6 feet to the left of the beam, which is perpendicular to

the wall. Find the stress in the two guy ropes and the thrust of the beam against the wall.

7. What is the theoretical horsepower that may be supplied by a waterfall 90 feet high and 84 feet wide when it delivers 16 cubic feet of water per second every foot of its width?

8. The waste gate in a sluiceway is 4.75 feet wide; the water on one side is 9 feet 6 inches deep and on the other 3 feet 3 inches deep. Find the amount and the position of the resultant water pressure on the gate. Find the theoretical velocity with which the water will begin to flow when the gate is raised six inches.

9. What fall must be given a waterway 2640 feet long, 123 feet wide at the top, 75 feet wide at the bottom, 12 feet deep, that it may convey 1800 cubic feet of water per second? (Take c = 88.)

10. A masonry reservoir wall weighs 120 pounds per cubic foot and is 28 feet high. It is 3.5 feet thick at the top and has a front batter of 1 in 6. What should be the least back batter and thickness at the bottom in order that it may safely retain water level with the top?

11. The following is a portion of a set of field notes for a stadia survey. One full space on the rod corresponded to a distance of 100 feet from the center of the instrument. The elevation of the instrument station was 131.57 feet. You are required to calculate the corrected distance, the difference in elevation and the elevation as called for by the columns below:

Stadia	Hor. Angle		Vert. Angle		Cor. Dist.	Dif. in Elev.	Elev.
.04	84°	37′	+8°	1′			
0.89	132	55	—1	33			
1.10	91	10	+3	12			
0.90	39	18	+4	52			
1.65	42	30	+3	22			
			etc.				

12. Plot to suitable scale the elevations you have determined in the above question and by them draw contour lines for each even foot of elevation. Do not erase the points plotted after drawing contours. (If you are unable to calculate the elevations called for above, assume a reasonable set of elevations and plot as directed.)

(Sheet 2.) In answering the questions on this sheet, any books of reference may be used. When such books are used for formulas, tables of constants, etc., candidates will give the name of the book and the page referred to.

1. A stream of surface width 50 feet and depth 4 feet, has side slopes 1¾ to 1. The sides and bottom are of coarse gravel and pebbles, with some large stone; the bed has a general fall of 0.5%. Find the height of the submerged weir that will double the depth, taking into account the velocity of approach.

2. Find the pressure of the stream against the weir in the above question and design the weir, illustrating by a cross-section diagram.

3. How far can 100 horsepower be transmitted by a 3½-inch new, straight, smooth iron pipe, laid on a grade of 1 in 2,000, with a loss of head of 25 per cent., under an effective pressure head of 750 pounds per square inch?

4. A dock wall, plumb at the rear with a face batter of 1 in 24, is 20 feet high and 9 feet thick at the base (bottom of stream). The water in front varies in depth from 10 feet to 18 feet. The masonry of the wall weighs 125 pounds per cubic foot and it is founded in earth weighing 112 pounds per cubic foot and having an angle of repose of 32° Find the least depth of foundation required.

5. A wall is to be 20 feet high and 4 feet thick at the top to retain earth with a surcharge of 10 feet, having a slope upward from the top of the wall of 1 to 1, the natural slope of the earth being 45 degrees. What should be the thickness of the wall at the bottom and the batter, if the back is to be vertical? Take the weight of the masonry at 130 pounds and that of the earth at 120 pounds per cubic foot.

6. Make an estimate of the total cost of excavating a ditch for a canal feeder, bottom width 5 feet, side slopes 1½ to 1, depth 6 feet, length 2¼ miles, through country approximately level. The soil for most of the distance consists of 3 feet of sand and gravel overlying stiff clay, but for ⅛ mile the feeder will pass through a ledge of hard shale and slate that comes to the surface with a dip of 45 degrees. The average overhaul is ½ mile and the time allowed for the work is 90 days. Give answer in detail.

New York City Municipal Civil Service.

Assistant Engineer—Group 3, fifth grade.

Age Limit—Not less than eighteen years of age.

Salary—More than $1,800, not more than $3,000.

Scope of Examination.

Subjects.	Relative Weights.
1. Handwriting (as shown on examination papers)	1
2. Arithmetic	1
3. Technical knowledge	6
4. Experience	2
Total	10

QUESTIONS GIVEN AT CIVIL SERVICE EXAMINATIONS.

Mathematics.

1. Extract the square root of 2030.4063036 to four places of decimals.

2. Reduce 35.2 inches per second to feet per minute and miles per hour.

3. Find the area of a triangle the sides of which are 50, 60 and 70 feet, respectively.

4. How many cubic feet of water per second will be discharged by a canal 125 feet wide at top, 75 feet at bottom, 10 feet deep and 2640 feet long, with a fall of 40 feet (take C equals 88)?

5. What is the weight of a cast-iron pipe 12′ long, 4 feet inside diameter, 1″ thick, allowing 2½ per cent. for increase of metal in hub and spigot?

6. Nat. sine = x; nat. cosine = y; nat. tangent = ?

7. Given the 4 sides of a quadrilateral and one diagonal, what is the area?

8. $\frac{90}{x} - \frac{90}{x+1} - \frac{27}{x+2} = o$ solve for x.

9. $\sqrt{629,514,455,084}$ to 4 places of decimals.

10. What is the area of a curved part of a street 60 ft. wide, of which the center line subtends 30° of a circle of which the R. is 530 ft.?

11. Add 6 ft. 7½″; 7 ft. 3 3/16 in.; 8 ft. 2 13/32″; 11 ft. 7⅛″; 6.5 in.; 12 ft. 7.35 in.; 21 ft. 11 9/16″ Reduce the result to decimals of a foot to 3 places.

12. How many cu. yds. of brickwork are there in a brick lining 2 ft. thick, of an elliptical arch of which the radii are 36 ft. and 20 ft. and the length of the work 100 ft.? (Arch is semi-elliptical, springing from abutments.)

13. A stream is 18′ wide on top, 6′ on bottom, 4′ deep. Slope 18″ per mile through a sandy loam. Calculate the flow at cross section.

14. A 24″ cast-iron pipe 3324 ft. long, without sharp bends, connects 2 reservoirs of elevations 72 and 340. Calculate discharge of pipe and make allowance or deduction for entrance head.

15. Area of a pond with nearly vertical banks is 1,200 sq. ft.; there is a wasteway 36″ wide and 1′ high, and it will at first discharge through a head of 3.68 ft. How long will it take to lower the surface of the same 12″?

16. Solve the following according to the algebraic signs:

$$\sqrt{\frac{(6\frac{2}{7} - 4\frac{5}{9}) \times 8\frac{7}{16}}{4\frac{4}{12}} \times 67873.367}$$

and show your work.

17. Major axis ellipse = 10′; minor axis = 6′. Find area.

18. The bases of a truncated cone have radii of 7 ft. and 11 ft., respectively. The height of the trunk is 17 ft.; find the volume.

19. The following are the cuttings on a borrow-pit: 40′ x 50′. Find by the easiest average method the volume of this pit, 6.3, 6.1. 5.8, 5.2, 5.4, 8.7, 6.7, 6.3, 5.8, 5.3, 6.4, 6.8, 6.7, 6.1. 5.6, 8.6, etc.; these cuttings were every 10′ each way in equal squares.

20. Soundings were taken across a river and found as follows: At 10 ft., 8.1 ft.; at 20 ft., 10.3 ft.; at 25 ft., 10.2 ft.; at 30 ft., 10.3 ft.; at 40 ft., 8.1 ft., and at 45 ft., 6.0 ft. The river is 50 ft. wide and the fall is 0.12.

Given the formula $V = C\sqrt{RS}$

find velocity of water in stream for C = 88.

21. If 12 men can shovel 90 yds. of earth in an hour and a half, how many men will be required to shovel 2,550 yds. in a working day of 10 hours?

22. A culvert must take drainage from 1,000 acres. How many cubic feet per second must be carried by the culvert?

Use formula $Q = cy\sqrt[4]{Sa^3}$,

where Q = cubic feet per second reaching culvert; c = proportion of rainfall reaching culvert; y = rainfall per hour; s = average slope of watershed in ft. (per 1,000 ft. of horizontal dist.), and a = acres of watershed. (Give values to c, y and s according to your judgment; exact quantities are not required.)

23. A grade of 1/270 is how much per 100? How much per mile?

24. A building is 50 ft. wide and the pitch of a peaked roof is 30°. What is the length of the rafter? With rafters spread 10 ft. apart, what is the strain in the rafter from a uniform load of 30 lbs. per sq. ft. of horizontal area?

25. $\sqrt{47,065,106}$

26. Extract the square root of 200703,-00783. (Log. not to be used.)

27. Find (by algebra) three numbers such that if the first be multiplied by the sum of the second and third, the second by the sum of the first and third, the third by the sum of the first and second, the products shall be, respectively, 35, 50 and 56.

28. A board 12 feet long is 16 inches wide at one end and 8 inches at the other. How far from either end must it be cut transversely so that each part may contain one-half of the board?

29. The population of a certain town in 1880 was 7085; it having increased 25% in 10 years; what was it in 1870?

30. How many feet B.M. in the flooring of a room 20′ x 30′ x 2 1/3″ thick?

31. Find values of x in $2x + 37 = 33 - 4x + 7$ and $x^2 - x - 40 = 170$.

32. Explain meaning of the expressions $A^{\frac{3}{4}}$ and $C^{\frac{1}{2}}$.

33. What is the logarithm of a number?

34. What is the base of the common system?

35. In making what calculations are they useful?

Technical.

1. Adjustment of level.

2. What error in level and distance would you allow in running a line a mile long? (How in New York City?)

3. How would you carry a line of levels over a stream 1800′ wide if you could not depend on the surface of the stream?

4. Width of road, 30′; radius = 750′; angle at center = 35°; how many sq. yds. paving needed?

5. In grading a new street when would you give levels, and when slope stakes are to be given, when and where would you put them, and how?

6. Give Kutter's Formula for flow.

7. Making and Laying Cement Concrete.—State kinds of cement and sand.

8. Give prismoidal formula.

9. What is specific gravity?

10. What are similar triangles?

11. How proportional to each other?

12. The sides of a polygon prolonged, what is the sum of the exterior angles?

13. How do you pass a circumference through 3 points?

14. How do you inscribe a square in a circle?

15. How do you inscribe a regular hexagon in a circle?

16. What proportion do circumferences and areas of circles bear to each other?

17. Find contents of wedge 20′ x 30′; h = 10′; edge 15′

18. State prismoidal formula. Would you use it in calculating earthwork?

19. Calculate the following sections, cutting being shown by —;

fill by $+$; both being written for the distance from center. Slope 1:1.

0	$+\frac{13}{37}$	$+\frac{3.7}{14}$	$+5.9$	$+\frac{6.0}{22}$
50	$+\frac{16}{31}$	$+\frac{6.9}{10}$	$+3.7$	$\frac{0.0}{15}$
100	$+\frac{14}{28}$	$+\frac{5.3}{12}$	6.0	$+\frac{4.0}{19}$
150	$\frac{00}{15}$	$-\frac{5.7}{8}$	-8.2	$-\frac{12}{27}$

20. How many and what parts of a triangle must be given to find the rest?

21. Two sides and included angle of plane triangle given, how do you find the rest?

22. Two sides and opposite angle of plane triangle given, how do you find the rest?

23. Express algebraically the value of sin., cos., tg. and cotg. of angle A in terms of a, C, c, being the altitude, base and hypothenuse of the triangle.

24. What is the law of gravitation?

25. What do you understand is the difference between gravitation and gravity?

26. What is the law of falling bodies?

27. Express velocity, space, etc., algebraically.

28. What is the center of gravity of a body?

29. How is it found?

30. What is the center of gravity of a homogeneous body?

31. What is the specific gravity of a body?

32. What laws govern the pressure of liquids at rest?

33. What is the pressure per square inch 10′ below surface of water?

34. What will be the theoretical velocity of discharge per second from a reservoir through a pipe of 1″ diameter, discharging at a depth of 100′ below surface of water?

35. How many gals. of water will be discharged through a pipe 1″ diam. 328′ long, 13½ ft. fall, coefficient of flow .007?

36. State how many men are needed to make up a full party for a survey of a preliminary line of location of a public work, such as a railroad or an aqueduct.

37. State their several duties.

38. How do you set out a circular curved line upon the ground?

39. If an obstacle occurs, state how to pass it on straight lines and curves.

40. The radius of a curve being given and the angle of intersection of 2 tangents, how would you find length of curve, beginning of curve and length of tangents?

41. Describe an engineer's transit and name its adjustments.

42. Describe a "Y" level and name its adjustments.

43. How many kinds of leveling rods do you know of?

44. State how they are graduated so they can be read to 1/1000 ft.

45. Show field notes for transit work.

46. Show form of level book.

47. What are cross sections?

48. How are slope stakes set for excavation and embankment?

49. What is a grade line?

50. What proportion of the breaking weight of a beam would you consider a safe load?

51. With load uniformly distributed, beam supported at both ends, what part of weight may be considered as being carried at the center?

52. Suppose a beam supported at both ends, $w =$ weight; $l =$ length; $b =$ breadth; $d =$ depth; $s =$ brk. weight; express s in terms of rest.

53. Sectional area being the same, which section would be stronger, 6 x 6 or 4 x 9?

54. Make a design for a pair of rafters connected by a tie-beam for a roof 30′ span, showing dimensions for the several parts and the manner of connecting them. State in detail your method of obtaining the several dimensions.

55. How do you apply the principles of the parallelogram of forces in determining the strains in the various members of a structure. Illustrate graphically.

56. What should be the thickness of the top and bottom of a retaining wall 15′ high built to retain or maintain earth? Show your method of obtaining the required dimensions and make a sketch of the wall, showing how it should be founded.

57. A reservoir is to be built; depth of water 10′; if walls are built of masonry show section, etc.

58. Reflection and refraction—how do they affect the line of sight through a level?

59. Properties of cast iron, wrought iron, iron and steel, tests and their uses in engineering structures.

60. Bearing power of different soils.

61. Different methods of sheet piling.

62. Methods of driving piles and specifications for a good pile.

63. Retaining wall. When does earth pressure act?

64. If backing of earth horizontal where should resultant pressure pass?

65. Where should line of resistance in arch fall?

66. What is an arch? How many forms and of what may it be constructed?

67. Can you state how you would find the thrust of an arch of stone, span and rise being given?

68. Define the intrados and extrados of an arch.

69. Where should the line of resistance to pressure be found in an arch in order to retain its stability?

70. Can you find the thickness of abutments, the rise and span being given?

71. In a semi-circular arch where is the horizontal thrust greatest and where least?

72. Name the common kinds of stone used in buildings.

73. Define the terms quarry-faced, rough-dressed, pointed, fine-axed, bush-hammered, as applied to dressing a stone.

74. Describe rubble masonry, ashlar masonry and broken ashlar masonry.

75. What are headers and what stretchers?

76. What should be proportion of headers and stretchers?

77. How would you prepare the foundation for a heavy wall, and how deep should foundation be excavated?

78. How are walls founded on soft or yielding material?

79. Describe good quality brick, and state how you would know good from poor brick.

80. In how many courses is brickwork bonded to make good work in heavy walls?

81. What is hydraulic cement, and how many kinds do you know of?

82. Which do you consider the better quality, Rosendale or Portland, and why?

83. What is mortar composed of, and how mixed?

84. What kind of sand should be used and how do you test quality?

85. What is the meaning of the word setting as applied to cement?

86. Name common kinds of wood used in building.

87. What kind of timber resists decay longest underground?

88. How may timber be preserved from decay?

89. What do you understand by the term limit of elasticity as applied to a beam under strain or pressure? What is meant by the neutral axis of a beam?

90. What is tensile strength of good quality wrought iron per square inch?

91. For what parts of a structure may cast iron and wrought iron be used, in reference to tension and compression?

92. Make a sketch of a cast-iron beam best form to resist a transverse strain.

93. Describe the operation of cleaning the aqueduct.

94. State how the daily flow in the aqueduct is gauged.

95. In the application of Kutter's formula in computing the flow, state what the value of the coefficient (n) is found to be.

96. In computing the flow in an open channel by the ordinary formula, what quantities are required?

97. Suppose a dam is to be founded on rock, and on uncovering the rock it is found to be seamy, with water coming up at points, what would you do?

Suppose there is a spring with considerable head, what can be done?

98. Draw a section of an earthen dam to cross a valley; the depth to rock below surface of the ground being 25 feet at the deepest point and the water surface to be 30 feet above the ground at the same point. Give every detail to produce a safe work and give reasons.

99. What can be done to prevent water from following along a pipe which passes through a reservoir wall?

100. In laying up a masonry dam, what measures as to shape of stone, cutting same, bonding same and laying same will tend to make the tightest work?

101. (a) Describe the material which makes the best puddle. (b) Suppose you have to prepare puddles from materials on a work, how would you do it?

102. (a) What do you understand by the "angle of repose" of earth? (b) At what point above the base of a retaining wall will the

center of pressure from earth behind it be located, when the earth is level with the wall?

103. (a) Within what limits of the base of a retaining wall on the abutments of an arched bridge must the line of thrust fall to insure stability? (b) What proportion of headers to stretchers should be used in an abutment wall?

104. About what pressure per square foot can safely be placed on the following materials in founding structures upon them where liable to be continually wet: (a) Stiff clay; (b) loam; (c) gravel? (d) In taking loamy earth from a "borrow-pit" what difference in cubical contents will there be between the "borrow-pit" and the complete "fill"?

105. What is the difference between "refraction" and "reflection" as applied to light, and how does the former affect the line of sight taken through a level?

106. Suppose you had in leveling to take 1,600 feet sights, how can you do it with reasonable accuracy and eliminate the effects of "curvature" and "refraction"?

107. What are the several qualities of cast and wrought iron and wrought steel which make them useful for various classes of engineering construction? Describe briefly the work for which each is well adapted, and in general the methods by which you would test them.

108. Outline briefly the essential points to be covered in specifications for timber piles and pile driving.

109. Describe the various ways of shaping and using sheet piling to keep water from work in progress.

110. What is hydraulic cement, and what are its properties and uses as regards engineering construction? Outline briefly the method by which you would test the relative merits of two or more samples.

111. Show a form for monthly estimate to a general contractor, including at least six items of material or labor on some kind of city engineering construction; assume quantities and set unit values agreeing approximately with present market values; carry out the arithmetic, show percentage retained and previous payments.

112. Make up triangulation party and duties of each.

113. What is a well-conditioned angle, and why is it better than any other?

114. What is the system of setting monuments in New York City?

115. Given a series of levels, how would you check reduction of them?

116. How would you inspect the field notes of a closed survey to find out if the survey was well made?

117. Give a page of your notebook for 50′ street in excavation, showing ground transversely sloped 1½ to 1.

118. By careful work the sum of the foresights exceeds the sum of the backsights by ½ minute. What error is there in the work, and would it be plus or minus?

119. Will earth measurements in place be the same when placed in embankment? What difference and which is greater?

120. Show sketch of retaining wall, giving angle of repose of soil, prism of maximum pressure, when pressure is applied, and when resultant pressure should pierce base; relation between the angle of repose and prism of maximum pressure. (When is computation uncertain?)

121. Give sketch of retaining wall 30′ high, such as you would advise to build.

122. Show how sewer should be constructed across marshy or uncertain foundation.

123. In designing a sewer and during construction give all the information you can think of to insure proper cleaning of it.

124. Give all precautions to insure water-tight work.

125. In making masonry wall of dam give all precautions to insure water-tight work.

126. In a double-pitched roof composed of 2 rafters butting against walls, 25′ rise, 14′ rafters, 3′ c. to c., weight of roof 12 lbs. per sq. ft. and snow 20 lbs. per sq. ft., give thrust on each rafter and horizontal thrust at each wall.

127. Find pressure of water on gate 2½ x 3′, upper edge 5′ below surface of water.

128. Give Chezy's formula and explain terms of it.

129. Show intersection of streets paved (diagonally).

130. If instructed to locate a pier of H. R. R. R., state how you would take soundings and locate same.

131. How may the storage of cement affect the stability of masonry for hydraulic construction?

132. Give in detail the proper method of storing cement. Give reasons.

133. What do you understand by the mechanical examination of sand, gravel and stone?

134. Describe in detail the method of making and placing concrete to insure water-tight work. Give reasons.

135. Make sketch showing method of joining the foot of a slope paving of a reservoir with the bottom of the reservoir.

136. How would you determine the proportion of sand, cement and broken stone to make best and most economical mixture? Give the method you would use of determining the voids in broken stone and sand.

137. Write a clause of specifications for the stone, cement, sand, etc., and the method of laying rubble masonry for hydraulic work. Also the size of stones, bonding, etc.

138. What are the objections to grouting in laying up a wall?

139. Describe in detail from the time of picking up stone with derrick to the time the stone is finally laid in wall, of placing a large stone in rubble masonry work for hydraulic masonry.

140. Suppose you found quicksand in laying up a core wall, how would you make foundation safe?

141. When will quicksand make a good and stable foundation?

142. (a) How would you measure the amount of water flowing in a brook, and also in a large, deep river? (b) Show form of weir to give perfect end contraction; state length of approach, height of water, form of crest, etc.

143.

$V = C\sqrt{RS}$. What factor in this formula is affected by Cutter's formula? What factor in Cutter's formula is affected by surface in contact with water? What is the approximate value of n in Cutter's formula for aqueducts?

144. What is the average run-off, maximum run-off, minimum run-off for streams in the vicinity of New York? What is the average rainfall, maximum rainfall and minimum rainfall for same? How many years in succession do the dry or low cycle years extend? Give data approximately for the run-off of this period of years.

145. In laying up the embankment of a reservoir on the earth without digging to rock foundation, how would you treat the natural earth before beginning the embankment?

The following eight questions were given at an examination for engineer of highways:

146. Macadam Roads.—How are the foundations made through swampy ground, light sand, loam, woods, tenacious clay and soft ground too deep to excavate? Describe in detail.

147. What are the relative merits and demerits of the following materials for macadam roads: Trap, granite, shale, sandstone, silicious rock, limestone, river pebbles (a) for foundation course; (b) for binder course?

148. Write a brief specification for rolling the binder course for a macadam road.

149. State in details as regards size, shape and quality of the different ingredients for making a first-class macadam road.

150. In what three ways do macadam roads fail? Give a remedy for each.

151. When should a Telford be made for a macadam pavement, and how should ingredients be placed?

152. Write specification for concrete arch culvert, giving full instructions from beginning to end of the work.

153. Write specification for a first-class Portland cement.

The following fifteen (15) questions were given at an examination for engineer-inspector:

154. State all the ways in which you would examine the notes of a closed survey to test its accuracy, both as to angular and linear measurements.

155. What is the fundamental requirement in all foundations to insure equality of settlement in all parts?

156. (a) What points have to be considered in determining the bearing power of piles in any given case? State this fully and clearly. (b) State the circumstances, if any, where other than vertical forces must be considered.

157. Aside from construction and the wearing surface of streets and roads, what important considerations govern the design and execution of such works to insure permanency and freedom from unevenness in wear?

158. Suppose you were ordered to make hydrographic survey for several piers on the North River side of the city; state clearly how you would lay out the work, what you would do, and how you would do it. Illustrate by sketches if you so desire.

159. (a) Describe the method of making a survey of a reservoir site. (b) State clearly the method of computing the cubical contents of such reservoir to the flow line.

160. (a) The radius and length of tangents given, show how you would lay out a simple railroad curve by use of transit instrument. (b) Show at least one way of doing the same by use of chain or tape only.

161. A culvert to care for certain surface drainage is to be built under a road; how would you determine the necessary site?

162. Describe the proper construction of a weir for measuring the flow of water, and the precautions to be observed in its use.

163. Suppose the upper face of a dam to be vertical, (a) state at what depth the center of pressure of the water would be located; (b) state what the pressure per ft. of length of the dam would be, taking the depth as a.

164. What other ways are there for measuring the flow of water than by the use of weirs?

165. (a) Describe the action of the internal forces in a beam under transverse load. (b) Give the relation between a central load, P, and the unit strain, s, on a beam of length, l, depth, b, and width, w.

166. Describe what is meant by the term "shear" in computing the strength of bridges and how it is determined.

167. In riveted work the bearing value and the shearing value

have both to be considered. (a) Describe the cases in which each may be the governing element in computing the strength of the work. (b) About what values would you take for each?

168. Why is it desirable that the center of pressure in masonry (as in the keystone of an arch) or in a foundation (as that of a retaining wall) should not lie outside of the middle third of the joint? Is this condition necessarily fatal to the work?

The following fifteen (15) questions were given at an examination for superintendent of dam construction:

169. Suppose a temporary earthen dam is to be built upon the ground, would any preparation of the surface be made in advance of the fill, and, if so, what?

170. What is a puddle wall? What is it used for, and how is it constructed? (Describe clearly and fully.)

171. For what purposes and in what locations is sheet piling used? (b) Describe the several ways in which it is made. (c) Describe the several methods pursued to keep the piles in close contact while driving.

172. Suppose seams to be found in the rock in the last question, with water flowing from it at a low head, what would you do? (b) Suppose the water to flow with considerable head, what would you do so as not to have it affect the masonry?

173. Suppose a masonry dam is to be founded on rock, and the rock is uncovered, what is next to be done?

174. Describe everything to be done and precaution to be taken in building an earthen dam to get the most impermeable job.

175. What pipes are to be carried through an earthen dam? State everything to be done to prevent leaks along the pipe?

176. Where a dam is to be built of large blocks of rubblestone, state how you would proceed (a) to secure the best possible adhesion of the mortar to the surface; (b) to insure that the beds and joints shall be completely filled with mortar.

177. Describe clearly the character of the bond you would require in such work.

178. In laying out stonework in a dam, how much space (measured along the wall) would you require between the joints in the course below and those in the course you are laying; that is, should it be equal to the rise of the course you are laying, may it be more, or should it be less? Give your reasons.

179. Give the rules you would adopt where water-tight work is required, (a) for mixing and placing of concrete and its subsequent care, (b) for joining new work to old.

180. Describe the careful inspection of the following materials: (a) Bricks for hydraulic purposes; (b) sand; (c) cement (by a rough

and ready test); (d) stone; (e) timber for sheet piling; streets in excavating, etc.

181. Describe the proper way of doing the work of laying a heavy, water-tight brick wall.

182. In the construction of dams, (a) for what purpose may cribs be required; (b) describe the construction and sinking of a crib.

183. In the construction of a retaining wall, what may be done in the design and in placing the filling behind the wall to diminish the danger of overturning?

The following fifteen (15) questions were given at an examination for **Assistant Engineer** (General) in the Board of Water Supply (October 12, 1905):

Note.—Elaborate discussions are not required. Clear, concise answers covering the essential points will receive highest rating.

184. State (without describing in detail) the several adjustments, in their proper order of (a) the surveyor's transit; (b) level.

185. Explain briefly the method of stadia survey and show a form for notes, filling in such as are taken in the field and leaving blank those which are worked out in the office.

186. Explain how a drainage area is determined.

187. (a) How is the total rainfall for any area determined? (b) How the available rainfall? (c) State the full information for both necessary for purposes of securing water supply.

188. Describe briefly the several methods of measuring the flow of water in streams.

189. Name the watersheds available for supply of New York City and state briefly the advantages and disadvantages of each.

190. (a) Give the Chezy formula and explain its terms; (b) in what does Kutter's formula modify it?

191. State the causes of failure of retaining walls and the precautions necessary to prevent failure.

192. State briefly the important points regarding mixing (by machine) and placing heavy concrete masonry.

193. Describe briefly the several methods of sinking wooden piles and the conditions under which each would be used.

194. Discuss briefly the methods of handling quicksand in constructive work.

195. Draw a section of an earth dam, the depth of rock below earth surface, 20 feet; the water surface to be 25 feet above the ground. Show every detail to produce safe work.

196. Explain briefly the method of tunneling through rock.

197. Same through soft mud below water.

198. State the prismoidal formula and show by sketches its use in calculating the contents of an earth embankment.

The following fifteen (15) questions were given at an examination for **Assistant Engineer** (Intermediate) in the Board of Water Supply (October 17, 1905):

199. Describe making of an accurate triangulation survey.

200. State principles of (a) pressure of water; (b) of the siphon.

201. Discuss the use of steel to reinforce concrete.

202. Explain the method of designing a steel water tower 100 feet high and 40 feet diameter.

203. Outline the specifications for brick, mortar and work of construction of a large brick conduit.

204. Show by sketches and describe the method of carrying a large conduit over a marsh.

205. Describe clearly the method of accurately determining the character of foundations for an important dam.

206. Explain in detail the most accurate way of gauging the velocity of a stream.

207. Explain the method of designing a masonry arch viaduct to carry an aqueduct.

208. State what you know regarding evaporation from water surface, from snow and from earth.

209. State what you know regarding sedimentation in reservoirs and its prevention.

210. State important points to be considered in construction of an open canal for carrying water supply.

211. Describe theoretical design of masonry dams.

212. State the important details of construction of a heavy masonry dam; features peculiar to concrete.

213. Show proper form for intermediate monthly estimate; clearing and grubbing, with reasonable cost price; earth excavation (convenient waste); concrete 1-3-5 in masonry; rock excavation (quantity); random coursed rubble; sheet piling in place.

The following ten (10) questions were given at an examination for **Assistant Engineer** (Advanced) in the Board of Water Supply (October 17, 1905):

214. Describe how the flow of water in circular pipes is affected by friction, stating the various authorities and assigning values for each and quoting your authority.

215. Explain how to ascertain the time required to empty a reservoir.

216. State all the important points which should be considered in the choice of a reservoir site for gravity water supply.

217. Design an "aqueduct" section conduit of concrete, reinforced with steel.

218. Describe fully a system of water filtration suitable for a large city.

219. A stone arch bridge must take the drainage of 5,000 acres. How many cubic feet per second must be carried?

Use formula $Q = cy\sqrt[4]{a^3S}$, in which $c = .50$; $y = 1.5$; $S = 12$.

220. Assume bridge semi-circular for 50 ft. roadway. Design opening and show by sketches approximate construction.

221. State briefly the successive steps necessary to secure information regarding availability of a watershed for purposes of water supply.

222. Explain the theory of rain and that of the amount of rainfall as affected by wind and mountain ranges.

223. Discuss the important points to be considered in the construction of a large distributing reservoir, as is proposed near Yonkers.

The following questions were given at different examinations for **Assistant Engineer** in the Board of Rapid Transit (now the Public Service Commission):

224. Retaining wall. Give cross-section, angle of slope and maximum pressure. Give the angle of forces and state where the line of pressure should pass for stability.

225. Arrange soils according to thrust, starting with the one giving the least thrust.

226. Application of pressure to a wall in case of water. Find total pressure.

227. What is hydraulic radius?

228. Resistance in pipes.

229. Limit of elasticity.

230. Failure of riveted joints.

231. Degree of curve.

232. How is a curve laid out if you know degree of curve?

233. How would you locate property in surveying for a tunnel through a street?

234. How would you locate transit points?

235. How close should levels check in running a mile in a busy street.

236. What instruments are used in surveying for a tunnel?

237. Field book for a tunnel where line runs under 2 blocks closely built up.

238. Difficulties in surveying through busy streets.

239. In building a retaining wall where there is a superimposed bed of earth and where the earth is dry and stable, the load might be far enough back of the wall to have no effect upon the thrust. About how far in terms of the height should this distance be and why?

240. In what ways do retaining walls fail? Give conditions.

241. When a trench is made to the right depth for wall, what is the next step?

242. In formulas for flow of water in open channels, how is the fall in water surface taken account of?

243. Suppose a tank of a given height and diameter to be filled with water. How would you determine the tensile strain on the hoops which resist the pressure on the lowest part of height.

244. Make a sketch of an iron beam uniformly loaded and show the reaction at each support in terms of weight d, and the moment at the center.

245. State as far as you can the difference between long and short columns and the way each fails.

246. Where two tangents are to be joined by a curve state what field notes are taken and in general the way the elements of curve are determined.

247. A shaft having been excavated on the center line of a (subway) tunnel, how would you transfer the line from the surface to the bottom?

248. What would you consider the best way of fixing C. L. and grade in the tunnel?

249. In building a sewer how would you leave the unfinished brickwork over night?

250. Give sketch, carefully drawn, with dimensions of parts of a center for the support of a heavy brick arch 20′ span.

251. A column is to support a heavy load and to rest on a stepped granite foundation. Suppose the blocks to have a rise of 1′ each and to be 3 in number, the top one being 18″ square. Show how you would make sizes below.

252. Precautions to be taken in laying brickwork where great strength is to be required. Give a report of rebacking and refilling a pavement.

253. What are the precautions necessary to accurately measure a line such as a baseline for a triangulation survey?

254. What are the important points to be observed by a leveler in doing accurate leveling?

255. Same for rodman.

256. State, in their order, the adjustments of a transit which also has an attached bubble for leveling.

257. Except for base line measurements, describe how an important triangulation would be done across the East River, for purposes of tunnel construction.

258. In the designing of coursed ashlar masonry retaining walls, state briefly the theory of pressure and the methods of calculation.

259. In the construction of the same, state the precautions necessary to minimize the pressure and to secure sound, permanent work.

260. In concrete work, what are the important points (a) in fixing the proportions; (b) in mixing, and (c) in placing; all for high class, heavy work?

261. Outline briefly the principle involved in the reinforcement of concrete with steel.

262. Show by sketches with dimensions a pile foundation for heavy masonry walls.

263. Describe briefly a good roadbed and track construction suitable for the Rapid Transit subway.

264. State the prismoidal formula and illustrate its use by cross-sections of an earth railroad embankment.

265. Show an intermediate, monthly. contractor's estimate covering open cut earth and rock excavation, steel built columns and beams; use approximate ruling prices.

266. Describe the method of tunnel construction in gravel and clay, 50 ft. below water level.

267. Show by sketches giving general dimensions and sizes the method of construction of the lumber work of a double tunnel shaft, 8 ft. by 20 ft., 60 ft. deep in earth.

268. As applying to rapid transit subway construction, describe how you would transfer a bench mark from a point 200 feet off the line to and down a tunnel shaft to a point 400 feet within the tunnel.

269. The same for the transfer of a transit line to a P. C. in tunnel.

270. Show by sketch and describe a typical, vertical cross-section of a double-track tunnel, where it is necessary to keep close to the surface, including important dimensions and indication of materials used.

271. Show by plan horizontal cross-section just above track, the arrangement of answer to (270), assuming a curve to tangent at a right angle.

272. State, or show by sketch, why side clearance on curves in tunnel must be greater than on tangents, and calculate how much for 60 ft. cars, with 40 ft. center of tracks.

273. Describe how the standard railway track is best maintained in line and grade.

274. Describe "damp-proofing" and the methods of application.

275. Describe the method of transmission of electrical power by a third rail, stating briefly its advantages and disadvantages.

276. What are the important details of mixing and laying concrete to secure good work? (This does not refer to proportions of materials.)

277. Assume data you consider reasonable and determine the external pressure upon a steel tunnel tube 12 feet diameter, running beneath 100 feet of water, but supported above bottom.

278. Outline specifications for stone and laying of first-class ashlar granite masonry in a heavy retaining wall.

279. State briefly what you know of concrete-steel construction and its advantages.

280. How is condensation prevented on station walls and roofs?

281. State everything necessary to secure the best cement mortar joints in brickwork, such as important sewer construction.

282. Show by sketches and describe the method of sheeting a tunnel shaft (vertical) 12 feet wide, and 60 feet deep through earth.

The following ten (10) questions were given at an examination for **Assistant Engineer** in Buffalo, N. Y. **(February 26, 1898.)**

Answer questions on blank paper provided, not on this sheet. Make sketches wherever it is possible to more fully illustrate the answers.

283. State the foundamental law or equation upon which the flow of water in pipes, conduits, streams, etc., is based, and what modifications in it are necessary when applied to particular cases.

284. Give your reasons for the use of an oval sewer.

285. (a) Describe the combined system and the separate system of sewerage and name some advantages and disadvantages of each. (b) State under what circumstances it is well to adopt the combined system. (c) Give a general description of the sewerage system of one city or town using the combined system.

286. What is an intercepting sewer? Name two classes of intercepting sewers and describe the use of each. Design the cross-section

of an intercepting sewer to take the sewerage of an urban district of 150,000 inhabitants, the available fall being 1:2000.

287. (a) Make a sketch for a catch basin for a paved street. (b) Describe the usual method of ventilating and cleaning sewer. (c) What is the cause of explosions that sometimes occur in sewers? (d) What is the so-called sewer gas?

288. (a) Write a brief specification for bricks to be furnished for sewer work. (b) Write a brief specification also for Portland cement; for natural cement; for sand, for gravel or broken stone to be used in concrete.

289. Describe the preliminary work necessary to prepare a common dirt road to receive a first-class pavement.

290. (a) What are the advantages and disadvantages of different kinds of pavement in this city (Buffalo)? (b) In naming different kinds of pavements, place them in their order of merit for use in the City of Buffalo.

291. (a) What do the terms "positive" and "negative" in electricity mean, and what is meant when a water main is said to be "negative" to a rail in the street railway track above it? (b) To prevent electrolysis of water pipes, should the pipes be "positive" or "negative" to the rail, and why? (c) What do you understand "electrolysis" to be?

292. It is proposed to extend a street in South Buffalo across the river. The subsoil is a light, bluish clay, with considerable quicksand in it. It is not known at just what depth rock will be found. The foundations must be provided to carry the load of the piers and bridge of 150′ clear span, with its load. Total width of bridge = 50′, including sidewalks 5′ on each side. Across the bridge will run a double-track electric street railway. Write a description, stating fully the manner of constructing the foundations of this bridge, assuming that rock will be found 50 ft. below the level of the water in the river. Also method of building foundations, if rock proves to be 100′ deep, with same kind of material for laying it. Specify as fully as time will permit the masonry for the piers. Write a general specification for the steel superstructure, giving sufficient data and directions for the bridge company to make proposals. The bridge company to manufacture the steel structure only, to erect it, leave structure complete and ready for use. The street railway company to lay its own tracks.

It is desired that the question will be answered very fully, describing, as far as possible, not only the design of the structure, but the character of the material and the labor which may be put into it.

The following questions were given in Atlanta, Ga.:

293. (a) What are the main elements governing the design of a sewerage system, other than the method of disposal, of flushing and ventilation? (b) Describe in detail an ordinary method of giving lines and grades for large water mains or sewers. (c) Name three common methods of driving piles and state under what conditions each may be used.

294. Describe in detail the duties of an inspector on a public work.

295. (a) A circular steel bar, 4″ diam. and 20′ long, equals in weight a hexagonal bar 30′ long; find side of hexagon to nearest 1/100 of an inch. (b) Show a form of field notes used for running curves and a form of level book.

296. (a) What are proper proportions of sand, broken stone and cement in a concrete (a) for building foundations; (2) an abutment under water; (3) a sewer arch.

297. (a) What is the office and influence of a waste weir. Discuss the conditions which give length of overfall. (b) Indicate by sketch some common form of weir cross-section.

298. Write a specification for a concrete suitable for one of the conditions of 296.

299. (a) Enumerate the precautions that must be taken in order to obtain the best results in leveling with a "Y" level and ordinary rod.

REPORTS.

Assume reasonable facts and write a suitable report of not less than two nor more than three pages on any one of the following subjects:

1. The general availability of the watershed as a source of supply for New York City.

2. The Hudson River as a possible source of supply.

3. A proposed filtration system.

4. A proposed storage reservoir adjacent to New York.

5. The comparative merits of a steel reinforced concrete conduit and a brick conduit.

6. Possible methods of reducing water consumption in New York.

7. Make a report of not less than two pages nor more than four pages on the exploration of an unexplored stream for water supply purposes, entailing the construction of a dam for storage purposes; discuss the proposed site for the dam, the amount of water available; discuss also a period of low water years in which the supply of the stream is inadequate to supply the demand, and method of further increasing the supply by additional storage.

To pass—75% on technical paper; 70% on entire exam.

8. Report of at least two pages. Location and design of some public work with which you are acquainted, giving the reasons why the design was adopted in the form given. Also give the progress of the work up to some given time, containing such items as a progress report to your chief ought to contain.

9. Write a report of not less than three, nor more than four, pages, covering the examinations and method of shoring and placing in sound condition a building which has partly settled, due to open trench excavation of rapid transit tunnel.

10. Write a report covering at least two pages on the location and design of some public work with which you are acquainted, giving the reasons why the design was adopted in the form given. Also give the progress of the work up to some given time, containing such items as a progress report to your chief ought to contain.

11. Write a report on your examination of a valley for the best location of a dam. Describe fully the examination made, the result of the examinations, and your reasons for the location you have made.

12. Make report on trench sewer, part of which is in rock, unknown material and salt meadow to its outlet. Make detailed report.

SCIENTIFIC AND TECHNICAL REQUIREMENTS.

All what precedes (see Book I., The Axeman; Book II., The Chainman; Book III., The Rodman; Book IV., The Leveler; Book V., The Transitman; Book VI., The Inspector, and Book VII., The Draughtsman). Also Water Supply, Surveying.

WATER SUPPLY.

Amount of Water Per Capita.—From 100 to 120 gals. a day per person is more than ample for all family purposes. It has been computed that, were it not for waste, an average of 44 gals. would meet all requirements.

Sources of Waste—Willful waste, defective plumbing, leaks in service pipes; and of these the first is the greatest.

Parts of a Public Water System—Storage of water, conduction, reception and distribution.

STORAGE OF WATER.

Total Amount of Water Required.—Draw the curve of population of the town from all the census figures obtainable and continue it for a period of, say, 20 or 50 years to ascertain what may be the population 20 or 50 years hence. Multiply by 100 (or 120) and you will have the daily consumption to be provided for. If the figures of only two or three censuses are known, a simple calculation will answer.

Amount of Water Storage Required—Storage for 4 or 5 months' supply seems necessary.

Sources of Water Supply—Springs, underground water, artesian wells, surface or rain water.

Springs—A little basin, square, oblong or circular, may be built at

each spring. Pipes of sufficient diameters to carry the flow lead the water to a main or to a reservoir. The basins are made deeper than the invert of the pipes to allow for sedimentation.

Underground Water—A system of trenches deep enough to reach below the wet strata is dug; their bottoms are prepared for pipe drains with open joints or are filled with broken stones before they are filled up. These drains lead the water to reservoirs or to wells, from which it is pumped out.

Artesian Wells (from Artois, a French province, where such wells were first dug)—These are deep wells, bored through all kinds of materials until sufficient water is met, which often comes to the surface. They are expensive, uncertain and generally require pumping plants besides.

Surface Water—The rainwater falling within a certain territory is impounded, together with the water of all the streams flowing within that territory.

Conditions of a Good Surface Water Supply—It should be pure and abundant.

Pure Water—Free from bacteria or other contamination. The water of the several streams intended to be used must be chemically analyzed at different times of the year.

Abundant Water—The flow of rivers and streams must be gauged at different stages of dryness and flood in order to obtain the probable daily output.

Watershed—Area or territory from which the rainwater and that of the brooks and rivers is impounded for purposes of water supply.

Dam—A wall built across a valley for the purpose of retaining water on the upstream side. The face of a dam is on the downstream side. The back of a dam is the upstream side.

Reservoir—Body of water retained by a dam, or by a wall forming a closed circuit.

Purposes of a Dam—1° Storing water for water supply purposes. The reservoir back of it is then called a **storage reservoir.**

2° Regulating the flow of a river, the character of the territory below the dam, or the business done in it, requiring a uniform (or nearly so) supply. The reservoir is then called a **regulating reservoir.**

3° Sedimenting water before it is allowed to proceed further. The reservoir in that case is a **settling reservoir** (or **basin**).

4° Diverting the waters of a watershed into a reservoir in a neighboring watershed, through a weir and channel. The dam is then called a **diverting dam** and the reservoir a **diverting reservoir.**

Distributing Reservoir—A reservoir built within (or near) a city or town, which receives its supply from the impounding or storage reservoirs, and where the pipe lines begin which distribute the water to the houses.

Where a Dam Should Be Built—1° In the lower part of a valley, as near the mouth of the river as practicable, so as to secure almost the totality of water flowing in the watershed.

2° In a narrow gorge where indications point to rock being of good quality and near the surface; this for purposes of economy.

How to Determine a Watershed—If no surveys have been made, Government maps, to the scale of a mile to the inch (near), giving 10 or 20 ft. contours, will be found valuable. The location (prospective or final) of the dam being determined and plotted, run a pencil line from the left end of the dam (looking upstream) normal to the contours and up to the nearest summit. Follow (towards the right) the line of summits and low hills at the head of all streams tributary to the river across which the dam is to be built, until you return to a summit near the other end of the dam, which you then connect by a line normal to the contours. The watershed so determined may be planimetered in sections to obtain the area.

Yearly Rainfall in a Watershed—Multiply the area of the watershed in square feet by the average annual rainfall in feet. The product will be the total yearly rainfall in cubic feet.

If no data are at hand, establish rain gauges at a number of stations in the watersheds, where they will be regularly observed.

Rain Gauge—A U. S. rain gauge is composed of a circular, funnel-shaped receiver, 8″ in diam. leading to the open top of a measuring tube 2.53 inches diam. and 20″ long. The receiver serves as a cover to a cylindrical vessel, also 8″ diam. and 20″ long, used as an overflow receiver of the tube. When the cover and tube are removed it serves as a snow gauge.

It is accepted that 10″ snow = 1″ rain.

A rain gauge should be placed vertically on a firm stand about 4 ft. from the ground.

Observations should be taken after every rain and entered in a book with the data, the duration of the rain, the reading of the gauge and, in the case of the reading being from a tube and not from an open rain gauge, the correct rainfall. It is 1/10 the reading in the case of the U. S. gauge just described.

In any gauge it is given by formula $D = d\frac{r^2}{R^2}$ in which

R = Radius of receiver ; r = Radius of tube ; d = Depth of water in tube as measured ; D = Depth of water in gauge had there been no tube, or actual rainfall.

After a month, or a year, the monthly or yearly average rainfall may be computed.

The continuance of these observations for many years will furnish valuable data.

Table of Average Yearly Rainfall.

Alabama	53.6	Nebraska	26.9
Arizona	10.9	Nevada	7.6
Arkansas	50.6	New Hampshire	44.1
California	21.9	New Jersey	47.3
Colorado	14.8	New Mexico	12.7
Connecticut	46.8	New York	36.5
Delaware	40.8	North Carolina	53.7
District of Columbia	41.8	North Dakota	17.1
Florida	54.9	Ohio	40.
Georgia	51.4	Oregon	44.
Idaho	17.1	Pennsylvania	42.5
Illinois	38.1	Rhode Island	46.7
Indiana	42.7	South Carolina	45.4
Indian Territory	36.2	South Dakota	22.9
Iowa	32.9	Tennessee	50.7
Kansas	31.0	Texas	30.3
Kentucky	46.4	Utah	10.6
Louisiana	53.9	Vermont	42.1
Maine	45.	Virginia	42.6
Maryland	44.	Washington	39.8
Massachusetts	46.6	West Virginia	42.8
Michigan	33.8	Wisconsin	32.5
Minnesota	26.2	Wyoming	11.6
Mississippi	53.	U. S.	36.3
Missouri	38.		
Montana	14.0	All in inches.	

Amount of Rainfall Useful for Water Supply—This amount depends upon the character of the ground. It is not safe to reckon on more than half the average yearly rainfall.

Height of a Dam—Several considerations lead to the determination of the height of a dam, and they are the same which serve to establish the flow-line of a reservoir.

The character of the conduits, whether pipes or masonry aqueducts.

The elevation of the distributing reservoirs, which may be reached, perhaps, simply by an hydraulic grade. The amount of water which it is deemed necessary to impound. The existence of low, marshy flats, which it would not be judicious, for sanitary reasons, to submit to frequent flooding and draining.

Structures That Accompany Every Dam—An overflow weir, a spillway or waste channel and a gatehouse. Conditions may require more than one gatehouse.

Overflow Weir—Depressed portion of a dam which allows the water to escape when it reaches a certain elevation (that of the flow-line) in the reservoir.

The overflow may be within the length of the dam, or at one end, or in another part of the reservoir, such as at a low, rocky col, where it may be economical to let the water escape in a different

valley. When the overflow is a portion of the dam, its section must be that of the dam and stepped, the tops of the steps being large stones. There must also be a wall on either side, separating the overflow from the dam proper.

Spillway—A channel dug in the side hill or through a low hill next to the overflow; its bottom is generally stepped and sometimes wholly or partly paved; it receives the overflow water and carries it to the natural river channel below the dam.

Gatehouse—A structure containing chambers with inlets from the reservoir at different elevations and outlets through pipes or aqueducts. Such outlets are provided with gates for regulating the flow. Each outlet should have a double set of gates, and near each inlet, in a narrow portion of the chambers, there should be provided a double row of vertical **grooves** about 5″ x 6″, distant about 2 ft., into which **stop-planks** may be lowered when it is necessary to clean the chambers, or repair the gates. Clay is rammed between the two walls of planks, thus forming a coffer dam. Draining pipes are also provided below the floor of the chambers with valves opening in the floor. An iron ladder may be permanently put up in one chamber or more. A hoisting apparatus for actuating each gate completes the equipment of a gatehouse.

Inlets—Inlets may simply be openings left on the water side of the gatehouse or they may be ducts extending a long distance into the reservoir.

Overflow Dam—A dam entirely built up to, the elevation of the flow-line. The water falls on the face of the dam, which may or may not be stepped.

Principal Kinds of Dams—Timber dam, puddle dam, core wall dam, masonry dam.

Timber Dam—A crib-like construction, composed principally of heavy planking, close jointed, reaching from the top of the dam to rock bottom, where it may be buried in concrete. This planking is propped up with heavy braces, and these in turn are steadied with horizontal timber. The downstream side is generally built at a very acute angle with the horizontal and covered with heavy planks, closely laid and breaking joint. Sometimes a curved apron is built, extending to a level flooring some distance downstream in order to break the force of the current and protect the toe from being undermined.

The upsteam side is built at a lesser acute angle and is backed with gravel, often topped with large paving blocks to bear the erosive action of the water.

If rock is not to be found, sheet piling must be resorted to on the upstream toe; a double row may be necessary.

Large stones are placed between the timbers of the dam.

'Again, one or two coffer dams may be built, in a system of cribs, up to a certain elevation, where a masonry dam may be erected on them and on the rock filling of the cribs.

Every precaution should be taken to prevent the water from finding a way under the dam.

Puddle Dam—A trench is dug to several feet below the bottom of the reservoir and clay is rammed in it carefully. A clay wall is thus erected while, at the same time, an embankment is raised on each side.

The puddle wall should reach an elevation of not less than 3 to 4 ft. above the flow-line. It is better yet to carry it nearly to the top of the embankment, reserving only space for enough good soil to raise a good lawn, or, if a path or roadway is to occupy the top of the embankment, to sub-grade of the roadway.

Size of a Puddle Wall—Top about 6 ft. thick, gradually increasing to the bottom by offsets at the rate of about 1 ft. offset on each side for every 8 ft. vertical. These offsets bind the wall with the embankment much better than would a regular batter.

Core Wall Dam—Instead of a puddle wall, a masonry core may be built, with a batter on each side of about 1 in 12; width at top about 6 ft., and vertical when below natural ground or when it attains a thickness of about 18 ft.

Embankments are also raised on either side.

Masonry Dam—When a dam is to be more than 80 ft. high above the bottom of the valley it is wise to build an all-masonry dam, with stones of large dimensions carefully laid in cement mortar, breaking joint horizontally and vertically, except in the face and back, where regular courses with headers and stretchers are built above the natural surface for esthetic reasons. Each stone should be brushed and washed before being laid.

Transportation and Handling of the Materials—In order the better to handle the materials used in the construction, cable-ways are generally stretched from one end of the dam to the other. Narrow gauge railroads are also built from the works to the storerooms, quarries, sand and borrow-pits, dumps and alongside the works.

Cyclopean Masonry Dam—A kind of masonry dam in which the coursing, both horizontal and vertical, and the rectangular size of stone blocks are done away with, to be replaced by irregular, very large blocks of stones, as large, in fact, as it is possible to quarry, transport and handle them. The blocks are simply dropped in a mass of concrete, where they take any position which surrounding blocks may allow. A quantity of smaller stones is also dumped in to fill (more or less) the spaces between the larger blocks.

The face and back of such a dam may be cut stone, but they are now more generally made with concrete blocks molded and laid in advance of the filling, with headers and stretchers. Such a construction is evidently inferior to a well-built masonry dam, on account of the low compressive strength (2000) and modulus of rupture (150) of the concrete, of the much greater percentage of joint material which it contains, on account also of the blocks not presenting normal surfaces to the lines of pressure, a fact which certainly produces irregular stresses in the mass. As this kind of construction requires that the bottom layers shall not have set before the next

layers are placed, work must be carried on at a regular and rapid rate of progress. In order to facilitate the handling and depositing of the materials, steel towers are sometimes erected on the site of the dam and the work is carried on from platforms laid on top of the towers, which support mixers, derricks, etc. These steel towers are left in the work, which, by the reinforcement thus received, become partly reinforced.

Cyclopean masonry is more economical.

Triangular Dam Just Balancing Pressure of Water (Fig. 440).

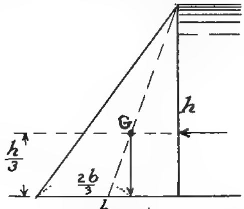

Fig. 440

Data: Apex is at level of water; wet side, **h**, is vertical; **g** = aver, weight of a cu. ft. of material in dam.

Required: Find base **b** level with bottom of water.

Moment of pressure $= \frac{2h^3 \times 62.5}{9}$. *Moment of resistance* $= \frac{b^2gh}{3}$

Hence. $\frac{b^2gh}{3} = \frac{2h^3 \times 62.5}{9}$ and $b = h\sqrt{\frac{2}{3} \times \frac{62.5}{g}}$

With factor of safety $(1+c)$ $b = (1+c)h\sqrt{\frac{2}{3} \times \frac{62.5}{g}}$

Rectangular Dam Just Balancing Pressure of Water (Fig. 441).

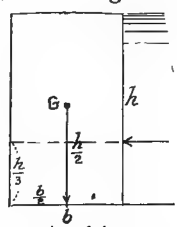

Fig. 441

Data: h = height; **g** = weight of a cu. ft. of material in **dam.**

Required: Find base = **b.**

Moment of Pressure $= \frac{2h^3 \times 62.5}{9}$ *Moment of Resistance* $= \frac{b^2gh}{2}$

Hence. $\frac{b^2gh}{2} = \frac{2h^3 \times 62.5}{9}$ and $b = \frac{2}{3}h\sqrt{\frac{62.5}{g}}$.

With factor of safety. $b = (1+c)\frac{2}{3}h\sqrt{\frac{62.5}{g}}$.

Trapezoidal Dam Just Balancing Pressure of Water (Fig. 442).

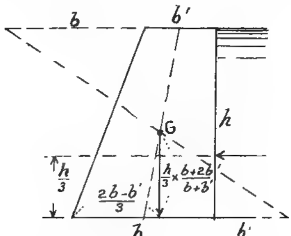

Fig. 442

Data: Vertical side **h**; smaller base **b**, at top of dam and at level of water; **g** = aver. weight of a cu. ft. of material in dam.

Required: Find larger base, **b**, at bottom of water.

$$\text{Moment of Pressure} = \frac{2h^3 \times 62.5}{9} \qquad \text{Moment of Resistance} = \frac{(b+b')(2b-b')hg}{6}.$$

$$\text{Hence: } \frac{(b+b')(2b-b')hg}{6} = \frac{2h^3 \times 62.5}{9} \text{ and } b = \frac{-3b' \pm \sqrt{81b'^2 + 96h^2 \times \frac{62.5}{g}}}{12}.$$

$$\text{With factor of safety} \quad b = \frac{(1+c)\left[-3b' \pm \sqrt{81b'^2 + 96h^2 \times \frac{62.5}{g}}\right]}{12}.$$

Cross-Section of a Masonry Dam (Fig. 443)—It should be so designed that it shall be able, by gravity alone

1° To support its own weight.

2° To resist the pressure of the water impounded.

3° To resist the pressure of ice.

4° To resist the pounding of waves.

These last two conditions affect only the top portion of the dam, say, the upper six feet. Therefore, the top of the dam must have a sufficient thickness, even if it is not to be used as a path or a highway.

A tentative profile is first assumed and drawn. Divide it by horizontals into sections of equal heights. Determine the center of gravity of the first top section, that of the first two sections, of the first three, etc. Apply to these points forces representing weight of dam and pressure of water—both calculated for 1 ft. length of dam—and compose them into a resultant which will show, on the lower joint, the point of application of the line of pressure. Connect these points of application with a continuous curve, which will be the line of pressure of the dam full. If this line passes within the middle third of every joint, the section will be that of a stable structure. If it deviates from the middle third, the section must be modified accordingly.

Depth of Dam Foundations—Generally to good bed rock, which may require blasting of from 5 to 15 ft. of the top, disintegrated

layers. If caverns or large, deep fissures are encountered, they may be arched or domed over. Small fissures are grouted.

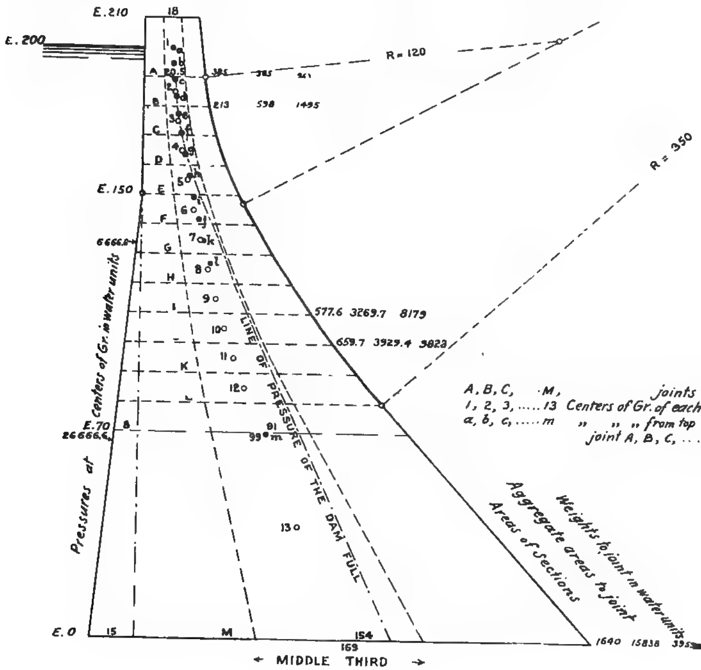

Fig. 443

Protection Against Sliding—Several large pits or trenches may be dug or blasted in the foundations, and these, when filled with masonry, will act as spurs or anchors against sliding.

Embankments—Their slopes may be 2 to 1 or 2½ to 1 on the inside and 1½ to 1 or 2 to 1 on the outside. They should be built in layers of 6 or 8 inches and rolled. A pavement not less than 18″ deep, laid in cement mortar and resting on a concrete bed 6″ or 8″ thick, should revet the inside slope from the bottom of the reservoir to above wave line (not less than 3 ft. vertically above flow-line). From this point up, thence across the top of the bank (say, 20 ft. wide) and down all along the face slope, the embankment should be sodded to prevent being washed off by rain.

This paving may be a double layer of concrete.

Stepped Banks—On the downstream face the embankment should be stepped 2 or 3 ft. every 10 or 15 ft. vertical; and the berms so made, sloped about 1 in. in 5 ft. from the center towards the sides, should form paved gutters leading to a larger gutter or to drain pipes at the junction of the bank with the side hills.

Capacity of a Reservoir—A careful survey of the reservoir site having been made and the whole leveled and cross-sectioned, a contour map is plotted to a scale of, say, 100 feet to 1 inch, which may require several sheets, showing contours every two or every five feet vertical. The contours, up to and including the flow-line contour, are carefully planimetered in sections and reduced to square feet. The average of two adjacent areas, multiplied by the vertical distance of the contours, will give the capacity in cubic feet of the particular zone under consideration. Beginning at the lowest point of available water, which is the elevation of the lowest intake invert, the capacity of the zones is successively added, thus giving the capacity from zero to each contour and finally to full reservoir.

Condemnation of Land—It is evidently necessary to acquire, by private purchase or through condemnation proceedings, not only the lands that will be flooded when filling the reservoir, but a certain strip bordering on the same. A width of from 250 to 300 ft. outside of the flow-line is deemed necessary for different purposes.

Land Maps—Maps of all the parcels intended to be taken shall be made to a uniform scale; they shall show the boundaries, with bearings and distances, the buildings, barns, outhouses, fences, the indication of the crops raised, and whether meadows, woods, orchards or the like. The parcels are numbered in sequence, for indexing purposes, and they show the names of the owners and the area.

When parcels border on a stream or on a highway and no record is found of the exact boundary along the same, the practice is to give each parcel one-half the stream or the highway.

Monuments—It is important that a number of monuments be placed, at intervals, at the principal corners of the reservoir boundary of the taking.

New Roads—All highways within the flooded area have to be replaced by other roads along the margin of the reservoir, and a number of culverts, arches and bridges will have to be provided at each stream, river or gully crossing. These roads should be fenced, on the reservoir side at least.

Private Rights of Way—Private roads or rights of way should be provided, as outlets to the new roads, for all properties left isolated by the taking of lands.

Clearing—Grubbing—The whole space within the flooded area should be cleared of all trees, brush-timber or any material which might decay and endanger the purity of the storage. The strip of 250 or 300 ft. outside the flow-line should also be cleared of timber, on account of the falling leaves in autumn. The roots of trees should be removed, which is called grubbing. Some engineers, however, prefer to leave the timber outside of the flow-line standing, claiming that it partly prevents evaporation and produces a fresher supply.

CONDUCTION.

Conduction is the means of conveying the water from the storage reservoirs to the points of distribution (distributing reservoirs).

Modes of Conduction—Canals, pipes, aqueducts.

Canal—A canal is a trench of regular cross-section and slope designed to carry water.

When a Canal May Be Used—A canal may be used to carry water for manufacturing purposes.

Structures Which Generally Accompany a Canal—Sluice-gates for the regulation of the flow.

Locks for sudden changes in elevation, when boats or conveyances have to navigate the canal. Gates have to be provided at the entrance of the lock and at the exit.

Garages or enlargements of the canal to allow of floats passing one another when going in opposite directions.

Flumes or timber portions of the canal when crossing roads or ravines; if made of a smaller section than the canal, they are built with a steeper grade, so as to accommodate the same amount of flow as the canal in its normal section.

Outlet (with gates) for each factory.

Towpaths to be used by the horses pulling the boats.

Pipes—One or more pipes. Their number, diameter and slope must be such as to enable them to carry the full supply required.

Kinds of Pipes—Concrete Pipes—They may be molded in the trench and require a carefully prepared bed to prevent unequal settling and breaks.

Cast-Iron Pipes—They may be 30, 36 and rarely over 48 inches diameter, with spigot end and bell; joints filled with lead.

Steel Pipes—Of late years riveted steel pipes have been extensively used; some are 5/8 in. stuff, 60 in. diam. and 30 ft. long. Larger pipes have been made.

How Pipes Should Be Laid—With a uniform grade, if possible.

Gate Valves—Should be provided (in duplicate is better) at the inlet of the main pipes and all branches and blow-offs.

Overflows—Overflows should be provided at different points to reduce undue pressure in the pipes. This may be done by pipes connected vertically to the conduit pipes and opening in a reservoir, or waste-weir, at the elevation deemed necessary to limit the pressure.

Vents—If it is not possible to lay pipe conduits to a uniform grade, vents are to be placed at all summit grades.

Blow-offs—Pipes fitted with gate valves, opening at the bottom of the conduit pipes, must be provided in order to empty sections in case of obstructions and needed repairs.

Aqueducts—Although the term applies to all structures destined to convey water, we here reserve it to masonry-built conduits. These

are two kinds, distinguished by their relative position under the surface of the ground:

1° Surface aqueduct.

2° Tunnel aqueduct.

Surface Aqueduct—One just deep enough for the water line to be below the frost line. It is built in an open trench and covered with the materials of the excavation, carefully packed and rammed all around it. It must, therefore, follow the side hills in a more or less circuitous way, which results in bends retarding the flow.

Weak Point of a Surface Aqueduct—The foundations of such an aqueduct will be variable; rock in spots, sand and gravel, shale, clay, etc., which materials will certainly settle unequally, producing cracks in the structure and consequent loss of water, however carefully the masonry is built.

Vents and Manholes—Covered with cast-iron cover. They must be built every mile or half mile for purposes of repairs and water ventilation.

Gate Valves, Overflows and Blow-offs must also be provided.

Drains—Should be laid on either side and outside of the bottom of the aqueduct to carry away infiltrations which might permeate the embankments or the filling and endanger the stability of the aqueduct. These drains empty in the nearest watercourse.

Tunnel Aqueduct—A tunnel is dug on a uniform grade from a certain elevation in the reservoir to the point of distribution. In that tunnel a masonry conduit is built, having the required section, and the space between it and the rough sides of the tunnel is carefully packed with rock debris and grouted, in order to make a solid mass with the surrounding rock. In a tunnel aqueduct rock is more likely to be encountered throughout the greater part of the length and danger of unequal settlement is thus minimized.

The length of the aqueduct is also much shorter than that of a surface tunnel, because it may be carried in a more direct line.

The temperature of the water is more uniform than in a surface aqueduct.

Selection of the Route—This is determined by a number of borings drilled along the prospective line, beginning with the low grounds.

Sections of the Aqueduct—Circular Section—In solid rock of good quality it is generally circular if under pressure.

Horseshoe Section—When the quality of the rock is doubtful and when the aqueduct is partly in rock and partly in softer material, also when it is not under hydraulic pressure, the section generally adopted is semi-circular at the top, with sides drawn on a curve of larger radius (about three times that of the crown), and a curved invert of about the same radius as the sides. It is called a horseshoe section.

This section should be so calculated as to have the same flow as the circular section.

Some aqueduct sections have straight, battered sides.

Materials—Generally brick, laid in cement mortar, with the length parallel to the axis of the aqueduct, the thickness in a plane perpendicular to it showing on the inside, and the width of the brick in the work. Two, three or four concentric rings, according to the size of the aqueduct, are so built. When the invert is on a good foundation—generally a concrete bed—two rings only may form the invert

Special bricks may and ought to be used at the junction of the sides and invert so as to avoid a joint at that place.

The bottom of the tunnel should be drained.

Instead of brick, a concrete ring may be and is often substituted.

Shafts—Every mile or so a shaft, about 8 ft. diam. and brick lined, should be sunk for the following purposes: For ingress and egress of the working force; for working the tunnel in two directions; for disposing of the excavated materials by means of a hoisting apparatus, and to serve as an accessible place for inspection and repair when the aqueduct is in operation.

Should a shaft be in a low location and subjected to hydraulic pressure, inverted arches must be built at the top and bottom of the manholes, closed with heavy and strongly set iron covers.

Head-Houses—A head-house should cover each shaft to protect the machinery, shelter the operators and serve as a toolhouse.

Waste Banks—Enough land should be purchased around or near the sites of shafts to allow of the excavated materials being dumped in waste banks without requiring overhaul.

Gate valves, overflows and blow-offs are to be provided where required.

FILTERING PLANT.

When a Filtering Plant Is Necessary—If the water supply is contaminated by sewage flowing into the reservoirs, or by decomposition of vegetable matter in quantities, such as that produced by stagnation of water in swamps, such water should be filtered and purified.

Filtering—Filtering is the process of clarifying the water, or depriving it of the solid matter which it contains. Filtering is done mechanically.

Purifying—Purifying is the process of neutralizing the effect of the contamination. Purifying is done chemically.

Best Location for a Filtering Plant—An ideal location would be the vicinity of a large stream, the waters of which are not used for alimentation, and the bed of which is at a lower elevation than the bottom of the intended basins of the plant, with rock not far below the surface.

Parts of a Filtering Plant—Boiler house, engine and pump house, mixing tanks (when chemicals are used), diluting basins, filters, clear water basins, and all the piping pertaining thereto.

Chemicals Mixed Before Filtering—If the water is to be purified, the chemicals are applied before the water reaches the filters.

Chemicals Used—Mostly sulphate of alumina. A concentrated solution is first made, and it is afterward diluted to the required degree and thoroughly incorporated by air agitation (from rotary blowers).

Restoring Alkalinity to Water—This is done in times of heavy rainstorms, when such alkalinity is much reduced. Solutions of soda ash are then used. These solutions are prepared in **iron** tanks by the application of hot and cold water. The solutions should contain about 5% of soda ash.

Filter Tanks Not Too Large—Filter tanks should not be too large, for the reason that they have to be cleansed, sometimes frequently; they are then out of commission. It is better to have a greater number and clean them in rotation. A size that has proved satisfactory is 15 x 24 x 8 ft. deep.

Filtering Materials—At the Little Falls filtering plant of the East Jersey Water Co. the bottom layer is 2 in. thick, of broken quartz that has passed screens of 4 meshes to 1 inch and been retained on screens of 8 inches to 1 inch. It is covered with a second layer 5" thick of finer quartz that has been retained on screens of 12 meshes to 1 in.

This is covered in turn by 30" of sand, clean, as uniform in size as possible, and about 1/64 inch in size.

Filtering—The water is admitted on top of the sand beds; it emerges at the bottom, leaving its impurities in the sand.

Cleaning the Filters—Wash water is forced through the sand beds by means of a number of small pipes covering the bottom of the tanks and fitted with strainers, placed about 6¼ in. from centers. having 1/16 in. perforations. When the tank to be cleaned is full, the flow of wash water is stopped and air is admitted, under pressure, through the same system of pipes, which stirs the sand (without disturbing the layer formation) and liberates the particles that may have adhered to the grains, forcing them up, where they float away with the wash water, when, substituting water for air in the strainer system, the level of water in the tank is raised to that of an overflow. The tank is then ready to operate again.

Clear Water Reservoirs—This reservoir (there does not seem to be need of more than one) may properly be located under the filters. which empty directly in it. The water mains connect with this basin and carry the water to the consumer.

Water That Is Not Filtered—For economic reasons, water for fire and manufacturing purposes is not filtered if it can be carried in a different conduction than water for home consumption.

RECEPTION.

Distributing Reservoirs—One or more distributing reservoirs are to be provided in high locations at different points of the locality to

be supplied, and, if possible, on a divide, each controlling the service of a certain district. Their capacity should be sufficient for the storage of an 8 or 10 days' supply at least. This would allow repairs to be made in the aqueducts or gate-houses. A distributing reservoir is a curvette or basin dug to the elevation which is required at the gate-house outlets, with a slope of about 1 inch in 50 feet from that point up toward the center of the reservoir. The sides of that basin are formed either with a core-wall and embankment, with a retaining wall or with a face wall when good rock prevails.

Paving of Bank—As has been explained above, for puddle wall and core wall dams.

Treatment of Bottom—Rock should be dug out to not less than 4 inches below finished bottom; crevices should be filled with concrete and the whole rock covered with a concrete of not less than 4 inches thick (6 inches would be a better job). Muddy portions should be excavated to a good stratum of sand or clay, and refilled with good material from the excavation, carefully rammed. All soft material should be excavated to 6 or 8 inches below finished bottom and covered with a concrete 6 or 8 inches thick. The whole area should be finished with an inch layer of a concrete made with Portland cement and good, sharp sand.

Gatehouses—There should be one gatehouse for the service of each valley and watershed section of which the reservoir is the common apex. Each should have gate chambers with pipe inlets, gate valves, stop-plank grooves, overflow and drain pipes, and a superstructure for the protection of the lifting machinery, the housing of tools and stop-planks, etc.

Pumping Station—If there is not a sufficient head for fire purposes, or for the service of high buildings, a pumping station is to be established of adequate capacity for the service required. A water tower or high tank, in which the water is forced to a sufficiently high level by the pumps, may be built or the water may be forced directly into the service pipes.

DISTRIBUTION.

Combined System—When all water users draw their supply from the same system of mains.

Separate System—When a system of mains is set apart for house use, another system for fire purposes and another for manufacturing needs. This last service may be merged with that for home consumption.

Mains—The principal pipes of the distributing system are of the same diameter as the outlet pipes at the gatehouses of the distributing reservoirs.

Their diameter is calculated according to the population which they are to supply, the rate of consumption per capita and the slope of the pipes.

Branches—Secondary pipes leading from the mains to lateral

streets or sections of the town. Their size is regulated by like conditions as in the case of mains.

Secondary and tertiary branches of diminishing diameters may be used.

For instance, the mains are 48 inches.

Branches may be, successively, 36, 30, 24 or 18 inch pipes.

A branch is rarely less than an 8″ pipe in short streets.

Number of Persons Supplied by a Pipe—Deducting water needed for manufacturing, washing streets, flushing sewers, fire purposes, etc., a pipe with a slope of 1 in 100 of the following diameters will practically supply 100 gals. a day of 12 hours (the supply is generally drawn within a shorter period) to the number of persons given in table below:

48″	diam. will supply	180,000	inhabitants
36″	diam. will supply	87,000	inhabitants
30″	diam. will supply	55,000	inhabitants
24″	diam. will supply	32,000	inhabitants
18″	diam. will supply	15,500	inhabitants
12″	diam. will supply	5,600	inhabitants
8″	diam. will supply	2,000	inhabitants

For a different slope of pipe, express the slope per hundred feet in a decimal number, of which take the square root, and multiply this by the population given in the table.

Mains and Branches Crossing Each Other—They should be connected, either directly or by means of a by-pass pipe. Valves should be provided at intervals for purposes of repairs. The position of the valves, with their dimensions, should be carefully mapped down, and a copy of that map by districts might properly be left in each fire house. The same map might show also the position of fire hydrants.

Coating of Pipes—All water pipes, before being used for water supply, should be coated with a varnish of coal-tar, applied to them hot, after they have been thoroughly cleaned, in order to prevent concretions and therefore reduction of flow and rapid destruction.

Pipe Joints—A packing of rope yarn is rammed around the joint and driven home evenly to the spigot with a calking hammer; a shield of clay is then applied to the open space around the spigot end, and holes left on the sides and on top. Lead is poured by two men from the side holes the better to distribute it evenly; the holes are plugged when the metal nearly reaches them, and the metal is allowed to harden before the filling is completed from the top hole.

After the packing of yarn has been placed, the operation of leading the joint may be made with a special pipe-jointer, which dispenses with the use of clay.

Dead End Sections—If pipes are to be laid in new streets to accommodate future town expansion and are not connected at their further ends with other mains of the system, such dead ends should be shut off from the supply by valves near their points of connection with the mains.

House Connections—House pipes or service pipes connect with the mains with **corporation stops.**

Sizes of Service Pipes—Single house, 2 or 3 stories, ½ in. to ¾ in. Larger house, 1 in. to 1¼ in. Apartment house and large tenements, 1½ to 2 in. Factories, hotels, 2 in., or one size larger if pressure is low.

Stop Valves—A stop valve and waste should be placed under the sidewalk, with access through a removable metallic cover.

A stop-cock and waste should also be placed inside the cellar wall, to be used in case of general repairs.

Sizes of Water Pipes in Buildings—

Wash basin	⅜	to	½	inch
Bathtub	½	to	¾	inch
Foot bath	½			inch
Flush tank (W. C.)	½			inch
Flush valve (W. C.)	¾	to	1	inch
Sink, kitchen	½	to	⅝	inch
Sink, pantry	⅜	to	½	inch
Sink, slop	½	to	⅝	inch

Stop-Cocks—A stop-cock should be provided for the supply pipe of every fixture, to be used in case of repairs to that particular fixture without cutting off the supply to the others.

SURVEYING AND DUTIES.

Who His Subordinates Are—The Assistant Engineer has under him the members of one or more engineering parties.

Engineering Party—It comprises: 1° Transit party. 2° Level party. 3° Topography party (q. v.).

His Duties—

1° His first care is the welfare of his men.

2° He is responsible for the property of the department.

3° He makes requisitions.

4° He directs the work of his men.

5° He studies the plans and projects, the routes, locations and structures before final adoption by the Division or Chief Engineer.

6° He prepares specifications.

7° He writes descriptions of properties and searches the titles.

8° He inspects the contractor's work.

9° He makes periodical reports of progress.

10° He makes monthly estimates.

Engineer's Duties to His Men—Before it is possible or advisable to secure permanent quarters he makes arrangements with hotel men or farmers for the double purpose of properly lodging and substantially feeding the men comprising his force.

When parties are widely separated and move rapidly he may instruct the chiefs of party in the manner of changing quarters and empower them to secure the necessary accommodations.

Payment for board and lodging is generally made by vouchers, which should be promptly paid.

In summer the parties may be camping, in which case he provides the necessary outfit, blankets, utensils, victuals, and secures the services of a cook.

He is responsible for the gentlemanly behavior of his men, a fact which he must impress on them.

Care of Department Property—He sees that the property taken to the field in the morning is returned in good condition in the evening; that the chains and tapes are cleaned and properly coiled; that the rods are wiped dry and laid in their proper places; that the levels and transits are in condition and good adjustment, boxed every evening and not left on tripods; that the instruments were not injured, and that they are returned to the lockers from which they were taken; that the stationery is used only for the purpose intended and in an economical manner.

Requisitions—He sees that the stock of stationery, drawing, cross-section and profile papers, tracing cloth, drawing inks, etc., is not completely depleted before sending a requisition for its renewal.

He should not delay in sending instruments back for repairs.

Field Work—Before detailing the men to any field work the Engineer must become familiar with the territory through which he and his corps will have to exert their labors. He generally precedes them to do a reconnaissance.

Reconnaissance Work—After ascertaining, from maps at hand, the most promising ground where the work intended to be erected may be established so as to meet the conditions imposed, he has a tracing made of that locality and goes over it, either on foot or in a carriage, remarking the peculiarities of the ground, the direction and size of streams, their fitness for water supply, the steepness of side hills, the size and position of towns, villages and settlements, etc.

He generally is accompanied by an axeman, sometimes one or two laborers, when important stations are to be established.

Direction of Field Work—Division of field work, prosecution of surveys, control of notes and mapping, gathering data.

Division of Field Work—He assigns a sufficient number of men to each party and gives every day instructions to be carried out.

He divides the territory to be covered into sections and has surveys made simultaneously in each section, provided his force is sufficiently large for the purpose.

When leveling, cross-sectioning or topography work is lagging, he detaches a transit party to do some leveling or a level party to cross-section or take topography.

Prosecution of Surveys—Consulting the notes every night, he ascertains if the level party is following the transit work with due celerity, and if the cross-sectioning and topography are keeping up with the rest.

Control of Notes and Mapping—He sees that the notes are kept in standard form; that they are plain, clear and sufficient. In the evening, and principally when bad weather prevents field work, he sees that the transitman calculates his traverses and plots his notes, assisted by the topographer; that the leveler checks his levels and prepares the profiles and cross-sections under dictation of a rodman; that the field notes not actually used are neatly copied in ink in notebooks for permanent record by chainmen and that the same are faithfully checked.

Gathering Data—He collects data of rainfall, as to time and quantity, and if necessary sets up a system of rain gauges for the purpose: he has regular observations made of temperature; of depth and flow of streams, principally after rains; establishes measuring weirs; inquires for high stages of water and duration of floods; notes pollution of streams and means to remedy it; inquires into the quality of local building stones, the existence or the possible establishment of quarries in proximity to the works; the possibility of securing laborers in the region, etc.

Paper Location of Works—When the preliminary map is completed he studies it and sketches on it (or on tracing paper laid over it and duly referenced) the lines of routes or the locations of works which appear in his judgment to offer advantages. He makes comparative estimates of the quantities and cost of such works on the various locations shown and writes a report on the same for submission to the Division or Chief Engineer. This report should be accompanied by a tracing showing the location he recommends, together with a profile and cross-sections. When a line has been approved, he has it plotted on the map, with alignments and curves calculated, and then he starts the work of locating it in the field.

In order to make comprehensible, and plausible comparative estimates, he has to study the many questions on hand under their several aspects. For instance:

Excavation, Earth—Is it possible and economical to excavate earth to a natural slope of 1½ to 1? Is there room to deposit (or waste) the material so excavated? If not, a different arrangement is to be adopted, perhaps a vertical trench, which may require sheet-piling. In fact, this mode is practically the only one adapted for excavations in streets for laying pipes or building sewers.

Rock—Rock cannot be blasted exactly to the lines of a structure. It is sometimes specified that only within such lines the rock excavation will be paid for; in this case the bid price is likely to be higher. More generally an allowance is made not to exceed, say, 6 or 9 inches from the work. Such an allowance is necessary in tunnel work to permit of building the lining properly.

On the other hand, too great an allowance will require considerable packing or rough masonry or grouting, which will have to be paid for.

Sheet-Piling—Piles are driven along the line of the work every 8 or 10 ft. and in two rows. Their tops are leveled off and two stringers are laid on them, with a clear distance between them equal to the thickness of the sheet piles. These are driven between the stringers and the piles and in close contact. In coffer dams they may be tongued and grooved.

Timber to Use—The kinds of timber which stand the weather best, or resist alternate conditions of wet and dry states, are fir, aspen and oak.

For **piles under water** fir, elm, chestnut, aspen and oak.

Fir is very lasting and very straight.

Elm shrinks, but is not liable to split, and bears bolts and nails best.

Aspen is as good on the edges as in the center or heart.

Chestnut stands bolts better than oak.

Piling—The sinking of piles close together (from 1 to 4 feet clear apart) over a certain area, which is to be used as the foundation for a structure, as a bridge pier.

Piles—They should be barked, in order to keep insects and worms away. They are from 9 to 12 and 14 inches diameter at the top. The foot may be sharpened and even iron shod. When driven by triphammer the head should be encircled with an iron hoop.

How Driven—The most general method is the use of a steam triphammer weighing from 800 to 1,500 lbs., and sometimes more.

Another method is the use of a capstan and the screwing down of the pile, previously shod with an iron screw.

When to Stop the Driving—When the sinking is only a small fraction of an inch and uniform at each blow.

Safe Load—Trautwine's formula:

$$L \quad \frac{.023\, C w \sqrt[3]{f}}{1+s} \qquad \text{in which}$$

L Safe load in tons;
C = Factor of safety ($\frac{1}{2}$ or $\frac{1}{3}$);
w = Weight of the hammer in lbs.;
f = Fall of hammer in ft.;
s = Sinkage due to last blow.

Sinking Cylinders in Water for Foundations—Two general processes are used:

1° The vacuum process.

2° The plenum process.

Vacuum Process—The cylinder (generally steel) is open at the bottom and closed on top, with a trap door in it, opening outward.

The top is connected by a flexible tube to a vessel at a distance.

The flexible tube may be closed by a cock.

The several steps are the following:

1° The cylinder is placed in position, the trap being open, and the water is pumped out.

2° The trap is closed, as well as the cock in the flexible tube.

3° Air pumps are set to work and draw the air out of the vessel.

4° The cock is opened and the air in the cylinder rushes out to fill the vacuum in the vessel. At the same time the atmospheric pressure acts on the cylinder, pressing it down; it also acts on the material surrounding the foot of the cylinder and pushes some of it inside.

5° The trap is again open, the water again pumped out and men go down to remove the material that has accumulated. The operation is then repeated.

Plenum Process—In this process the top of the cylinder contains an air-lock for the men and one for the material to be removed. Air is forced through pumps into the cylinder until the pressure is sufficient to drive back all the water it contains. This escapes through the foot of the cylinder. When the bottom is dry, workmen are sent down through the air-lock, which they close after them. They excavate and remove the materials of the bottom and around the edge of the cylinder. The men withdraw to the outside and the compressed air of the cylinder is allowed to escape, when the cylinder sinks by its own weight. The operation is then repeated.

In both cases the cylinder should be guided in its descent.

Tunnel—A tunnel should be straight, if possible.

It should be graded or have a slope.

If very long. one or more shafts should be excavated in order to have several points of attack. Shafts are not built for ventilation. Shafts are costly (two or three times as much per foot as tunnel work).

To work a tunnel, a heading is first started 6 to 8 ft. high, 5 to 10 or 12 ft. wide and maintained about 100 ft. in advance of the rest.

This heading is gradually enlarged until the full section is obtained, when the lining is built. The heading and successive enlargements should be supported by timbering, which consists in vertical posts (rough) supporting horizontal cross-pieces or caps, which in turn support planking. The planking is driven in to form the sides and the ceiling of the gallery. The timbering is gradually removed in order to effect the enlargements.

In soft material the heading is along the floor of the tunnel; it facilitates further enlargements. In rock. the heading is just below the soffit of the arch; this facilitates drilling.

The lining may be brick or concrete.

The space between the extrados of the arch, the back of the sides of the lining and the excavation should be completely filled with broken rock, earth and gravel; the cavities are sometimes grouted.

In a tunnel under water the excavation is carried on in an air-lock or shield specially constructed.

Treatment of Quicksand in Foundations—1° Quicksand may sometimes be removed by sand pumps.

2° It may be localized by sheet-piling around it.

3° It may be covered with large bundles of twigs and branches, called **fascines,** intertwined and covered with gravel, stones or concrete. They should extend a long distance, sometimes 100 ft., beyond the footings of the structure.

SPECIFICATIONS.

Specifications state:

1° What work is to be done.

2° How the site of the work is to be prepared.

3° What materials are to be furnished.

4° How the materials are to be prepared.

5° When the materials are to be delivered.

6° How the materials are to be laid.

7° When the work is to begin.

8° How the work is to progress.

9° When the work is to be completed.

No absolute rules can be given for writing specifications. The engineer knows what he wants, how and when he wants it, and he writes these down in as clear and concise a manner as possible, so as to avoid ambiguity and to leave no loophole for evasions on the part of the contractor.

Let the young man get copies of specifications of various works and study and compare them.

Principal Items to Be Specified—The items will relate to the different classes of excavation—earth, earth under water, rock, rock under water, tunnel excavation, the same under water; to the overhaul, refill, embankment, to the protection or economy of excavation; timbering, piling, sheeting; to the finishing of disturbed surfaces; clearing. sodding; disposition of materials; to the kinds of cement to be used, Portland. Rosendale; to the classes of concrete to be used for foundations; mass work under water, under ground, above ground, facing, coping; to the classes of masonry; dimension stone (or cut stone), rubble in cement, in mortar or dry, whether coursed or not. paving, in cement mortar or dry, rip rap; to the facing and finishing of joints; to the iron and steel, whether of stock dimensions or special castings or makes; to fencing. railing, or other enclosure of the finished work or the site of the same; to the removal of rubbish, unused materials, plant. etc., and the leaving of the work in a clean state and ready for occupancy or operation; or such other items as are or may be required.

Descriptions of Properties—A description is prefixed with the number of the parcel and the names of the owners. as, for example: Description of parcel No. 17, said to belong to Paul Jones. The starting point of a description must be clearly defined in order to be easily distinguished and determined in the field by any surveyor. It must be referenced to a monument, if possible, or to a known corner or road crossing, or to a point that will be permanently fixed during the prosecution of the work, as a station or a center line, a plug, etc. Examples: Beginning at a point distant 85.3 ft. from monument 64 on a course N. 53° 24′ W., and running thence, etc.; or: Beginning at a point distant 25 ft. easterly from Sta. 12 + 05.9, as marked by a stake set in the center line of Road No. 3, said distance being measured on a perpendicular to said center line, and running thence, etc., or again: Beginning at the most southerly corner of lands of Paul Jones, as shown by the intersection of a picket fence with the easterly line of the road or highway leading southerly to Bethel and northerly to Tamar, and running thence, etc.; then following the successive courses and distances to the point of beginning; the acreage in each town (if situated in more than one town) and the aggregate area. When the line crosses a natural feature. as a stream, a highway, or when a corner is well defined, as a post, a boulder, an intersection of fence, a monument, etc., mention is made of the fact.

Searches—Searches are first made with the actual owners, from whom maps, descriptions of properties and extracts of title are copied. The engineer inquires as to liens, mortgages and payment of taxes. If further information is necessary, search must be continued at the County Clerk's office, the Recorder's office or any office where such records are filed. There, by means of index books under the titles of Grantor and Grantee, the previous owners may be traced back, the

satisfaction of previous liens, mortgages or judgments ascertained, and the engineer makes abstracts of such facts and copies any portion of descriptions contained therein and of maps referred to. He also ascertains from the proper town officers if the taxes have been duly paid.

Inspection of Work—He is provided with a copy of the specifications and sees that the work under his direct control is carried on according to the terms of that instrument, in quantity, quality and progress.

Excavation—Must be carried on to the proper slopes and with the berms specified, with precautions necessary to insure safety to life and property on account of slides, unprotected or excessive blasts, etc.

When rock is reached at any point, levels must be taken at once. Rock should not be excavated recklessly, but within a specified distance from the lines of the proposed structures and no allowance should be made beyond that distance. Nor should it be excavated below a point where its hardness and its texture are thought to offer a good bed for the foundations.

Embankment—Must be of the required width and the slopes must reach the slope stakes. The elevations must be made true when sinkage has taken place. Heads of embankments or dumps should be rounded with quarter-cones.

Foundation—The bottom is properly stepped up if necessary; fissures are filled with fragments of stone and grouted or arched over; the emplacement is made clean before the foundation beds or courses are laid.

On large works the inspection of construction work is usually severally delegated to inspectors of cement, of concrete, of masonry, of ironwork, etc., but the engineer sees that they perform their duties faithfully.

Reports of Progress—Every month he makes a report of progress —of the surveys and location and later of construction. These reports are to be concise but clear, and are exemplied by diagrams, sketches, profiles and maps, or, rather, by tracings of the same, the originals being preserved in the office of the engineer, where they are duly dated and signed by him before being copied and filed.

Monthly Estimates—At a fixed date every month parties are detailed to take cross-sections of the work. These are plotted in the office and calculations are made showing the total quantity of work done in the several classifications since the beginning of the contract. These quantities are entered on two special sheets and under their proper heads; they show the total work done to date. A second column is used for the quantities of the same classifications which were the totals of the previous estimate. In a third column the differences of the present and last total estimates are written as the estimated quantities for the current month. If the contractor is to receive money for materials delivered (although not used as yet) it should be stated and entered here. These quantities multiplied by the contract prices give the amounts, which are entered in a fourth column; their sum is written at the bottom of that column. Usually a retenue or deduction

of 10% is made; if that be the case, this retenue is written below the total amount, and under it the difference is written, prefixed with the caption, **Amount to Be Paid to Contractor.**

Each monthly estimate is dated and also numbered in rotation. These numbers are used for reference and they also serve as a clue to the duration of the work. The engineer signs one or both of the sheets, keeps one in his files and sends the other to the Chief Engineer.

If he is under a Division Engineer, it is this last officer who signs the monthly estimates.

The engineer is to use all diligence in order to have the monthly estimates prepared and expedited as soon as possible, bearing in mind that the contractor needs his money to pay for his material and labor.

Surveys He Must Personally Supervise—1° The measuring of a base line.

2° Triangulation.

3° Determination of magnetic declination.

4° Settlement of disputed corners and lines by adjacent property owners.

Base Measuring—On an extended work of some magnitude, triangulation is resorted to and precedes surveys. The first operation in triangulation is the selection and measurement of a base line.

Base Line—A line very carefully measured, from the position and length of which the positions and lengths of many other lines are calculated.

Selection of a Base Site—Select a sketch of relatively level and unobstructed ground as long as possible and about equal to a side of the first intended triangle, which we suppose to be equilateral.

Ways of Measuring a Base—1° With steel tape.

2° With rods.

What Tape to Use—An ordinary standardized steel tape with thermometers at both ends, a spring balance at one end, and the ends of the chain marked by shoulders, both pointing in the same direction.

End Monuments—It is desirable that the ends of a base be well defined on an immovable spot; it is also necessary that they should be recoverable. These ends should be taken on solid rock or on very large boulders. A copper bolt is sunk and sealed into the rock and a mark as a dot or a cross is nicked into its head.

Ground Monuments—If no rock is found, stone monuments of large size are set several feet in the ground and below freezing point; bolts are sunk and sealed into them as stated. Careful referencing is made. They are often covered by a glass jar, inverted the more surely to preserve them intact from any motion of the ground. A large peg is set at the surface and over the monument, and a nail with a pointed or nicked head is driven in it exactly on a vertical with the monument.

The Ends of a Base to Be Visible—The ends of a base line must be in open ground and visible from the principal points of the country.

Lining Up a Base Line—Set up the transit very accurately over one monument and sight to the other. Interpolate large plugs a little less than a chain length apart. Nail a piece of zinc about 2" x 2" on top of each plug and draw a fine line in a direction perpendicular to the base. The plugs are "shot" in line with the transit and their tops are not less than 6" from the ground.

Chain Supports—Interpolate stakes between the plugs at a distance of 20 feet from one another, with one face on the line, and into this face drive a nail to support the chain, or suspend hooks for that purpose. The nails must be on the line of the plug tops.

Measuring a Base with Tape—The tape is laid flat on the nails or hooks. The rear chainman plumbs up the zero of the chain over the monument, while the head chainman gives a pull of 12, 16, etc., lbs., as agreed, and a third man reads the tape at the mark on the first plug and measures the portions of a tenth with a steel ruler to 0.001 of a foot; a fourth man records the distance, notes the readings of the two thermometers and the angle of the slope if the line is not level. The measurement is continued in the same manner between the successive plugs to the end of the base at the other monument.

Length of the Base Line—Each distance is corrected for expansion, sag, stretch and slope. The sum of the corrected distances is the length of the base line. The base may be measured a second time reversely, when the average of the two corrected lengths will be a better approximation of the true length of the base.

Permissible Error in Base Measurement—The permissible discrepancy or error is

$$E. = \frac{\text{Length of Base}}{100\ 000} \quad \text{or about 6 in. in 10 miles ,}$$

and this approximation is sufficient for all practical purposes.

Use of Rods—For very extensive works, as the mapping of a state or a coast line, etc., the base line is measured with rods.

Materials of Rods—Brass, steel, wood, glass. Metallic rods are very sensitive to sudden changes of temperature; they are affected even before the attached thermometers will show such changes. Wooden rods are affected by moisture. Glass rods are not affected as much by these two causes.

Glass tubes will not sag as much as glass rods, and are therefore preferable. But glass is fragile and requires more attention. Wood is more convenient. Rods of white pine, straight grained, well seasoned, kiln dried, dipped in boiling oil, painted and varnished, have given satisfaction. To prevent sag, they may be trussed or they may be supported by plank laid on edge between supports.

Length of Rods—Ten to fifteen feet.

Attached Thermometers—Each rod carries a thermometer.

Rounded Ends—When the measurement is to be by contact, both ends of a rod are made semi-cylindrical, but in a quadrantal opposition,

so that two adjacent rods will come in contact at a single point, where the two cylindrical ends meet, forming a cross.

Standardizing the Rods—The rods selected must be measured separately with a duly standardized chain and the fractional reading taken with a vernier, a graduated wedge or a micrometer.

Identification Marks on Rods—The rods should have different marks on them for purposes of identification. If wooden rods are used they may be painted in different colors.

Rod Supports—Stout horses or trestles provided with a strong vertical screw governing a horizontal piece or rest for the rod are convenient. Instead of supporting the rod, these rests often support the ends of a stout plank laid on edge between them and destined to support the rods directly.

Measuring a Base with Rods—Three or four trestles are lined up in the direction of the base and on them the rods are laid. The first rod, projecting a little beyond the first trestle, is set on line with the transit and its end is brought to be in the vertical of the monument. The second and third rods are also lined up and either or not (as previously agreed) brought in contact carefully and without shock.

If intervals are left between the rods, they are measured with graduated ruler and vernier, wedge or micrometer. The first rod and horse are then carried ahead and also lined up, and the operation continues to the second monument or the end of the base line.

Notes Taken—1° Mark of the rod; 2° its temperature; 3° its slope (if not horizontal); 4° distance between a rod and the next; 5° slope of that distance (if the tops of the rods are not on the same line).

Length of the Base Line—Each rod length is corrected for expansion and slope. Each distance is also corrected for expansion of metallic rule used and for slope. The sum of the corrected rod lengths and distances is the length of the base line.

Best Time to Measure a Base Line—In calm and cloudy weather. This facilitates the lining up and prevents too sudden variations in the length of tape or rods.

Reduction of Length of Base to Sea Level—

$$l' = l\frac{r}{r+h} \quad \text{in which}$$

l = Corrected length of base ;
l' = Length of base at sea level ;
r = Radius of the earth = 20 886 455 ft. ;
h = Average elevation of base above sea level .

Triangulation—From the ends of a base line, distant objects are sighted and their deflection from the base is noted. These sights, joined to the ends of the base and to each other, form triangles, which will have to be calculated. The instrument is then carried to these points and further distant objects are sighted and their deflections

from any of the previous sights are noted. A system of triangles is thus made to cover the whole territory.

Check Base—One of the last triangles formed is given a side on a piece of level ground, and this side is both calculated and measured with the same care as the original base; a comparison of the two results will show the care with which the first base was measured.

Stations—The angle points of a system of triangulation are stations.

Conditions for a Good Station—It must be visible from at least three other stations.

Station Monuments—Like base line monuments, they are established on solid foundation and in many cases their tops are placed below the surface, even below frost line, so as to be absolutely undisturbed. They are protected by a glass jar before being covered up.

Signals—The stations being at great distances from one another, a simple monument marking them is not sufficient to render them visible. They are surmounted with signals ordinarily built in two parts independent of each other. 1° A mast and flag tower; 2° an instrument platform.

Mast Tower—Usually constructed with three or four stout legs, well braced and solidly secured in the ground, where they may rest in concrete blocks, or where they are buried in heaps of stones. The legs form a pyramid carrying a mast, at its apex. The mast is straight. and carries a flag at the upper end; it is set vertical and its axis is instrumentally centered over the station monument.

Instrument Platform—A strong platform is erected within the pyramid formed by the mast tower, and is made to rest on independent supports. The platform should be large enough for setting the instrument and making the observations with convenience. This platform is often dispensed with and the instrument is set on the ground. The advantages of a platform are the greater height at which the observer is placed, which enables him to find distant stations more readily; it also keeps the instrument and the observer away from the dampness of the ground; and, moreover, in a flat country, it removes the instrument from the low strata of disturbed air, in which observations cannot be made accurately. The platform may be sheltered by a tent. On the floor of the platform a nail shows the position of the monument below and of the center of the pole above, which are on a vertical.

Masts—Some are in one piece and held up by the outer tower, so that between their lower end and the instrument platform there is a free space for the erection of a tent. Others are in two sections, joined with bolt and collar; the lower portion is set in the ground and the upper portion may be turned down, thus allowing the instrument to be placed exactly over the monument. The flags should be light in color, if they are to be observed against the ground, and dark if observed against the sky. Flags are almost useless in calm weather.

Night is a good time for observations, as the air is undisturbed.

Signals are lights shown through an aperture in a disc, with or without reflectors. We recommend the Belgian lamp for that purpose; it is safe, simple of construction, and gives a powerful light, almost white.

Heliotrope—It consists in a telescope, with two rings attached and held vertically above the telescope, and a circular mirror mounted on pivots in a frame set near the eyepiece in such a manner that the center of the mirror and of the two rings are on a line parallel to the axis of the telescope.

Perforated discs are preferable to rings.

How to Use the Heliotrope—The heliotroper sets the heliotrope over a station a little before a time agreed upon with the engineer, or when motioned to do so, if he can see such motion. He sights at the station where the observer is placed with the transit or theodolite and turns the mirror so that the beam of light passes through both rings.

Observing Angles—Horizontal angles are repeated several times and in different portions of the graduated circle, then an average is taken.

Vertical angles are also measured in order to calculate differences of elevations.

Reduction of a Station Distance to the Level of the Sea—or to an arc of a great circle at that level (Fig. 444).

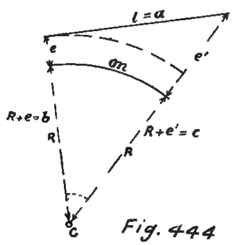

Fig. 444

The sides of the triangle so formed are, respectively:

$$l, \quad R+e \quad \text{and} \quad R+e'.$$

$$\text{Writing} \quad l = a; \quad R+e = b; \quad R+e' = c,$$

The angle may be calculated from formula

$$\sin C \quad \frac{2}{bc}\sqrt{p(p-a)(p-b)(p-c)}; \quad \text{and arc } m$$

$$\text{from formula} \quad m \quad \frac{\pi RC}{180} \quad \text{if } C \text{ is in degrees}$$

$$\text{or} \quad m = \frac{\pi RC''}{648\,000} \quad \text{if } C \text{ is in seconds}.$$

Reduction of an Angle to the Center of the Station (Fig. 445)—When a station is a tower, a spire or other structure, and it is not possible to set the instrument over its center **A**, it is set as near the station as possible, as at **O.** Angles **m** and **n** are measured, and also the distance **d** from which angle **A** is to be determined. Sides **b** and **c** may be known as belonging to sets of triangles already calculated, or they may be approximately computed by using angle **n** instead of angle **A** in the triangulation.

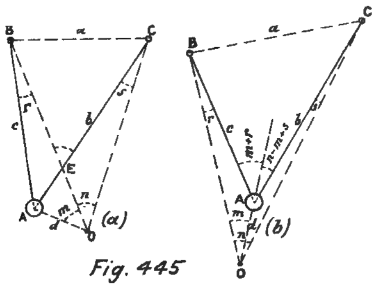

Fig. 445

We have: **A + r = E; n + s = E;** therefore **A + r = n + s** and **A = n + s — r,** in which **s** and **r** are to be computed.

In triangle ACO we have $\sin s = \frac{a}{b} \sin(m+n)$
" " ABO " " $\sin r = \frac{a}{c} \sin m$

which may be calculated, thus giving angles (or arcs) s and r
The above value of A becomes:

$$A = n + \text{arc} \sin\left[\frac{a}{b} \sin(m+n)\right] - \text{arc} \sin\left[\frac{a}{c} \sin m\right].$$

The third term may be positive, as in Fig. 445 (b), in which
A = n + s + r

Distance from a Point to the Center of a Tower [Fig. 446 **a**]—The distance **d** has often to be calculated. In the case of a tower the bisection of the angle formed by the tangents gives the direction of the station and the radius may be obtained by measuring the circumference; this added to the shortest distance from the temporary station **O** to the wall of the tower will be **d.**

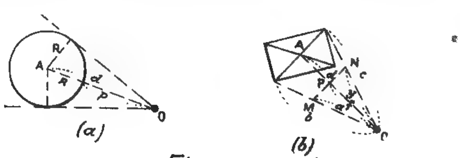

Fig. 446

Distance from a Point to the Center of a Building [Fig. 446 **b**]—

In the case of a house, measure the extreme tangents **b** and **c** and assume **a** on **b**; measure **y** such that

$$y = \frac{ac}{b}$$

Take the middle P of MN, it will be in the direction OA

Measure f; then $$d = \frac{fb}{a}$$

Primary Triangulation—The set of triangles determined by the base and the monuments chosen and covering the country over which works are to be built or studies made, and the sides of which generally are many miles long, is called the primary system or primary triangulation.

Secondary Triangulation—It is a second set of triangles, determined also by the monuments sighted on from one or more angles of primary triangles, the sides of which are used as new bases.

When the triangulation is complete, surveys are begun and transit lines are tied to the monuments wherever possible, and the distances are adjusted accordingly.

Determination of the Meridian—(See Book V., The Transitman, p. 60.)

Determination of the Magnetic Declination—When the true meridian has been obtained, the reading of the cômpass will be the magnetic declination, and it shall be recorded as east or west, as indicated by the instrument.

Settlement of Lines and Corners—When adjacent owners don't agree as to the correct common lines of their properties, the Engineer supervises the adjustment of such lines and corners, as explained hereinbefore.

THE ENGINEER.

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